Activity 25

Objective

- To investigate relationships in figures related to area

Cabri® Jr. Tools

Investigating Area Relationships

Introduction

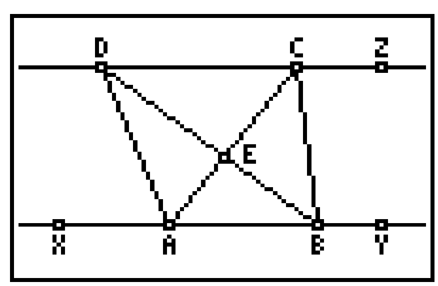
Interactive geometry tools such as the Cabri Jr. application make it easy to measure the area of triangles and quadrilaterals. Many of the activities in this book suggest that you investigate area relationships in the figures constructed as one aspect of the activity. In this activity, you will be exploring some interesting area relationships in quadrilaterals.

Part I: The Diagonals of a Trapezoid

Construction

Construct a trapezoid and its diagonals.

- Draw a line $\overleftrightarrow{XY}$ in the lower half of the screen.
- Construct a line parallel to $\overleftrightarrow{XY}$ passing through and defined by point *Z*.
- Construct trapezoid *ABCD* as shown with points *A* and *B* on $\overleftrightarrow{XY}$, and points *C* and D on the line parallel to $\overleftrightarrow{XY}$ containing point *Z*.
- Construct the diagonals of trapezoid *ABCD*. ($\overline{AC}$ and $\overline{BD}$.)
- Construct point *E* at the intersection of $\overline{AC}$ and $\overline{DB}$.
- Use the **Triangle** tool to overlay triangles formed by the diagonals (for example, ΔABD) or the intersection of the diagonals (for example, ΔEDC).



Exploration

Explore the relationships between the areas of the triangles formed by the diagonals $\overline{AC}$ and $\overline{DB}$ of trapezoid *ABCD*. Be sure to investigate the relationships while dragging the vertices and sides of the trapezoid.

Adjust the position of points *X*, *Y* and/or *Z* and repeat the above Exploration.

Questions and Conjectures

1. Explain why for any trapezoid *ABCD*, the triangles $\triangle ABC$ and $\triangle ABD$, must have the same area but are not necessarily congruent.
2. Make a conjecture about any other triangles formed by the diagonals or intersection of the diagonals of a trapezoid. Justify your answer and be prepared to demonstrate.
3. Under what conditions are the triangles that have equal area also congruent? Explain your reasoning and be prepared to demonstrate.

Part II: Dividing a Parallelogram into Two Equal Areas

Construction

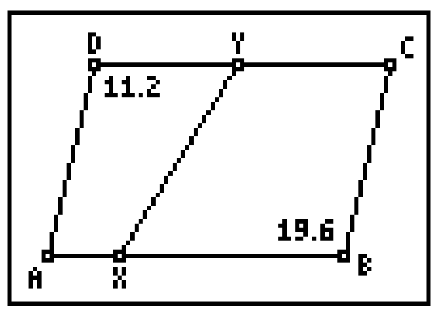
Construct a parallelogram with a segment that divides the parallelogram into two trapezoids.

Clear the previous construction.

Construct parallelogram *ABCD*. (If necessary, see Part I of Activity 24.)

Construct $\overline{XY}$ so that point *X* is on side $\overline{AB}$ and point *Y* is on side $\overline{CD}$ of parallelogram *ABCD*.

Construct and measure the area of quadrilaterals *AXYD* and *BXYC*.



Exploration

Adjust the location of points *X* and *Y* until the two quadrilaterals have equal area. Observe whether or not there is a unique location for the points *X* and *Y* that will satisfy the condition of equal areas.

Repeat the above exploration for quadrilaterals formed by a segment $\overline{WZ}$ where point *W* is on side $\overline{BC}$ and point *Z* is on side $\overline{DA}$.

Consider point *P* to be defined as the point of intersection of $\overline{XY}$ and $\overline{WZ}$ when each segment meets the condition defined above (that is, each segment divides parallelogram *ABCD* into two polygons having equal area). Use various construction and measurement tools to investigate the properties of point *P*.

Questions and Conjectures

1. Describe the relationship between quadrilaterals *AXYD* and *BXYC* when these quadrilaterals have the same area. Are these quadrilaterals the only quadrilaterals defined by parallelogram *ABCD* and $\overline{XY}$ that have these properties? Explain your reasoning and be prepared to demonstrate.

2. Make a conjecture about the properties of point *P* as defined in the exploration. Explain your reasoning and be prepared to demonstrate.

Part III: Overlapping Squares

Construction

I. **Construct a square and its center.**

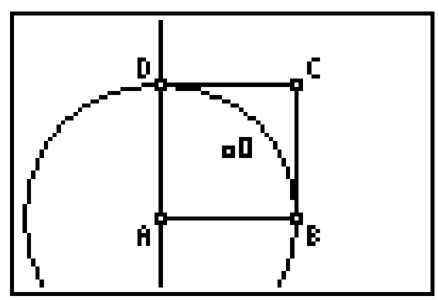
Clear the previous construction.

Draw a horizontal segment $\overline{AB}$.

Construct a line perpendicular to $\overline{AB}$ at point *A*.

Construct a circle centered at *A* using point *B* as a radius point.

Construct a segment $\overline{AD}$ so that point *D* is at one of the intersection points of the circle and the line perpendicular to $\overline{AB}$.



Construct a point *O* at the midpoint of points *B* and *D*.

Use the **Symmetry** tool to construct the image of side $\overline{AB}$ through point *O*. Construct the image of side $\overline{AD}$ through point *O*. Label the intersection of these images *C*.

Construct quadrilateral *ABCD.*

Verify that quadrilateral *ABCD* is a square.

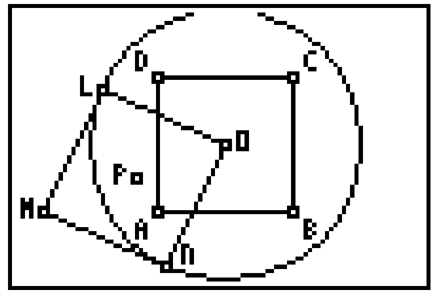
II. **Construct a second congruent square having a vertex at the center of the first square.**

Hide $\overleftrightarrow{AD}$ and the circle centered at *A*.

Use the **Compass** tool to create a circle centered at *O* having a radius equal to $\overline{AB}$.

Construct a segment $\overline{OL}$ so that point *L* is on the circle centered at *O*.

Construct a line perpendicular to $\overline{OL}$ at point *O*.



Construct a segment $\overline{ON}$ so that point *N* is at one of the intersection points of the circle and the line perpendicular to $\overline{OL}$.

Hide the line perpendicular to $\overline{OL}$.

Construct a point *P* at the midpoint of points *L* and *N*.

Use the **Symmetry** tool to construct the image of sides $\overline{OL}$ and $\overline{ON}$ through point *P*. Label the intersection of these images *M*.

Exploration

Explore the relationship between the area of square *ABCD* and the area of the region formed by the overlap of squares *ABCD* and *LMNO*. Be sure to investigate this relationship for different lengths of segment $\overline{AB}$ and for different locations of point *L*. (Point *L* can be animated.)

Questions and Conjectures

Make a conjecture about the relationship between the area of square *ABCD* and the region formed by the overlap of squares *ABCD* and *LMNO*.

Teacher Notes

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Investigating Area Relationships

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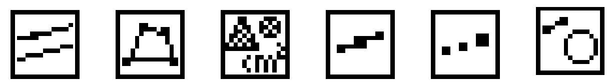


Part I: The Diagonals of a Trapezoid

Answers to Questions and Conjectures

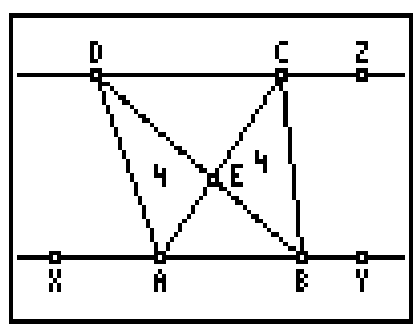
1. Explain why for any trapezoid *ABCD*, the triangles $\triangle ABC$ *and* $\triangle ABD$, must have the same area but are not necessarily congruent.

   $\triangle ABC$ and $\triangle ABD$ are such that they share a common base $\overline{AB}$. Since the line containing $\overline{CD}$ is parallel to the line containing $\overline{AB}$, the distance from point *C* to $\overleftrightarrow{AB}$ is equal to the distance from point *D* to $\overleftrightarrow{AB}$. This distance is the height of these triangles. Triangles that have equal bases and equal heights have equal areas even though the triangles are not congruent.

2. Make a conjecture about any other triangles formed by the diagonals or the intersection of the diagonals of a trapezoid. Justify your answer and be prepared to demonstrate.

   $\triangle ACD$ and $\triangle BCD$ will have the same area because they have the same base ($\overline{CD}$) and the same height.

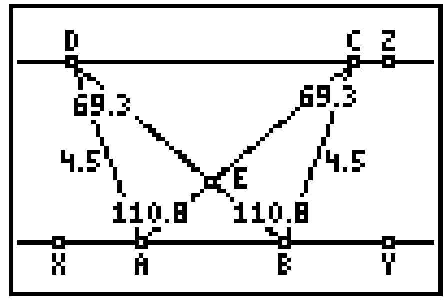
   Since the area of $\triangle ABD$ is equal to the area of $\triangle ABC$, it can be shown that the areas of $\triangle ADE$ and $\triangle BCE$ are equal. $\overline{AE}$ separates $\triangle ABD$ into $\triangle ABE$ and $\triangle ADE$; therefore, the area of $\triangle ABE$ added to the area of $\triangle ADE$ will equal the area of $\triangle ABD$. $\overline{BE}$ separates $\triangle ABC$ into $\triangle ABE$ and $\triangle BCE$; therefore, the area of $\triangle ABE$ added to the area of $\triangle BCE$ will equal the area of $\triangle ABC$. (Equal quantities subtracted from equal quantities give equal quantities.)

   $\triangle ABE$ and $\triangle CDE$ are similar but will not have equal areas unless trapezoid *ABCD* is a parallelogram (or a special parallelogram).

3. Under what conditions are the triangles that have equal area also congruent? Explain your reasoning and be prepared to demonstrate.

   The minimum condition to make triangles with equal areas congruent is to require quadrilateral *ABCD* to be an isosceles trapezoid. It would also hold true for a parallelogram or special parallelogram.

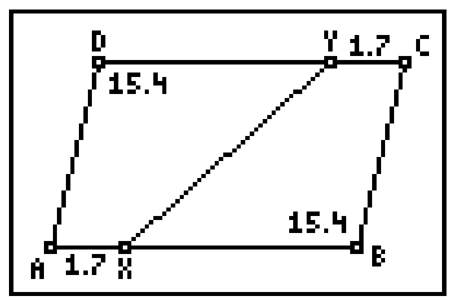


Part II: Dividing a Parallelogram into Two Equal Areas

Answers to Questions and Conjectures

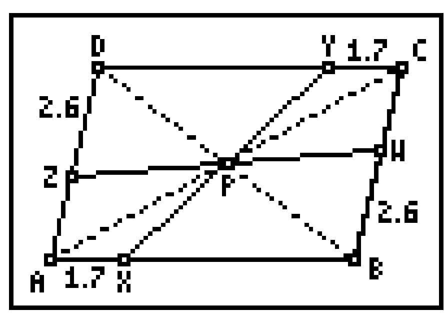
1. Describe the relationship between quadrilaterals *AXYD* and *BXYC* when these quadrilaterals have the same area. Are these quadrilaterals the only quadrilaterals defined by parallelogram *ABCD* and $\overline{XY}$ that have these properties? Explain your reasoning and be prepared to demonstrate.

   Quadrilaterals *AXYD* and *BXYC* are congruent when they have equal areas. There are infinite possible orientations of $\overline{XY}$ that will divide parallelogram *ABCD* into two congruent quadrilaterals as long as the distance of $\overline{AX}$ is equal to the distance of $\overline{CY}$.



2. Make a conjecture about the properties of point *P* as defined in the Exploration. Explain your reasoning and be prepared to demonstrate.

   Point *P* will be located at the intersection of the diagonals of parallelogram *ABCD*. Any line passing through point *P* will divide parallelogram *ABCD* into two congruent polygons (trapezoids or triangles).



Part III: Overlapping Squares

Answers to Questions and Conjectures

Make a conjecture about the relationship between the area of square *ABCD* and the region formed by the overlap of squares *ABCD* and *LMNO*.

The area of the overlapping quadrilateral is always one fourth of the area of the square. The answer to the question can be visualized better when point *P* is coincident with a vertex of square *ABCD* or when $\overline{LO}$ passes through a vertex of the original square. If either of the overlapping sides of the second square is extended to cross the first square, then it is cut into four congruent areas, each with an area that is one fourth of that of the whole square. The congruence of these four areas is assured by the fact that the cutting lines are perpendicular and pass through the center of the square.