TI-73

NUMBER LINE
APPLICATION SOFTWARE

Walking the Line: Activities for the TI-73 Number Line

Christine A. Browning Dennis St. John

EXPLORATIONS™

TEXAS INSTRUMENTS

Walking the Line: Activities for the TI-73 Number Line Application

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Preface

These activities are intended to help students use the number line and the fraction line to develop both an operations and number sense. Several activities focus on operations with the integers, such as the *mystifying* process of subtracting negative integers, providing students a tool for interpreting processes and understanding results. Other activities focus on using the number and fraction line for various types of skip counting; it's not just for whole numbers anymore! The fraction line activities allow for examining relationships between ordinary fractions, decimal fractions, and percentages.

The teacher notes that accompany each activity provide the teacher with instructions for using the number line application, as well as providing sample responses and solutions. We realize that there is a lot of reading for the busy classroom teacher, but sometimes we just had so much to say and didn't know what to edit out. We hope you will initially skim through the teacher notes for key points in the lesson, questions to ask, and paths to take, and perhaps later take some time to read through them carefully. The majority of the activities can be completed in a 50-minute class session, depending upon the time spent on whole class discussion. Some activities are written in a sequential nature, such as *Skip Counting* and *More Skip Counting*, and it is recommended that they be completed in the intended order.

We would like to thank Elisa Brueningsen-Kerner, Ellen Johnston, and Mary Dezelsky, who took time away from their busy schedules to review these activities and provide their very helpful comments. We would also like to thank Joshua Browning, an 8th grade mathematics student, who tried some of the activities and provided his insightful comments from a student viewpoint. We also appreciate the help of our colleagues at Texas Instruments who assisted us in the preparation of these materials.

The activities do not even begin to address all of the capabilities of the Number Line application for addressing important upper elementary and middle school mathematics topics. Other topics include probability, properties of the number line, and examining irrational numbers. But time was short and we decided to only give you a taste of what *is* possible. We hope that you and your students enjoy and learn from these activities. We hope further that we have you hooked on the Number Line application and that you'll send us the activity ideas you develop!!

— *Christine A. Browning*

— *Denny St. John*

About the Authors

CHRISTINE A. BROWNING is an Associate Professor at Western Michigan University and *DENNY ST. JOHN* is an Associate Professor at Central Michigan University. Both are former middle school mathematics teachers with continuing interests in mathematics curriculum, instruction, and technology use in the middle grades. Both enjoy a great laugh at each other's expense.

Christine tries to ride her two sons' dirt bikes (only one at a time) but has trouble with the jumps!

Denny enjoys refereeing battles between his three sons. Referees **CAN** lose matches!

EXPLORATIONS

Activity 1

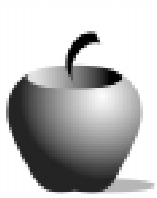
Integers!
It All Adds Up!

Objective

- To explore addition of integers using a number line.
- To develop strategies for adding integers.

Materials

- TI–73 Calculator



Teacher Notes

Introduction

Integers (the collection of whole numbers, the opposites of the nonzero whole numbers, and 0) are quite familiar to people today, but at one time they were not accepted as true numbers in the mathematical community. Many mathematicians referred to integers as "absurd," "ugly," or "fictitious" numbers even though they would arise as solutions to problems. For example, an East Indian text of about 1150 A.D. indicates a solution to an equation as -5, but then makes fun of such a useless response!

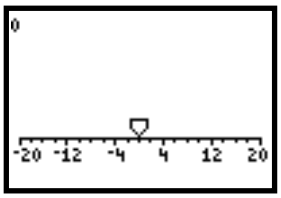
In 1545, the Italian mathematician Girolamo Cardano, published the rules governing operations with negative numbers in his text *Ars Magna* (*Great Art*).

If earlier mathematicians questioned the validity of these numbers, then it is understandable that many middle school students struggle with making sense of the operations with integers! The following activity provides students the opportunity to construct some rules for themselves based upon using the Number Line application.

Procedure

1. Distribute the Student Activity pages. Have students work in pairs using one calculator between them. Roles that can be assigned are that of reader/recorder and Number Line "Operator."

2. Have students access the Number Line application by pressing [APPS] and selecting the NUMLINE application. (It will typically be the third option, but this depends on other applications that you have installed.) Press [ENTER] twice to get to the NUM/FRAC LINE menu and select option **1: Number Line**.

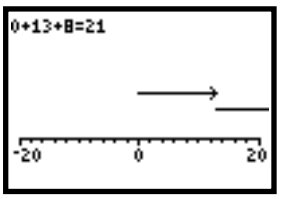


   If the standard or default window doesn't appear with the line indicator at 0 and a view of the number line from -20 to 20, have students either press [WINDOW] and change the **Min**, **Max**, **Scale**, and **Start** values (default values are -20, 20, 2 and 0, respectively) OR press [ZOOM] and select option **2**.

   If no values are displayed on the Number Line other than **Min** and **Max,** check to be sure that **LabelOn** is selected in the WINDOW settings. If previous work is displayed, press [CLEAR] **1** to clear the window.

   Note: The first option under [ZOOM] is **ZoomFit**. This activity does not make use of this option since the intent is to have students do mental arithmetic as much as possible. Students will certainly see **ZoomFit** and test its capabilities, letting you know that option is much faster than setting the viewing window! You may wish to postpone the use of **ZoomFit** for this activity to encourage the development of mental arithmetic skills and number sense.

3. Students can initially investigate something familiar to them on the Number Line by adding positive integers or whole numbers, and then explaining how addition of two or more numbers can be represented on the number line.



   When students enter 13 + 8, the screen will show 0 + 13 + 8. The starting value will always appear and you do not need to enter + 13 + 8. As you begin to type in numbers, the application automatically assumes addition.

   If students make errors while typing in the entry, they can use the arrow keys to backspace and type over the error, or use the [DEL] key. If they have already pressed [ENTER] and find an error, they should press [ENTER] again, press [◄] or [►] to move to the error and correct it, then press [ENTER] or [GRAPH].

4. Ask students to explain how the "number rays" or vectors represent the operation of addition. Introduce students to the term *vector* and how it is used throughout the activity.

The responses should have some reference to the length of the vectors being the same as the absolute value of the integer. The vectors should be pointed in the positive direction on the number line, with one vector beginning at the end of the other, and the sum being at the endpoint (arrow head) of the final vector.

5. Ask students to add 15 to the previous sum of 21 and determine an appropriate **Max** for the viewing window before they press GRAPH. Encourage students to share their mental arithmetic strategies. They only need an estimate to determine a new **Max**, but many will try to find the exact sum. A sample strategy might be 20 + 10 is 30 and 5 + 1 is 6 so the new sum is 36. If you view many "36's" for the maximum value as you roam the room, you know students were searching for the exact sum. You might want to talk about why the max could be 38, 39, 40, etc.

6. Students clear the window and draw a sketch of their prediction for 0 + ⁻12. Sketches should show one vector, starting from zero and ending at ⁻12.

Make sure that students know the difference between the − and (−) keys. The TI-73 displays the negative sign as a raised character.

7. Students add ⁻8 to ⁻12. Ask how the vectors show addition of negative integers. An explanation similar to the one for positive integers could be used, but the vectors are pointed in the negative or opposite direction.

8. Students add ⁻5 to ⁻20 and select a new **Min**. They should notice that the continued sum of negative integers will produce vectors that appear further to the left, away from zero. As long as you are adding negative integers, the vectors will not change direction. Addition of negative integers is similar to adding positive integers in that you continue moving in the same direction on the number line as you add. A difference is the direction in which you move on the line; one in a positive direction, the other in the negative direction.

You might want to talk about the idea that although the absolute value of the sum is increasing, the sum is getting smaller. A movement towards the left on the number line will produce a number smaller than the starting point (and movement towards the right on the number line will produce a number greater than the starting point). Students usually believe this idea on the positive side of the number line, but struggle with it on the negative side.

You could make "real world" comparisons

- to temperature: Which is colder, 10 degrees below zero or 20 degrees below zero?
- to money: Would you rather be $10 in debt or $50 in debt?
- to sports: Would you rather have a 5-yard loss or 15-yard loss in a football play?

Each example may help illustrate the idea that negative integers closer to zero have a greater (or more preferred) value than those that are further away, or, as you move to the left, the integer value gets smaller.

If students have had some previous experience with integer arithmetic, some might say that the vector could point in the positive direction since "a negative and a negative equals a positive." They are recalling multiplication rules without meaning. Repeatedly adding ⁻5, for example, would model *n x ⁻5*, which would be a negative value. They can use that thinking when they discuss multiplication.

9. Here you can observe from the student sketches whether students understand that addition of a negative number moves one in the negative direction. Instead of zero being the starting point, we have moved first to 15. From there, we will add ⁻9, which should move us 9 spaces to the left.

10. Encourage students not to change their sketches in step 9 if they don't exactly match what is produced by the application. Have them discuss the differences and why they are acceptable or not. The sketches provide important information for you to use in determining their understanding of the addition.

11. Students are asked if ⁻19 added to the previous sum would produce a sum that is still in the viewing window. This question focuses on number sense. Students might think "If I'm on the right side of zero, it would take adding a number less than ⁻20 to get me out of the viewing window. Negative 19 is bigger than ⁻20 so I shouldn't be out of the window range." Here is another opportunity to encourage appropriate thinking. Students might say "I need to add a number bigger than ⁻20 to move out of the viewing window" when they need to think *smaller* than ⁻20.

12. The feeling students might get when adding positive and negative integers is one of "changing directions," "going back and forth," or being "pulled in opposite directions" on the Number Line. Remind students to use only the ◄ and ► keys when "taking a ride." Using the ▼ and ▲ keys produces a different kind of "ride."

13. Through their own examples, students should see the commutative property at work and determine that order doesn't matter. Some are surprised when this happens. "How can you start off in a positive direction one time and a negative direction the other and still end up at the same place?" The calculator allows them to explore many examples to help see why this is the case.

14. Students may write that to find opposites, you simply "turn around and go back as far as you came" or "if you're pulled 13 away from zero, the opposite pulls you 13 spaces back."

Having them think about always returning to zero should help them understand that the sum of two opposites is zero. They can "build" several sets of opposites without having to press [CLEAR] since the starting point is always zero when adding the opposites.

15. Students may write that for the sum of positive integers, you always move in a positive direction or to the right on the number line. When adding negative integers, you always move in the negative direction or to the left. Adding positives and negatives moves you in both directions.

Some students may comment that adding positives and negatives is like subtraction. It is, but students must pay attention to the sign of the integer which is greater in absolute value in order to determine the sign of the sum. Having the visual image of the number line may help many students understand why the sum of one set of positive and negative integers can be positive while another sum can be negative.

16. Encourage students to share their strategies either orally or perhaps as a journal entry. Use writing activities with caution when sharing mental strategies. Having students write their thoughts often discourages mental arithmetic since "I have to write it down anyway!"

Student Activity

Name _____

Date _____

Activity 1

Integers! It All Adds Up!

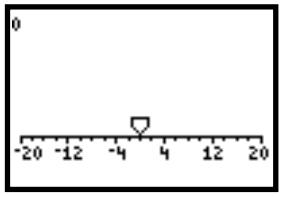
Objective: *In this activity, you will use the number line to investigate how to add integers and to justify and support the rules that you develop. You will use the Number Line application on the TI-73 calculator.*

Integers (the collection of whole numbers, the opposites of the nonzero whole numbers, and 0) are quite familiar to people today, but at one time they were not accepted as true numbers in the mathematical community. Many mathematicians referred to them as "absurd," "ugly," or "fictitious" numbers even though integers would arise as solutions to problems. For example, an East Indian text from about 1150 A.D. indicates a solution to an equation as -5 but then makes fun of such a useless response!

In 1545, the Italian mathematician Girolamo Cardano, published the rules governing operations with negative numbers in his text *Ars Magna* (*Great Art*).

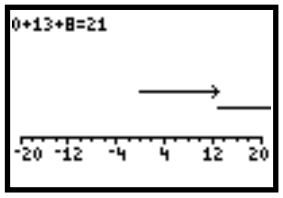
1. Access the Number Line application by pressing [APPS] and pressing [▼] to move down to the NUMLINE application. Press [ENTER] twice to get to the NUM/FRAC LINE menu and select the Number Line option.

Your window should look like the one at the right. If not, press [WINDOW] and set **Min= -20**, **Max= 20**, **Scale= 2**, and **Start= 0**. Press [GRAPH] to view the Number Line again. If the window needs to be cleared, press [CLEAR] and select **1:Yes**.

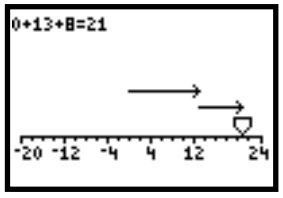


2. Find a solution to the sum 13 + 8 using the Number Line application.

3. Type in **13** [+] **8** and press [ENTER].



4. To capture the entire picture, press [WINDOW]. You need to select a new maximum value. There are many reasonable values that you could choose. Try 24 by pressing [▼] to move down to **Max** = and entering **24**. Press [GRAPH] to return to the Number Line. Explain how the "number rays" or vectors represent the operation of addition.

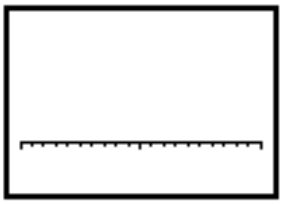


5. Suppose you add 15 to the sum from question 4. You know that the sum would appear off of the current window. Before finding the sum, press [WINDOW] and choose a new value for **Max** that will allow you to see all of the vectors. Again, there are many acceptable values, but try to select a maximum that is no more than 5 units greater than the new sum.

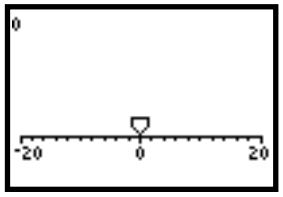
The value of **Max** we chose *before* viewing the sum is: _____ .

Now, add 15 to the sum by pressing [+] **15** [ENTER].

Draw a sketch of your Number Line window on the blank screen at the right.



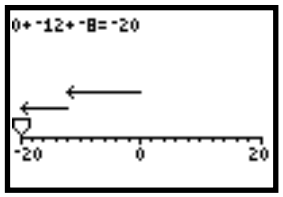
6. Press [CLEAR] to clear the window and press [WINDOW] to change the **Max** value to **20**. Press [GRAPH] to return to the Number Line window.



This time, add negative integers and see what happens. What is your prediction of how the application will show adding **0 + -12**? Draw a sketch of your prediction on the screen at the right.

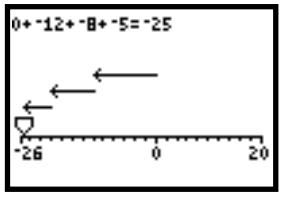
Press [+] [(-)] **12**,then [ENTER]. (The [(-)] key allows you to enter negative values or to "take the opposite of" the current entry. The [−] key is used for the operation of subtraction.) Does your previous sketch match the current Number Line view?

7. Now, add -8 to the current sum of -12. Press [+] [(-)] **8**, then [ENTER].



How do the vectors show addition of negative integers when using a number line?

8. Add -5 to the sum. By doing so, you will be out of the current viewing window. Press [WINDOW] to change the **Min** value so that you will be able to see all vectors. Since you're adding -5, a minimum of -25 would just capture all of the vectors, so -26 would allow them to be viewed more easily. Type in **-26** and press [GRAPH]. Add -5 if you haven't already done so.



a. If you continue adding negative integers, how would the vectors would look in the viewing window?

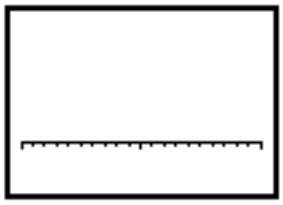
`

b. Would a vector ever be pointed in a positive direction if you continued adding negative integers? Explain.

c. How is adding positive integers similar to adding negative integers? How is it different?

9. Now try adding positive and negative integers.

What do you predict the sum of 15 + ⁻9 will be?



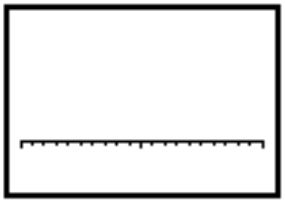
Draw a sketch of the Number Line representation of the sum on the screen at the right. Provide some labels on the Number Line to help with reading the vector length.

10. Press [CLEAR] to clear the window, and press [WINDOW] to reset the **Min** value to **⁻20**. Press [GRAPH] to return to the number line window. Press [+] **15** [ENTER]. The line indicator moves to 15 on the number line. Now, press [+] [(–)] **9** [ENTER]. Adding the ⁻9 produced a new vector beginning at 15, 9 units long and pointed in the negative direction. The sum is at the end of this vector.

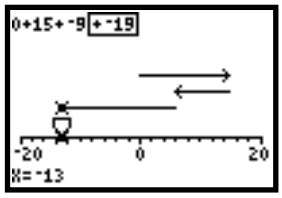
11. If you added ⁻19 to the current sum, would the new sum still be in the current viewing window or would you have to change the window settings? Explain.

In which direction will the vector for adding ⁻19 be pointed? Explain.

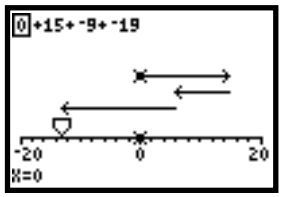
Test your prediction. Draw a sketch of the result of adding ⁻19 to the previous expression 15 + ⁻9. Notice that this is the same as adding ⁻19 to 6.



12. To help understand the movement along the Number Line, press the [TRACE] key to move along the vectors. Notice a box appears around the last addend of ⁻19 and flashing cursors appear on the end of the vector and on the number line.

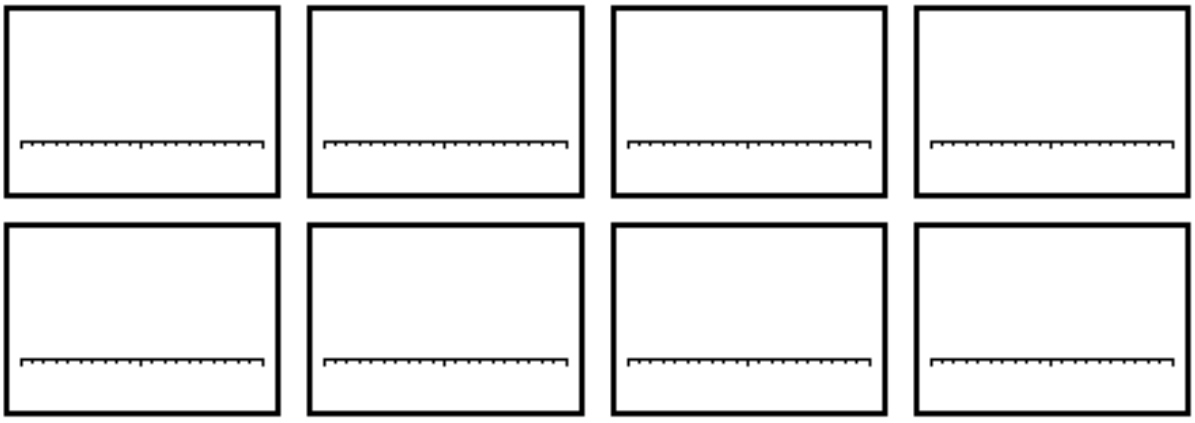


Press the [▶] and [◀] keys to "untrace the path" of the vectors until you are at the original starting point of 0. When you reach the end of a vector, continue pressing the [▶] and [◀] keys to move up (or down if you're moving forward through the sum) to the next vector in the sum. Press the [▶] and [◀] keys to "trace the path" of the sum.



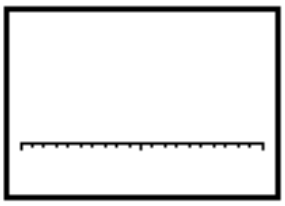
Imagine sitting on top of the cursor and taking "a ride" along the vectors (as you "move" through the expression **0 + 15 + ⁻9 + ⁻19**). Describe the "ride" of adding positive and negative integers.

13. Try a few more additions of positive and negative integers. Record your sketches of final sums in the boxes below. For one of your examples, enter in the addends in the opposite order and see what happens. For example, if you have done 15 + ⁻9; try ⁻9 +15. These examples help in understanding the *commutative property for addition*.



14. Predict what will happen when you add 13 to ⁻13.

a. Test your prediction and record a sketch of the window. Press the [TRACE] key to move along the vectors.



b. 13 and ⁻13 are called *opposites*. How does the Number Line application help make sense of the term *opposite*?

c. Test a few more opposites on the calculator. Do you always return to zero? (If you are trying very large numbers, remember to change the viewing window so you can see all of the vectors.)

15. Think about adding positive integers. How would you describe the general movement of the [TRACE] key in those sums? If you need to, enter a sum of positive integers to do the tracing.

a. Describe the movement of the [TRACE] key when adding only negative integers.

b. What is the biggest difference between

- adding just positive or just negative integers, and
- adding positive AND negative integers?

16. Find the following sums by using mental arithmetic and visualizing the movement on the Number Line.

a. 19 + 17

b. 13 + ⁻27

c. ⁻27 + ⁻13

d. 26 + ⁻5

e. 24 + 6

f. 15 + 15

EXPLORATIONS

Activity 2

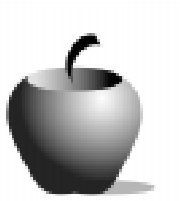
Integer Subtraction—What's the Difference?

Objective

- To explore subtraction of integers using a number line.
- To develop strategies for subtracting integers.

Materials

- TI–73 Calculator



Teacher Notes

Introduction

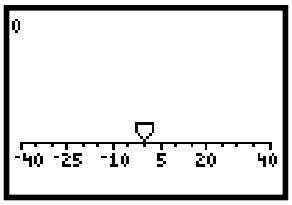
This activity builds on Activity 1 by having students use the Number Line to investigate integer subtraction.

Procedure

1. Distribute copies of the Student Activity pages. Have students work in pairs.

2. Have students access the Number Line application by pressing [APPS] and pressing [▼] to select the NUMLINE application. Press [ENTER] twice to get to the NUM/FRAC LINE menu and access the NUMBER LINE option by pressing [ENTER].

   To display more of the Number Line, have students access and change the window settings. Press [WINDOW] and change the **Min** to **-40**, **Max** to **40**, **Scale** to **5**, and **Start** to **0**. Highlight **LabelOn** and press [ENTER]. Return to the number line window by pressing [GRAPH]. Press [CLEAR] **1** (or [CLEAR] [ENTER]) to clear any previous work.



3. Present a quick review of adding negative integers, explaining the concept of addition and subtraction being opposite or *inverse* operations.

4. Have students compare number lines for adding ⁻18 and subtracting 18. Ask students why the number lines look the same even though subtraction and addition are opposites. The purpose of the question is to have students see the idea of "opposite" being conveyed in the sign of the number added. If one adds and subtracts the same number, then the number line views will look different. But when adding a negative number or subtracting the opposite of that negative number, the results will be the same (equivalent).

5. Students will begin to see a pattern, but should be cautioned to consider all cases.

6. The first variation to consider is when the largest number in absolute value is negative so the sum is no longer positive. If students merely subtract the largest number in absolute value from the smaller, their answer will have the wrong sign.

Students are asked to find a subtraction expression equivalent to **39 + ⁻45** and to verify their entry using the Number Line application. When students are asked to verify, no screen shot of the window will be provided. The calculator allows for student self-assessment. All too often, students get in the habit of asking "Is this right?" without first thinking for themselves. You can help minimize this habit by encouraging discussion and resolution in the student pairs. The activity encourages students to think ahead and then determine for themselves if their thinking is headed in an appropriate direction.

7. Here is a similar example. Depending upon the experiences of your students, you may not need to linger over several examples, but for those thinking about integer subtraction for the first time, the repetition of the thinking process is appropriate.

8. Here students should say they do subtract but the sign of the difference depends upon the sign of the number with the greater absolute value. You may want to use the language of absolute value or talk about the distance the number is from zero. So, for example, 39 + ⁻45 will have a negative answer since ⁻45 is farther away from zero than 39, not because ⁻45 is a bigger number than 39.

9. With this example, you can present the idea that subtracting a number is equivalent to adding the opposite of the number, an idea that arises from their viewing several examples and searching for a pattern.

		Equivalent Addition Problem	Prediction
a.	23 - 5	23 + (⁻5)	
b.	6 - 33	6 + (⁻33)	
c.	18 - 28	18 + (⁻28)	

10. The students begin to apply the rule. Notice that the calculator does not have a key similar to the (−) key to indicate positive numbers. Numbers without signs are taken as positive values.

Students are encouraged to stick with their mathematical reasoning even when it doesn't match their intuition. Recall the early mathematicians who thought negative numbers were "absurd"!

		Equivalent Addition Problem	Prediction
a.	18 - (-13)	18 + (+13)	
b.	32 - (-5)	32 + (+15)	
c.	20 - (-19)	20 + (+19)	

11. A "different point of view" is presented.

12. Again, students return to adding a negative integer, something they should be comfortable with by now.

13. Finally, students consider subtracting a negative integer using the "opposite direction" thinking.

Encourage students to share how they make sense of subtraction with integers.

14. These exercises are to help students develop and practice reasonable estimation skills in addition to developing some number line sense. For example, an estimate of the difference for **29 - 35** might be **-5**. The **-5** helps determine a **Min** for the window setting, but which number should the student focus on in determining a **Max**? 35 is larger than 29, but is that important for this subtraction? At what point on the number line will the vector for 35 start and stop? Which way will that vector be directed? These questions should help students understand the entire problem rather than just the final response.

		Min	Max
a.	29 - 35	-7	30
b.	45 - 18	0, 25	46
c.	16 - (-26)	0, 15	43
d.	-23 - 17	-41	0
e.	-35 - (-29)	-36	-5, 0

15. This is just another approach to helping students make sense of the "subtracting a negative integer" problem. Ask students if, based on the pattern they observe in the equations, the response they find for 18 - (⁻1) matches the response they would have found using previously developed rules.

16. Ask students to summarize their thoughts on subtracting integers.

Student Activity

Name_____

Date _____

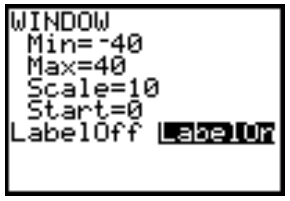
Activity 2

Integer Subtraction—What's the Difference?

Objective: *In this activity, you will use the number line to investigate subtraction of integers and to justify and support the rules you develop. You will be using the Number Line application on the TI-73 calculator.*

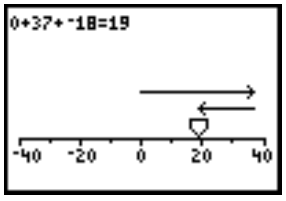
1. Access the Number Line application by pressing [APPS] and pressing [▼] to select the NUMLINE application. Press [ENTER] twice to get to the NUM/FRAC LINE menu and access the NUMBER LINE option by pressing [ENTER].

2. Press [WINDOW] and change the **Min** to **-40**, **Max** to **40**, **Scale** to **10**, and **Start** to **0**. Highlight **LabelOn** and press [ENTER]. Return to the Number Line window by pressing [GRAPH]. Press [CLEAR] 1(or [CLEAR] [ENTER]) to remove any previous work.

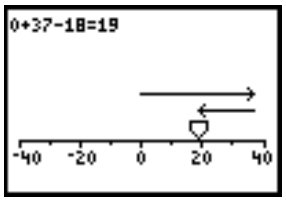


3. When adding a positive and negative integer, the vectors representing the two integers point in opposite directions. Vectors point in the *same* direction when you add two integers with the same sign.

Enter **37 + -18** to verify this. Adding a negative 18 is represented by a vector pointing to the left with a length of 18.



4. Have another member of your group enter **37 - 18** on the calculator. Compare the number line showing the addition expression **37 + -18** with the number line displaying the subtraction, **37 - 18**. Notice that subtracting a (positive) 18 is also represented by a vector pointing to the left with a length of 18.



Subtraction and addition are opposite operations, yet when you view the two number line windows showing the addition and subtraction problem, you see the *same* picture, not an opposite picture. Why is this so?

Press [CLEAR] **1** and use two calculators to compare number lines for these two problems:

29 + -16 and **29 - 16**.

Based on these two examples, it would appear that adding a negative integer to a positive integer looks just like subtracting a positive integer from a positive integer. Let's do more investigating to examine all possible variations of subtraction of integers.

6. Predict the sum of 39 and -45.

Verify your response using the Number Line application.

How does this example differ from **37 + -18** and **29 + -16**?

When working just with whole numbers (numbers greater than zero), you generally subtracted (took away) the smaller number from the larger number. If adding a positive and negative integer is just like your "familiar" subtraction, you might think that **39 + -45** is the same as **45 - 39**. Is that the case? You can verify your thinking using the calculator.

How is **39 + -45** similar to **45 - 39**?

Based upon your earlier observation about subtraction and addition being opposite operations, what subtraction entry would give the same results as **39 + ⁻45**? Verify your entry on the calculator.

7. Try another example. Before using the calculator , predict whether the sum of 17 and ⁻28 will be positive or negative. Explain.

How are you mentally determining the distance of the sum from zero?

Press [CLEAR] **1** and enter **17 + -28**. What subtraction is equivalent to **17 + -28**? Enter your prediction to verify.

8. What "new" information did these last two examples, **39 + -45** and **17 + -28**, provide? Is the addition of a positive and negative integer just like subtraction? Explain.

9. If **39 - 45 = 39 + -45** and **17 - 28 = 17 + -28**, then **19 - 35 =** ?

These examples suggest that subtraction is the same as adding the *opposite* of a number.

Write the equivalent addition problems and your predictions for differences (or sums) for the following problems.

		Equivalent Addition Problem	Prediction
a.	23 - 5		
b.	6 - 33		
c.	18 - 28		

10. Apply this idea to this problem: **15 - (-6)**. The previous reasoning would suggest that subtracting -6 from 15 is the same as adding the opposite of -6 (or +6) to 15. Rewritten, it looks like

15 - (-6) = 15 + (+6)

Verify this using the Number Line application. One person in your group should enter **15 - (-6)** while the other enters **15 + 6**.

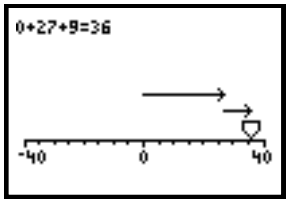
Try the following examples. Write your predictions for the differences prior to entering them on the calculator.

		Equivalent Addition Problem	Prediction
a.	18 - (-13)		
b.	32 - (-5)		
c.	20 - (-19)		

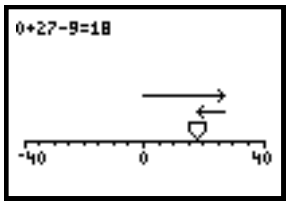
It is the subtraction of negative numbers that causes many students to look twice at their answer since the difference turns out to be greater than the numbers with which they started. For example, 18 - (-13). How can you start with 18 and subtract -13 and end up with 31! Many only think of subtraction as "taking a number away", and so the difference of 31 just doesn't make sense. Yet, mathematics is about finding patterns, developing rules to describe those patterns, and then applying those rules to other examples. *You have done just that!* You have reasoned through many examples using a number line model, observed patterns between the direction of vectors and the operations, developed rules describing those patterns (for example: subtraction is adding the opposite), and then applied those rules in other cases. The rules appear to agree with your intuition except perhaps in the case of subtraction involving negative integers. It helps to rely more on reasoning and logical thinking than intuition. Using other models for integer subtraction will help support thinking and modify intuition.

11. Here's another view of subtraction with negative integers. Think about subtraction and addition as the direction of the vectors on the number line. When you subtract a number on the number line, the vector points (or directs a move) in the opposite direction than it would when adding the number.

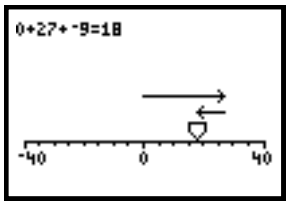
For example, enter **27 + 9**. (Press CLEAR **1** if needed.) The 27 is a positive value. The vector shows a move, or points, in the positive direction and has a length of 27. To add a positive 9, the next vector continues to point in the *same* direction and "travels" for a distance of 9 units to arrive at the sum of 36.



Clear the screen and enter **27 - 9**. Compare this to the previous expression. The first vector representing 27 is the same as before. The operation of subtraction, though, indicates a move in the *opposite* (or negative) direction for a distance of 9 units arriving at the difference of 18.



12. Press CLEAR **1**. Now consider **27 + (-9)**. Think through this first before entering the example.



The first vector will still have a length of 27 and point in the positive direction. The operation of addition does not change the direction of the vector. In this case, though, the vector changes direction because of the sign of the 9. We know a -9 is a vector of length 9 that points in the negative (or left) direction. We need to add this vector to 27 so it will start at 27 and move left 9 units landing at 18. Verify using the Number Line application.

13. Finally, think about **27 - (-9)**.

The starting vector is the same as before. To subtract -9, move in the *opposite* direction than you would if you were adding -9. The addition of -9 had the vector pointing towards the left, so you need to move in the opposite direction which would point the vector towards the right. The number line window should look like which of the previous three windows? Verify your prediction.

14. Without using the calculator, find **Min** and **Max** values that would allow you to see all of the vectors for each of the following problems. You can be no more than 3 units away from the final difference and/or the endpoint of any vector.

		Min	Max
a.	29 - 35		
b.	45 - 18		
c.	16 - (-26)		
d.	-23 - 17		
e.	-35 - (-29)		

15. Study the patterns in the equations below and then respond to the following questions.

18 - 7 = 11

18 - 6 = 12

18 - 5 = 13

18 - 4 = 14

18 - 3 = 15

18 - 2 = 16

18 - 1 = 17

18 - 0 = 18

Describe any patterns you see as you work through the equations.

The integers being subtracted decrease by 1 each time. What should be the next equation in the list if you continue that pattern?

Does the solution match what you would have predicted using the rules from #14.

16. You've examined the subtraction of integers from several different views. What patterns have you seen that will help you in subtracting integers?

EXPLORATIONS

Activity 3

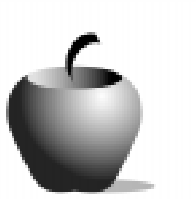
Multiplication of Integers—Repeated Addition and Subtraction

Objective

- To explore skip counting to complete patterns
- To develop multiplication

Materials

- TI–73 Calculator
- Red & Blue Chips



Teacher Notes

Introduction

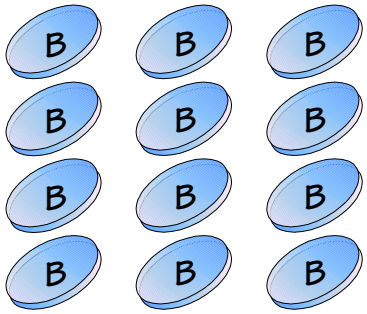
Repeated addition and subtraction can lead to ideas about multiplication. This activity provides students the opportunity to create expressions with repeated addition or subtraction using the Number Line application.

Procedure

1. Demonstrate how to use red and blue chips to multiply integers. Let the blue chips stand for positive integers and the red chips stand for negative integers. When the number of red and blue chips are equal then we have zero.

To show 3 x 4 use three groups of four blue chips.

- Start with no chips on the overhead.
- The positive three means you "put in" three groups.
- The positive four tells you that each group put in will contain four blue.

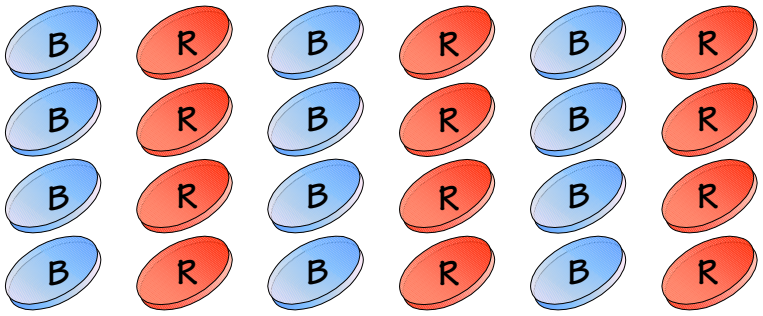


To show 3 x ⁻4, use three groups of four red chips.

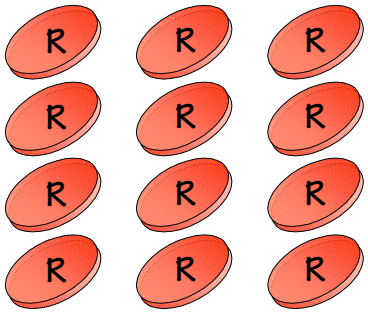
- Start with no chips on the overhead.
- The positive three means you "put in" three groups.
- The negative four tells you to use four red chips in each group.

To show ⁻3 x 4, use three groups of four blue chips and three groups of four red chips.

- Start with no chips on the overhead.
- The negative three means you "take away" three groups. Since there are no chips to take away we add enough zeroes so that there are enough blue chips to take away.
- The positive four tells you the groups being removed will each contain four blue chips.

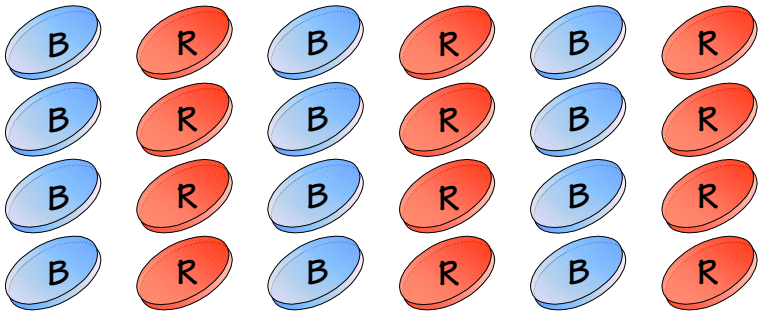


- Once the three groups of four blue chips are taken away, 12 red chips will remain.

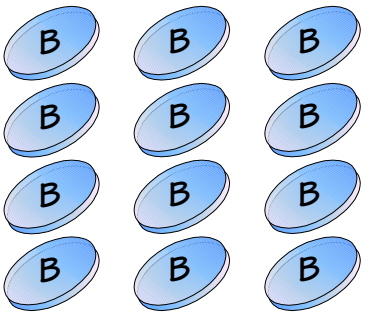


To show ⁻3 x ⁻4 use three groups of four blue chips and three groups of four red chips.

- Start with no chips on the overhead.
- The negative three means you "take away" three groups. Since there are no chips to take away we add enough zeroes so that there are enough red chips to take away.
- The negative four tells you the groups being removed will each contain four red chips.

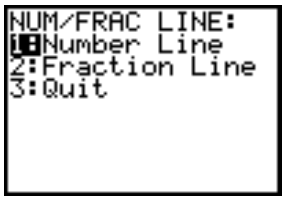


- Once the three groups of four red chips are taken away, 12 blue chips will remain.



2. Start the Number Line application by pressing the APPS key and selecting the NUMLINE application.

3. Once the App has started, select 1:Number Line.



4. Distribute the Student Activity pages and have students work in pairs using the Number Line application.
5. Discuss the answers for items 5 through 10.

Answer Key

4. ⁻8. Answers will vary.

5. The expression 0 – ⁻2 – ⁻2 – ⁻2 – ⁻2 is equivalent to 8.

This expression is equivalent to –4 x ⁻2.

6.

	Expression	Equivalent
a.	0 + 3 + 3 + 3 + 3 = 12	4 x 3
b.	0 + -3 + -3 + -3 + -3 = ⁻12	4 x -3
c.	0 – 3 – 3 – 3 – 3 = ⁻12	⁻4 x 3
d.	0 – -3 – -3 – -3 – -3 = 12	⁻4 x -3

7.

	Addition/Subtraction Expression	Multiplication Equivalent
a.	0 + ⁻8 + ⁻8 + ⁻8 + ⁻8	4 x ⁻8
b.	0 + ⁻9 + ⁻9 + ⁻9	3 x ⁻9
c.	0 – ⁻7 – ⁻7 – ⁻7 – ⁻7 – ⁻7 – ⁻7	⁻6 x ⁻7
d.	0 – 6 – 6 – 6 – 6 – 6	⁻5 x 6

Student Activity

Name_____

Date _____

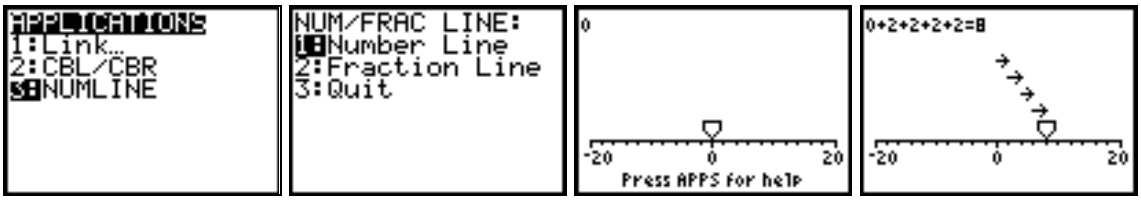
Activity 3

Multiplication of Integers—Repeated Addition and Subtraction

Objective: *In this activity, you will investigate how repeated addition or subtraction is related to multiplication. You will use the Number Line application on the TI-73 calculator.*

We have seen how red and blue chips can be used to model multiplication of integers. Are there other models? One that has been used extensively in the past uses the number line.

1. Access the Number Line application by pressing [APPS] and pressing [▼] to select the NUMLINE application. Press [ENTER] twice to get to the NUM/FRAC LINE menu and select 1: Number Line.



2. Now, press [+] 2 [+] 2 [+] 2 [+] 2 [ENTER]. Above you see the expression 0 + 2 + 2 + 2 + 2 is equivalent to 8. We could also write 2 + 2 + 2 + 2 or 4 x 2.

3. Now, press [CLEAR] **1** to clear the expression. Then enter 0 [−] 2 [−] 2 [−] 2 [−] 2 [ENTER].

As you see, the expression 0 − 2 − 2 − 2 − 2 is equivalent to -8. This expression has the opposite value of the first one computed.

4. Compare this to the expression 0 + -2 + -2 + -2 + -2.

Press [CLEAR] then 0 [+] [(−)] 2 [+] [(−)] 2 [+] [(−)] 2 [+] [(−)] 2 [ENTER]. What is the value of the expression?

Which of the expressions is it equivalent to? Explain.

5. Now press [CLEAR] **1**, then press 0 [−] [(−)] 2 [−] [(−)] 2 [−] [(−)] 2 [−] [(−)] 2 [ENTER].

The expression 0 − -2 − -2 − -2 − -2 has a value of:

Write an expression using multiplication:

6. Use the application to compute each of these expressions:

	Expression	Equivalent
a.	0 + 3 + 3 + 3 + 3 =	This is equivalent to __ x 3
b.	0 + ⁻3 + ⁻3 + ⁻3 + ⁻3 =	This is equivalent to __ x ⁻3
c.	0 – 3 – 3 – 3 – 3 =	This is equivalent to __ x 3
d.	0 – ⁻3 – ⁻3 – ⁻3 – ⁻3 =	This is equivalent to __ x ⁻3

7. Compute the values of these expressions using the Number Line application. Then write equivalent multiplication problems.

	Addition/Subtraction Expression	Multiplication Equivalent
a.	0 + -8 + -8 + -8 + -8	
b.	0 + -9 + -9 + ⁻9	
c.	0 – ⁻7 – ⁻7 – ⁻7 – ⁻7 – ⁻7 – ⁻7	
d.	0 – 6 – 6 – 6 – 6 – 6	

8. Explain the relationship between repeated addition and multiplication.

9. Explain the relationship between repeated subtraction and multiplication.

10. Explain the relationship between repeated addition and repeated subtraction using chips and the number line.

EXPLORATIONS

Activity 4

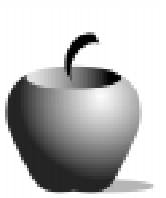
Objective

- To explore skip counting to complete patterns
- To develop multiplication

Materials

- TI–73 Calculator

Skip Counting—2, 4, 6, 8, Who Do We Appreciate?



Teacher Notes

Introduction

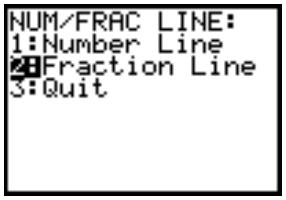
How often have we heard that chant? Can we exploit the idea of skip counting to enhance student understanding of multiplication and pattern completion? This activity will show how you might foster those skills.

This activity provides students the opportunity to construct patterns using the Number Line Application.

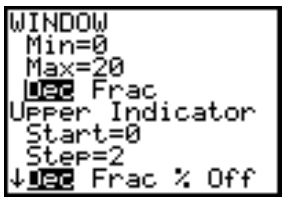
Procedure

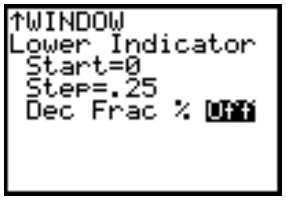
1. Demonstrate how to create the pattern 2, 4, 6, 8, 10, … using the Number Line Application.

To start the application, press [APPS] and press [▼] to select NUMLINE. Once the App has started, select 2:Fraction Line.

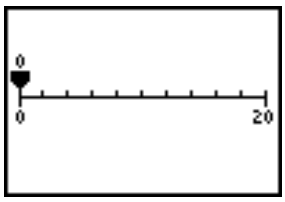


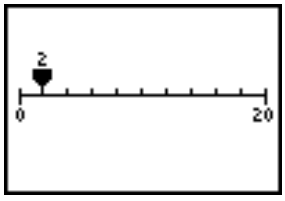
To view the pattern, change the window values to match those seen at the right. Press [WINDOW], change **Min** to **0**, **Max** to **20**, **Start** to **0** and **Step** to **2**. Also, select **Dec** and press [▼] to view the window for the Lower Indicator. Select **Off** for the Lower Indicator.





Press [GRAPH] to see the number line. Now press [▶] a few times to see the next terms in the sequence. You should see the indicator move from 0 to 2, then to 4, and finally 6. The value of each step is 2, matching the value entered into the window settings.

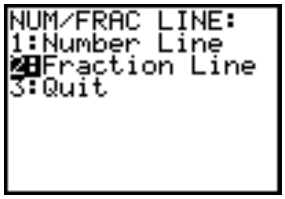




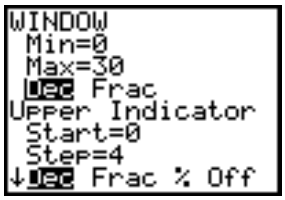
2. Distribute the Student Activity pages with the problem statement. Have students work in pairs using one calculator between them.

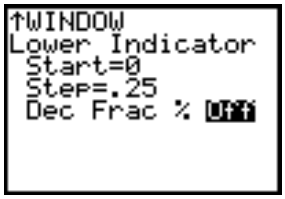
3. Have students access the Number Line application by pressing [APPS] and pressing [▼] to select the NUMLINE application.

Press [ENTER] twice to get to the NUM/FRAC LINE menu and select 2: Fraction Line.



In order for students to view the pattern change they must change the calculators' window values to match those seen at the right. Press [WINDOW], change the **Min** to **0**, **Max** to **30**, **Start** to **0** and **Step** to **4**. Also select **Dec** and press [▼] to view the window for the Lower Indicator. Select **Off** for the Lower Indicator.





The indicator will start at 0 and will jump by 4 each time [▶] is pressed. Eventually the value will exceed 30 but the window will automatically adjust to show the correct location for the indicator.

4. Students will first complete the multiplication portion, then the pattern completion part of the activity.

Answer Key

4. 0, 4, 8, 12, 16, 20, 24, 28, 32, 36.

5.

a.	0, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30	Start = 0 Step = 3
b.	0, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60	Start = 0 Step = 6
c.	0, 7, 14, 21, 28, 35, 42, 49, 56, 63, 70	Start = 0 Step = 7
d.	0, 9, 18, 27, 36, 45, 54, 63, 72, 81, 90	Start = 0 Step = 9
e.	0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100	Start = 0 Step = 10
f.	0, 13, 26, 39, 52, 65, 78, 91, 104, 117, 130	Start = 0 Step = 13

7. 1, 6, 11, 16, 21, 26, 31, 36, 41, 46.

8.

a.	4, 9, 14, 19, 24, 29, 34, 39, 44, 49	Start = 4 Step = 5
b.	5, 12, 19, 26, 33, 40, 47, 54, 61, 68	Start = 5 Step = 7
c.	7, 10, 13, 16, 19, 22, 25, 28, 31, 34	Start = 7 Step = 3
d.	9, 12, 15, 18, 21, 24, 27, 30, 33, 36	Start = 9 Step = 3
e.	10, 21, 32, 43, 54, 65, 76, 87, 98, 109	Start = 10 Step = 11
f.	13, 25, 37, 49, 61, 73, 85, 97, 109, 121	Start = 13 Step = 12

9. Answers will vary.

10. 161.

Student Activity

Name_____

Date _____

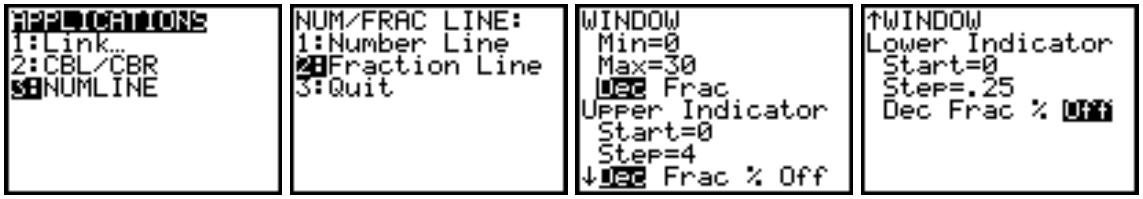
Activity 4

Skip Counting—2, 4, 6, 8, Who Do We Appreciate?

Objective: *In this activity, you will investigate how to complete patterns using the number line. You will use the Number Line application on the TI-73 calculator.*

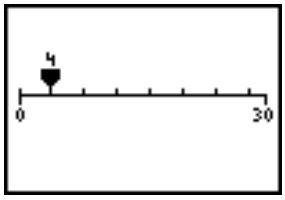
We have all heard that rhyme. Some have used that to memorize multiplication facts. What else can we learn from the rhyme?

1. Access the Number Line application by pressing [APPS] and pressing ▼ to select the NUMLINE application. Press [ENTER] twice to get to the NUM/FRAC LINE menu and select 2: Fraction Line.



2. In order to view the pattern change you must change the calculator's window values to match those seen above. Press [WINDOW], change **Min** to **0**, **Max** to **30**, **Start** to **0** and **Step** to **4**. Also select **Dec** and press ▼ to view the window for the Lower Indicator. Select **Off** for the Lower Indicator

3. Press [GRAPH] and then press ▶. Notice that every time ▶ is pressed the indicator increases by 4. Eventually the value will exceed 30 but the window will automatically adjust to show to correct location for the indicator.



4. Record ten of the values:
0, 4, ___, ___, ___, ___, ___, ___, ___, ___, ___

5. Use the Number Line Application to complete these patterns.

a. 0, 3, 6, 9, ___, ___, ___, ___, ___, ___, ___

b. 0, 6, 12, 18, ___, ___, ___, ___, ___, ___, ___

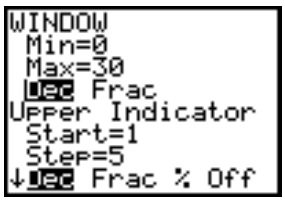
c. 0, 7, 14, 21, ___, ___, ___, ___, ___, ___, ___

d. 0, 9, 18, 27, ___, ___, ___, ___, ___, ___, ___

e. 0, 10, 20, 30, ___, ___, ___, ___, ___, ___, ___

f. 0, 13, 26, 39, ___, ___, ___, ___, ___, ___, ___

6. Some patterns are not all multiples of the first value. For instance, the sequence 1, 6, 11, 16, 21, 26, … starts with 1 and increases by 5 each term. Adjust your window to **Start = 1** and **Step = 5**. Leave the lower indicator off. Remember to press GRAPH to view the window.



7. Record ten of the values:
1, 6, ___, ___, ___, ___, ___, ___, ___, ___

8. Use the Number Line Application to complete these patterns.

a. 4, 9, 13, ___, ___, ___, ___, ___, ___, ___ Start =___ Step =___

b. 5, 12, 19, ___, ___, ___, ___, ___, ___, ___ Start =___ Step =___

c. 7, 10, 13, ___, ___, ___, ___, ___, ___, ___ Start =___ Step =___

d. 9, 12, 15, ___, ___, ___, ___, ___, ___, ___ Start =___ Step =___

e. 10, 21, 32, ___, ___, ___, ___, ___, ___, ___ Start =___ Step =___

f. 13, 25, 37, ___, ___, ___, ___, ___, ___, ___ Start =___ Step =___

9. Create a number pattern like those above. Be prepared to share it with your classmates.

___, ___, ___, ___, ___, ___, ___, ___, ___, ___ Start =___ Step =___

10. Predict the 20th term in this pattern:

1, 9, 17, 25, ?, …

EXPLORATIONS

Activity 5

Objective

- To explore skip counting to complete patterns with fractions.
- To develop addition of fractions.

Materials

- TI–73 Calculator

More Skip Counting—$\frac{2}{8}, \frac{4}{8}, \frac{6}{8}, \frac{8}{8}$, Who Do We Appreciate This Time?

Teacher Notes

Introduction

We can use the idea of skip counting to enhance students' understanding of addition and pattern completion. This activity will show how you might foster those skills.

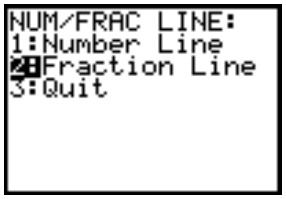
This activity provides students the opportunity to construct patterns using the Number Line application.

Procedure

1. Demonstrate how to create the pattern $\frac{2}{8}, \frac{4}{8}, \frac{6}{8}, \frac{8}{8}, \frac{10}{8}, \frac{12}{8}, \ldots$ using the Number Line application.

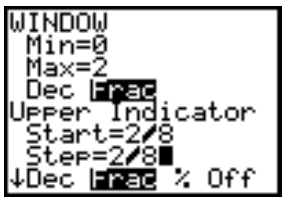
Start the Number Line application by pressing [APPS] and pressing ▼ to select the NUMLINE application.

Once the App has started, select 2: Fraction Line.

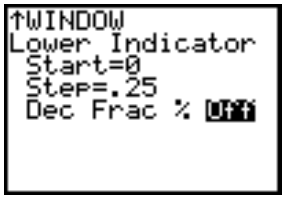


Press [MODE] to change the Mode setting to match the settings shown at the right.

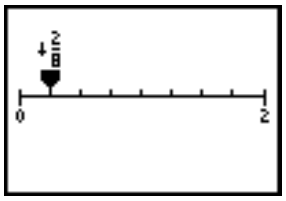
To view the pattern, change the window values to match those seen at the right. Choose **Frac,** and under Upper Indicator, change **Start** to $\frac{2}{8}$, **Step** to $\frac{2}{8}$. (To enter $\frac{2}{8}$, press 2 [b/c] 8.)

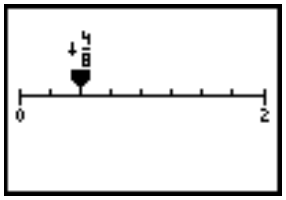


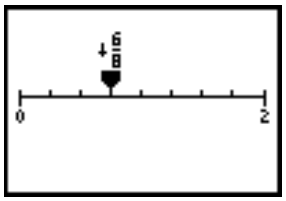
Press [▼] to view the window for the Lower Indicator. Select **Off** for the Lower Indicator.



Press [GRAPH] to see the Number Line. Now, press [▶] a few times to see the next terms in the sequence. You should see the indicator move from $\frac{2}{8}$ to $\frac{4}{8}$ then to $\frac{6}{8}$, $\frac{8}{8}$, $\frac{10}{8}$ finally $\frac{12}{8}$. The value of each step is $\frac{2}{8}$, matching the value entered into the window settings. The arrows in front of these fractions indicate that the fraction is not simplified.



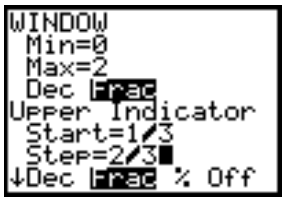




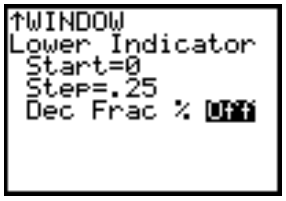
2. Distribute the Student Activity pages with the problem statement. Have students work in pairs.

3. Have students access the Number Line application by pressing [APPS] and pressing [▼] to select the NUMLINE application. Press [ENTER] twice to get to the NUM/FRAC line menu and select 2: Fraction Line.

In order for students to view the pattern change they must change the calculators' window values to match those seen at the right. Change **Min** to **0**, **Max** to **4**, and under Upper Indicator set **Start** to $\frac{1}{3}$, **Step** to $\frac{2}{3}$, and select **Frac.**



Press [▼] to view the window for the Lower Indicator. Select **Off** for the Lower Indicator.



The indicator will start at $\frac{1}{3}$, and will jump by $\frac{2}{3}$ each time ▶ is pressed. Eventually the value will exceed 4, but the window will automatically adjust to show the correct location for the indicator.

4. Students will first do the pattern completion activity with improper fractions and then with mixed numbers.

6. Students will see that changing an improper fraction to a mixed number is *not* the same as simplifying the fraction.

8. Students might notice that it is easier to determine the common difference between terms when working with improper fractions.

Answer Key

4. $\frac{1}{3}, \frac{3}{3}, \frac{5}{3}, \frac{7}{3}, \frac{9}{3}, \frac{11}{3}, \frac{13}{3}, \frac{15}{3}, \frac{17}{3}, \frac{19}{3}$

5.

a. $\frac{3}{7}, \frac{6}{7}, \frac{9}{7}, \frac{12}{7}, \frac{15}{7}, \frac{18}{7}, \frac{21}{7}, \frac{24}{7}, \frac{27}{7}, \frac{30}{7}$ Start = $\frac{3}{7}$, Step = $\frac{3}{7}$

b. $\frac{6}{11}, \frac{12}{11}, \frac{18}{11}, \frac{24}{11}, \frac{30}{11}, \frac{36}{11}, \frac{42}{11}, \frac{48}{11}, \frac{54}{11}, \frac{60}{11}$ Start = $\frac{6}{11}$, Step = $\frac{6}{11}$

c. $\frac{7}{3}, \frac{14}{3}, \frac{21}{3}, \frac{28}{3}, \frac{35}{3}, \frac{42}{3}, \frac{49}{3}, \frac{56}{3}, \frac{63}{3}, \frac{70}{3}$ Start = $\frac{7}{3}$, Step = $\frac{7}{3}$

d. $\frac{9}{12}, \frac{18}{12}, \frac{27}{12}, \frac{36}{12}, \frac{145}{12}, \frac{54}{12}, \frac{63}{12}, \frac{72}{12}, \frac{81}{12}, \frac{90}{12}$ Start = $\frac{9}{12}$, Step = $\frac{9}{12}$

e. $\frac{0}{8}, \frac{10}{8}, \frac{20}{8}, \frac{30}{8}, \frac{40}{8}, \frac{50}{8}, \frac{60}{8}, \frac{70}{8}, \frac{80}{8}, \frac{90}{8}$ Start = $\frac{0}{8}$, Step = $\frac{10}{8}$

f. $\frac{0}{9}, \frac{13}{9}, \frac{26}{9}, \frac{39}{9}, \frac{52}{9}, \frac{65}{9}, \frac{78}{9}, \frac{91}{9}, \frac{104}{9}, \frac{117}{9}$ Start = $\frac{0}{9}$, Step = $\frac{13}{9}$

7. $\frac{1}{5}, 1\frac{2}{5}, 2\frac{3}{5}, 3\frac{4}{5}, 5, 6\frac{1}{5}, 7\frac{2}{5}, 8\frac{3}{5}, 9\frac{4}{5}, 11$ Start = $\frac{1}{5}$, Step = $1\frac{1}{5}$

8.

a. $\frac{3}{7}, \frac{6}{7}, 1\frac{2}{7}, 1\frac{5}{7}, 2\frac{1}{7}, 2\frac{4}{7}, 3\frac{3}{7}, 3\frac{6}{7}, 4\frac{1}{7}, 4\frac{4}{7}$ Start = $\frac{3}{7}$ Step = $\frac{3}{7}$

b. $\frac{6}{11}, 1\frac{1}{11}, 1\frac{7}{11}, 2\frac{2}{11}, 2\frac{8}{11}, 3\frac{3}{11}, 3\frac{9}{11}, 4\frac{4}{11}, 4\frac{10}{11}, 5\frac{5}{11}$ Start = $\frac{6}{11}$ Step = $\frac{6}{11}$

c. $2\frac{1}{3}, 4\frac{2}{3}, 7, 9\frac{1}{3}, 11\frac{2}{3}, 14, 16\frac{1}{3}, 18\frac{2}{3}, 21, 23\frac{1}{3}$ Start = $2\frac{1}{3}$ Step = $2\frac{1}{3}$

d. $\frac{9}{12}, 1\frac{6}{12}, 2\frac{3}{12}, 3, 3\frac{9}{12}, 4\frac{6}{12}, 5\frac{3}{12}, 6, 6\frac{9}{12}, 7\frac{6}{12}$ Start = $\frac{9}{12}$ Step = $\frac{9}{12}$

e. $\frac{0}{8}, 1\frac{2}{8}, 2\frac{4}{8}, 2\frac{6}{8}, 3, 3\frac{2}{8}, 3\frac{4}{8}, 3\frac{6}{8}, 4, 4\frac{2}{8}$ Start = $\frac{0}{8}$ Step = $1\frac{2}{8}$

f. $2\frac{8}{9}, 4\frac{3}{9}, 5\frac{7}{9}, 7\frac{2}{9}, 8\frac{6}{9}, 10\frac{1}{9}, 11\frac{5}{9}, 13, 14\frac{4}{9}, 15\frac{8}{9}$ Start = $2\frac{8}{9}$ Step = $1\frac{4}{9}$

9. 6, $5\frac{2}{5}$, $4\frac{4}{5}$, $4\frac{1}{5}$, $3\frac{3}{5}$, 3, $2\frac{2}{5}$, $1\frac{4}{5}$, $1\frac{1}{5}$, $\frac{3}{5}$ Start = 10 Step = $-\frac{3}{5}$

10. Please be sure to have your students share these with classmates. Answers will vary.

Student Activity

Name_____

Date _____

Activity 5

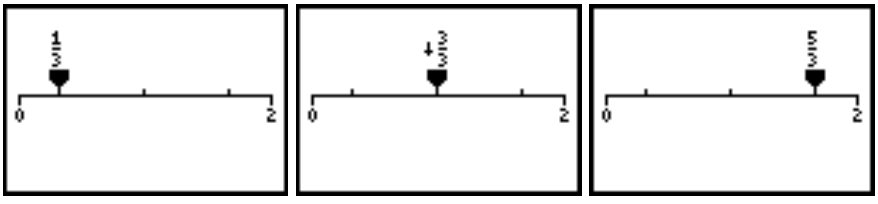
More Skip Counting—$\frac{2}{8}, \frac{4}{8}, \frac{6}{8}, \frac{8}{8}$, Who Do We Appreciate This Time?

Objective: *In this activity, you will investigate how to complete patterns using the fraction Number Line. You will be using the Number Line application on the TI-73 calculator.*

1. Access the Number Line application by pressing [APPS] and pressing ▼ to select the NUMLINE application. Press [ENTER] twice to get to the NUM/FRAC LINE menu and select 2: Fraction Line.

2. In order to view the pattern change you must change the calculator's window values to match those seen above. Press [WINDOW] and change **Min** to **0**, **Max** to **4**, **Start** to $\frac{1}{3}$, **Step** to $\frac{2}{3}$, and select **Frac**. Press ▼ to view the window for the Lower Indicator and select **Off** for the Lower Indicator.

3. Press [GRAPH] and then press ▶. Notice that every time ▶ is pressed, the indicator increases by $\frac{2}{3}$. Eventually the value will exceed 4, but the window will automatically adjust to show the correct location for the indicator.



4. Complete the next seven values: $\frac{1}{3}, \frac{3}{3}, \frac{5}{3}$, ___, ___, ___, ___, ___, ___, ___

5. Use the Number Line application to complete these patterns.

a. $\frac{3}{7}, \frac{6}{7}, \frac{9}{7}$, ___, ___, ___, ___, ___, ___, ___ Start = ___ Step = ___

b. $\frac{6}{11}, \frac{12}{11}, \frac{18}{11}$, ___, ___, ___, ___, ___, ___, ___ Start = ___ Step = ___

c. $\frac{7}{3}, \frac{14}{3}, \frac{21}{3}$, ___, ___, ___, ___, ___, ___, ___ Start = ___ Step = ___

d. $\frac{9}{12}, \frac{18}{12}, \frac{27}{12}$, ___, ___, ___, ___, ___, ___, ___ Start = ___ Step = ___

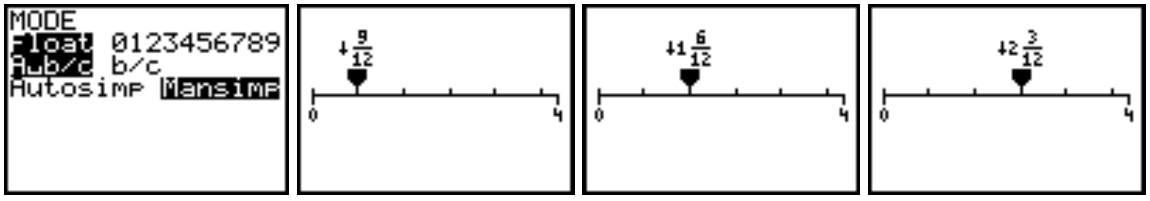
e. $\frac{0}{8}, \frac{10}{8}, \frac{20}{8}, \frac{30}{8}$, ___, ___, ___, ___, ___, ___, ___ Start = ___ Step = ___

f. $\frac{0}{9}, \frac{13}{9}, \frac{26}{9}, \frac{39}{9}$, ___, ___, ___, ___, ___, ___, ___ Start = ___ Step = ___

6. Some patterns are not written as fractions only. Sometimes the patterns include mixed and whole numbers. For instance, the sequence $\frac{9}{12}, \frac{18}{12}, \frac{27}{12}, \ldots$ could also be written as:

$\frac{9}{12}, 1\frac{6}{12}, 2\frac{3}{12}, \ldots$

To view mixed numbers, press MODE and change to the mixed number display. **Mansimp** means that the fractions are not simplified automatically.



7. Find the other values for this sequence:

$\frac{1}{5}, 1\frac{2}{5}, 2\frac{3}{5}$, ___, ___, ___, ___, ___, ___, ___ Start = ___ Step = ___

8. Use the Number Line application to complete these patterns.

a. $\frac{3}{7}, \frac{6}{7}, \frac{9}{7}$, ___, ___, ___, ___, ___, ___, ___ Start = ___ Step = ___

b. $\frac{6}{11}, 1\frac{1}{11}, 1\frac{7}{11}$, ___, ___, ___, ___, ___, ___, ___ Start = ___ Step = ___

c. $2\frac{1}{3}, 4\frac{2}{3}, 7$, ___, ___, ___, ___, ___, ___, ___ Start = ___ Step = ___

d. $\frac{9}{12}, 1\frac{6}{12}, 2\frac{3}{12}$, ___, ___, ___, ___, ___, ___, ___ Start = ___ Step = ___

e. $\frac{0}{8}$, $1\frac{2}{8}$, $2\frac{4}{8}$, $3\frac{6}{8}$, ___, ___, ___, ___, ___, ___, ___ Start = ___ Step = ___

f. $1\frac{4}{9}$, $2\frac{8}{9}$, $4\frac{3}{9}$, ___, ___, ___, ___, ___, ___, ___ Start = ___ Step = ___

9. Create a number pattern starting at 6. Subtract $\frac{3}{5}$ to get the next value, then subtract $\frac{3}{5}$ again to get the next value in the sequence. Complete the pattern.

6, $5\frac{2}{5}$, $4\frac{4}{5}$, ___, ___, ___, ___, ___, ___, ___ Start = ___ Step = ___

How many times can you take away $\frac{3}{5}$ before you reach 0?

10. Create a number pattern like those above. Explain how the pattern was created. Be prepared to share it with your classmates.

___, ___, ___, ___, ___, ___, ___, ___, ___, ___ Start = ___ Step = ___

EXPLORATIONS

Activity 6

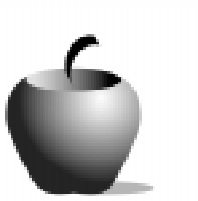
Comparing Decimals—Wider Isn't Always Larger!

Objective

- To compare and order decimals.

Materials

- TI–73 Calculator



Teacher Notes

Introduction

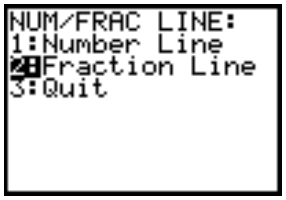
Sometimes students will say that 2.35 is greater than 2.3. While this is true, it is possible that a student might believe that 2.35 is larger than 2.3 simply because 2.35 has more digits. Comparing decimals can be difficult for some students. This activity will show how you might foster those skills using the Number Line application.

Procedure

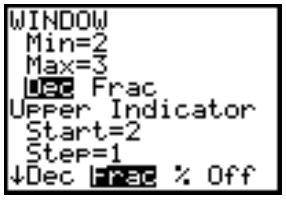
1. Demonstrate how to show that 2.35 exceeds 2.3 using the Number Line application.

Start the application by pressing [APPS] and pressing [▼] to select the NUMLINE application.

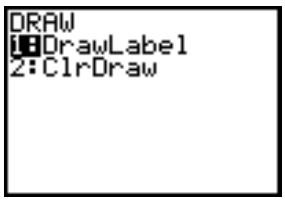
Once the App has started, select 2: Fraction Line.



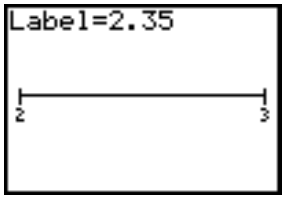
Press [WINDOW] to set the window to **Min** = **2** and **Max** = **3**. Choose **Dec** (decimal), change **Start** to **2** and **Step** to **1**. Set the Lower Indicator to **Off**.



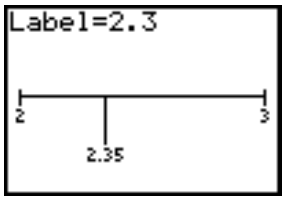
Press [DRAW] to mark the first number **2.35**. Press [ENTER] to select **DrawLabel** and enter the number **2.35**.



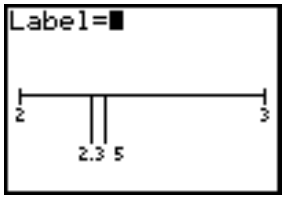
The number line will appear with the value 2.35 indicated.



Next, enter the second number **2.3**. Press [ENTER] to select **DrawLabel** and enter the number 2.3.



You can see that 2.3 is to the left of the 2.35. Notice that part of the label for 2.35 is overwritten by the label for 2.3.



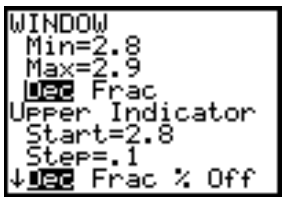
Discuss with the students the relationship between the image shown on the screen and the placement of the numbers. To clear the marks for 2.35 and 2.3, press [DRAW] and select **2: ClrDraw** to clear the drawing.

Now, compare 1.63 and 1.6289 using the Number Line application. Use **Min = 1.6**, **Max = 1.7**, **Start = 1.6**, and **Step = .01**. Repeat the steps above for these numbers.

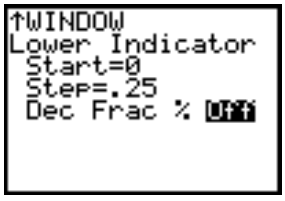
2. Distribute the Student Activity pages with the problem statement. Have students work in pairs using one calculator between them.

3. Have the students access the Number Line application by pressing [APPS] and pressing ▼ to select the NUMLINE application. Press [ENTER] twice to get to the NUM/FRAC LINE menu and select 2: Fraction Line.

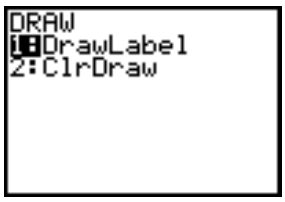
In order for students to view the pattern change they must change the calculators' window values to match those seen at the right. Press [WINDOW] and change **Min** to **2.8**, **Max** to **2.9**, **Start** to **2.8**, **Step** to **.1**, and select **Dec**.



Press ▼ to view the window for the Lower Indicator. Select **Off** for the Lower Indicator.



Press DRAW to mark the first number, **2.85**. Press ENTER to select **1:DrawLabel** and enter the number **2.85**. Press ENTER and the number line will appear with the value **2.85** indicated.



Now enter the second number **2.9** and press ENTER. You can see that **2.85** is to the left of the **2.9**.

4. Students will first complete the comparison activity with the Number Line application and will then discuss other methods for comparing decimals.

Answer Key

3. 15.762 is on the left.15.81 is on the right.

4. Compare these numbers. Circle the larger number.

a. 2.35, (2.4)

b. (0.98), 0.076

c. (14.95), 14.931

5. Write the smaller number before the larger number.

a. 1.23, 1.262

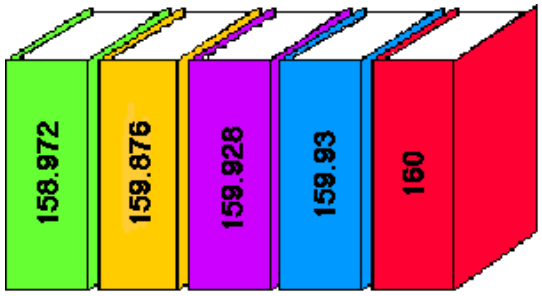
b. 12.3, 13.2

c. .056, .06

6. Answers will vary. Be sure to discuss students' answers to this question. Some may suggest digit by digit comparisons for each place value. Others might suggest placing enough 0's behind each number so that the numbers could be compared like whole numbers.

7. 3.221, 3.227, 3.229, 3.23, 3.235, 3.33

8. Displayed in order:
158.972
159.876
159.928
159.93
160



Student Activity

Name_____

Date _____

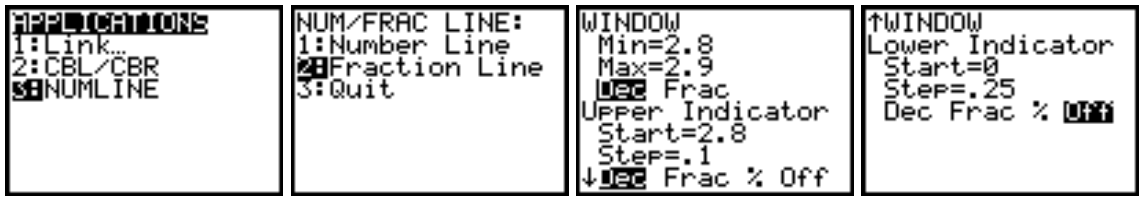
Activity 6

Comparing Decimals—Wider Isn't Always Larger!

Objective: *In this activity, you will investigate how to compare and order decimals using the fraction Number Line. You will use the Number Line application on the TI-73 calculator.*

In some libraries, books are ordered using the Dewey Decimal System. To do this, librarians and their assistants must be able to compare and order decimals. We will explore how to do this in this activity.

1. Access the Number Line application by pressing APPS and pressing ▼ to select the NUMLINE application. Press ENTER twice to get to the NUM/FRAC LINE menu and select 2: Fraction Line.



Press WINDOW and set **Min = 2.8**, **Max = 2.9**, and **Start=2.8**. Choose **Dec** (decimal), and turn off the lower indicator. Now, press DRAW to mark the first number **2.85**. Press ENTER to select **DrawLabel** and enter the number **2.85**. The number line will appear with the value **2.85** indicated. Next, enter the second number **2.9**. You can see that 2.85 is to the left of the 2.9. The largest of the two numbers is to the *right* of the smaller number.

2. To clear the marks for 2.85 and 2.9, press DRAW and select **2:ClrDraw** to clear the drawing.

3. Compare 15.762 and 15. 81. Press WINDOW to set the window to show the numbers between 15.5 and 16. Set **Min = 15.5**, **Max = 16**, **Start = 15.5** and **Step = .1**. Choose **Dec** (decimal) and turn off the lower indicator. Now, press DRAW to mark the first number **15.762**. Press ENTER to select **1:DrawLabel** and enter the number **15.762** on the screen. Next, enter the second number **15.81**.
Which number is on the left? _____.
Which number is on the right? _____.

4. Use the Number Line application to compare these numbers. Circle the larger number.

a. 2.35, 2.4

b. 0.98, 0.076

c. 14.95, 14.931

5. Use the Number Line application to compare these numbers. Write the smaller number before the larger number.

a. 1.23, 1.262:

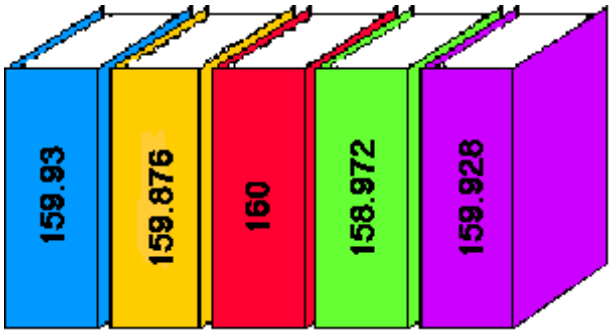
b. 13.2, 12.3:

c. 0.056, 0.06:

6. Can you and your partner think of other ways to correctly compare decimals? Be ready to share your ideas with your class. Write your explanation here.

7. Write these numbers in order from smallest to largest:
3.33, 3.23, 3.229, 3.221, 3.235, 3.227

8. These books are marked with numbers using the Dewey Decimal System. Sort them in order from least to greatest.



EXPLORATIONS

Activity 7

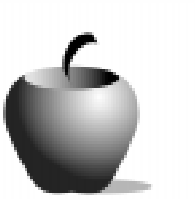
Partitions— How Many Ways Can We Split A Number?

Objective

- To explore multiple representations of positive integers
- To develop partitions of positive integers

Materials

- TI–73 Calculator
- Paper cups
- Marbles

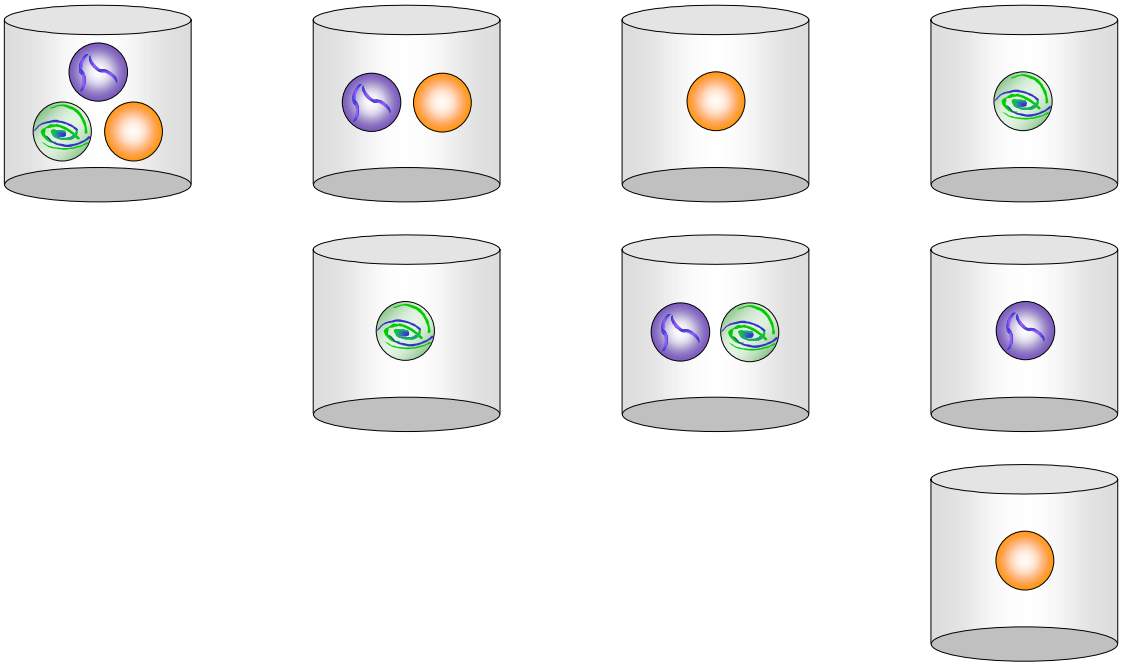


Teacher Notes

Introduction

How many ways can we place three marbles in cups? One way is to place all three marbles in one cup. Another way is to place each of the marbles in separate cups. Another way is to place two marbles in one cup and the third marble in another cup. These are illustrated below:

3 = 3 | 3 = 2 + 1 | 3 = 1 + 2 | 3 = 1 + 1 + 1



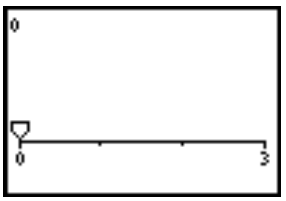
To simplify our counting, we can choose not to include one of the two ways to show 3 = 2 + 1 and 3 = 1 + 2. If the cups are indistinguishable, then we need only keep track of one way. Let's choose the first: 3 = 2 + 1. Thus, there are three different ways to *partition* three marbles into cups.

This activity provides students the opportunity to construct partitions using the Number line Application, paper cups and marbles.

Procedure

1. Demonstrate how to partition 3 marbles into cups.
2. Distribute Student Activity pages with the problem statement. Have students work in pairs using one calculator between them.
3. Have students access the Number line application by pressing [APPS] and pressing [▼] to select the NUMLINE application. Press [ENTER] twice to get to the NUM/FRAC line menu and select option 1: Number line.

Have the students press [ZOOM] and select option 2. Have the students change the minimum and maximum values of x in order to view the entire picture. To do this press [WINDOW] and move to **Min** and enter **0**. Now move down to **Max** and enter **3**. Set the **Scale** to **1** and select **LabelOn**.



4. Students will first investigate how many ways to partition 3 and 6 using the Number line application. They will then find and count the partitions for the numbers 1 through 7. Included are Extra Challenges 8 through 10.
5. Ask the students to share their method for predicting how many partitions there are for the numbers 1 through 7.

Answer Key

6. There are 11 partitions of 6: 6, 5 + 1, 4 + 2, 4 + 1 + 1, 3 + 3, 3 + 2 + 1, 3 + 1 + 1 + 1, 2 + 2 + 2, 2 + 2 + 1 + 1, 2 + 1 + 1 + 1 + 1, and 1 + 1 + 1 + 1 + 1 + 1.

8. Predictions may vary.

Numbers to be partitioned	Predicted number of partitions	Number of partitions
1		1
2		2
3		3
4		5
5		7
6		11
7		15
Extra Challenges		
8		22
9		30
10		42

Student Activity

Name_____

Date _____

Activity 7

Partitions—How Many Ways Can We Split a Number?

Objective: *In this activity, you will investigate how to partition integers using the Number Line. You will use the Number Line application on the TI-73 calculator.*

Suppose you have three pieces of hard candy. How many ways could it be distributed? How many ways can you distribute six pieces of candy? Some think that there will be twice as many ways to distribute 6 pieces as 3 pieces because 6 is twice 3. Is this true?

1. Access the Number Line application by pressing [APPS] and pressing [▼] to select the NUMLINE application. Press [ENTER] twice to get to the NUM/FRAC LINE menu and select the NUMLINE option.
2. To capture the entire picture, press [WINDOW]. You need to select new minimum and maximum values. All of the sums are positive, so move to **Min=** and enter **0**. Since we are computing sums that equal 3, move down to **Max=** and enter **3**. Set **Scale=1**, **Start=0,** and select **LabelOn**.
3. Press [GRAPH] to return to the Number Line. Press [CLEAR] to clear the expression. Select **Yes**. Now enter **1** [+] **1** [+] **1** and press [ENTER].
4. Clear this expression and show the other ways to get a sum of 3 using positive integers. Draw sketches of your Number Line windows.

1 + 1 + 1	2 + 1	1 + 2	3

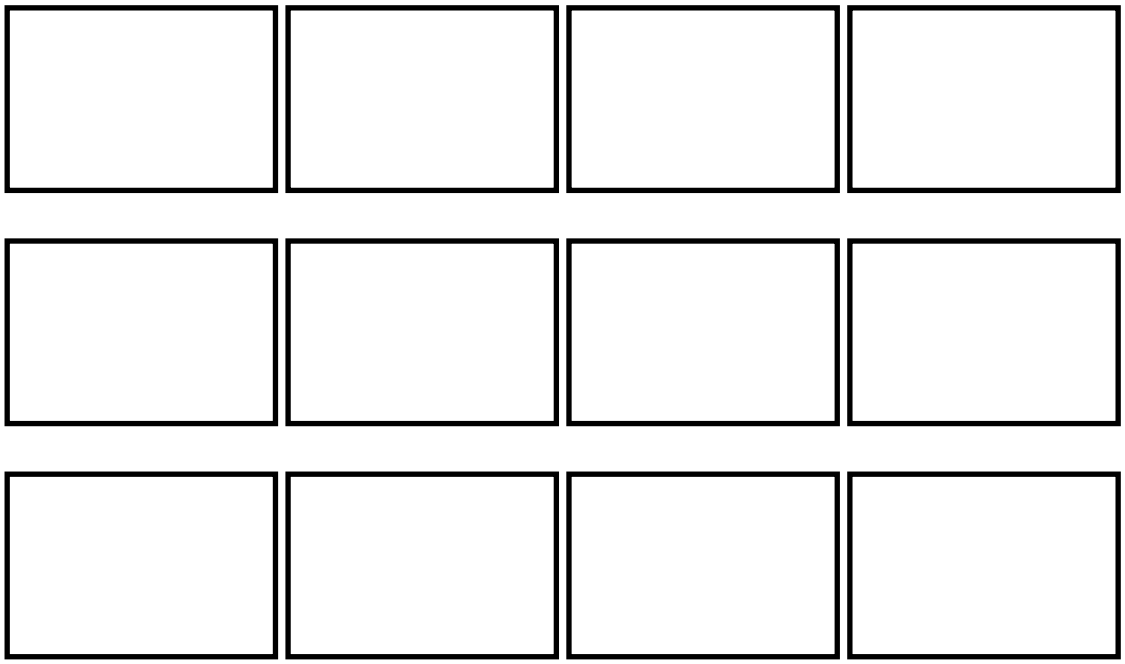
Since **2 + 1** and **1 + 2** are similar ways to get a sum of **3**, we will count only one of those ways. Now we have found three distinct ways to split or partition 3.

5. This time find all of the ways to get a sum of 6. What is your prediction for how many ways there are to partition 6?

6. Press [WINDOW] to reset the **Max** to **6**. Press [GRAPH] to return to the Number Line window. Press [CLEAR] to clear the expression. Select **Yes**. Now find as many sums as you can that equal 6.

Draw and label sketches of your expressions on the blank screens below. (There are more screens than you need.)

***Note**: The TI-73 Number Line Application can display up to five vectors. For the expression 1 + 1 + 1 + 1 + 1 + 1 = 6, your sketch should show six vectors of length one, but only five vectors are drawn.*



Did you find fewer or more ways to partition than your prediction?

Complete the table below using the Number Line application, cups, and marbles.

Number to be partitioned	Predicted number of partitions	Number of partitions
1		
2		
3		
4		
5		
6		
7		
Extra Challenges:		
8		
9		
10		

Were your predictions close? Mathematicians worked hard to find a way to predict how many ways there were to partition a given number. You can see that the number of partitions grows slowly at first, but then what happens? Explain.

EXPLORATIONS

Activity 8

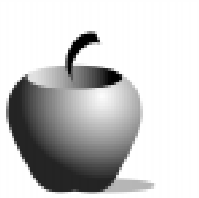
Somewhere In Between

Objective

- To develop estimation and mental arithmetic skills for integer addition and subtraction

Materials

- TI-73 Calculator



Teacher Notes

Introduction

In this activity, the Number Line application will be used for targeting estimations. Students will make predictions of sums and differences and try to land "somewhere in between" a target interval set up by their partners.

Procedure

1. Distribute Student Activity pages. Have students work in pairs using one calculator between them. Direct students to use the sum of the last three digits of their phone numbers to determine who will be Player 1 and Player 2.

2. After accessing the Number Line application, have students change the viewing window to make use of a larger portion of the number line. Window settings can vary depending upon the integer arithmetic experience of the players.

   Tables are provided at the end of the activity for students to complete as they work through the activity. Their entries will help you assess their "integer arithmetic sense."

3. While in the window settings, Player 1 also needs to set a new **Start** number. Player 1 then chooses a target interval using the **Draw Label** option of the DRAW key. Since the interval is to be small, the numbers showing the endpoints of the interval will probably overlap. Students have a table that they can complete so that the interval range is not forgotten.

   Player 1 also chooses the initial arithmetic operation for the start of the round. Each player will have the opportunity to choose the operation for the other player; both addition and subtraction are possible.

4. Have Player 2 choose a number that will put the sum (or difference) in the target interval. Encourage students to use estimation strategies other than rounding (such as compatible numbers) to determine responses.
5. If the vector lands within the target interval, Player 2 wins. Otherwise, Player 2 chooses the next operation and the play continues.
6. Continued description of play is provided.
7. Students are encouraged to write down strategies they used during the game.
8. Tables are provided for students to complete, showing their entries on the calculator.

Student Activity

Name_____

Date _____

Activity 8

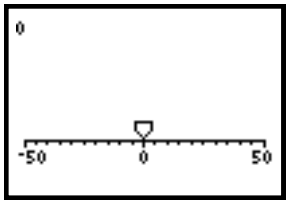
Somewhere In Between

Objective: *In this activity, you will use the Number Line application for "targeting estimations." You will make some predictions of sums and differences and try to land "somewhere in between" a target interval set up by your partner.*

1. You will need to have a partner for this activity and a calculator with the Number Line application. Mentally find the sum of the last 3 digits of your phone numbers. Whoever has the greater sum will be Player 1 and the other will be Player 2.

Access the Number Line application by pressing [APPS] and pressing [▼] to select the NUMLINE application. Press [ENTER] twice to get to the NUM/FRAC LINE menu and select the Number Line by pressing [ENTER]. Press [CLEAR] **1** (or [CLEAR][ENTER]) to clear any previous work.

2. Press [WINDOW] and change the **Min** to **-50**, the **Max** to **50**, and the **Scale** to **5**. Unless you and your partner decide otherwise, you won't need to change these three values again.

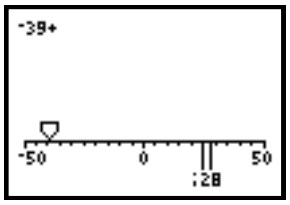


Complete the tables (see #9) as you follow through the activity.

3. **Player 1**: Change the **Start** number in the WINDOW settings to any integer in the range of **-50** to **50**. When you have selected that number, press [GRAPH] to return to the Number Line window.

Example

Player 1 chose -39 as the start number, a target range of 25 to 28, and addition as the starting operation.



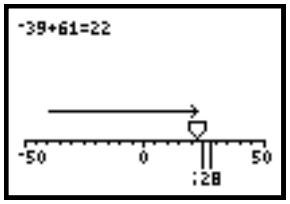
Now choose two different integers between -50 and 50 which have a small difference (or distance between). This will be the target range for the first round of the game. To enter these two numbers on the number line, press [DRAW] and [ENTER] to select the first option of Draw Label. Type one of the two numbers after **Label** = at the top of the window and press [ENTER]. A marker should appear on the number line at that number. Type your second number after the **Label** = and press [ENTER]. Press [DRAW] (or [ENTER]) to exit the Draw mode.

Before handing the calculator to Player 2, you need to determine the initial operation that Player 2 has to use: either addition or subtraction. To do that, simply press either the [+] or [−] key and then pass the calculator to Player 2.

4. **Player 2**: You need to use your mental arithmetic and estimation skills and choose a number that will put the sum (or difference) in the target range shown on the number line. Type your number and press [ENTER].

Example

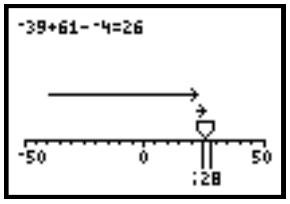
Player 2 added 61 to -39. The sum of 22 did not fall in the target range.



5. If the vector endpoint lies within the target range (or on one of the two boundary markers), Player 2 wins the round. If not, Player 2 presses either the addition or subtraction key, passes the calculator back to Player 1, and the round continues. Notice that in the example at the right, Player 1 does not start over, but continues to estimate from the current sum (or difference).

Example

Player 2 didn't win so subtraction is chosen and the calculator handed to Player 1. Player 1 subtracted -4 and landed in the range. Player 1 won.



6. **Player 1**: Think of a number that will put the sum (or difference) in the target interval. Type in that number and press [ENTER].
7. If the vector endpoint lies within the target range, Player 1 wins the round. If not, Player 1 chooses the next operation of addition or subtraction, hands the calculator to Player 2, and the round continues.
8. Describe two strategies that you used during the activity.

9. Keep a record of your choices in the tables below.

Target Range: _____ Selected by Player: _____		Start Number:_____
Operation Chosen	Number Entered	Sum or Difference

Target Range: _____ Selected by Player: _____		Start Number:_____
Operation Chosen	Number Entered	Sum or Difference

Target Range: _____ Selected by Player: _____		Start Number:_____
Operation Chosen	Number Entered	Sum or Difference

Target Range: _____ Selected by Player: _____	Start Number:_____	
Operation Chosen	Number Entered	Sum or Difference