Exploring the Parabola

ID: 8358

Time required
90 minutes

Activity Overview

This activity explores the key features of the parabola, both geometrically and algebraically. A variety of interactive representations support student learning as they build their understanding of this important curve and its real-world applications.

Topic: Conics

- *Write the equation of a parabola with vertex at (h, k) and axis of symmetry x = h or y = k and graph it.*
- *Derive the equation of any conic (just parabola here) using the focus-directrix definition.*

Background

In many high school curricula, the primary objective in the study of parabolas tends to be algebraic, moving quickly to the study of the quadratic function, though the key defining features of this function are geometric in nature. Students often misrepresent other curves as 'parabolic' simply because they have a similar appearance; the catenary is a classic example (A chain suspended from its ends forms a catenary curve—the word catenary is derived from the Latin word for chain). It is therefore important for students to understand some of the properties of a parabola, features that make this curve both unique and important. This activity supports students in actively linking some of the geometric and algebraic properties of a parabola.

Teacher Preparation

Prior to this activity, it is recommended that you take some time to build an understanding of locus concepts with your class. This might take the form of physical involvement on the part of the students, as they build from a simple locus to the more complex. A simple locus for students to 'construct' is a circle:

> *A volunteer from the class becomes the most popular person in the school—everyone wants to be close to this person. How could you arrange yourselves to all be equally close to the person?*

A more complex situation is the 'construction' of a perpendicular bisector:

> *Two volunteers are now equally popular but very jealous—everyone wants to be close to them, but never closer to one than the other! How could the friends of these two people arrange themselves?*

From here, you can model the locus to form a parabola:

> *Consider one very popular person, and the most popular group, lined along the wall at the school dance, hand in hand. How will others arrange themselves so as to be equally close to both the individual any single person from the group?*

Note: In this last example, care should be taken that students interpret the 'group' as a single, continuous identity. The linking of hands helps to create this view.

- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "8358" in the keyword search box.***

Classroom Management

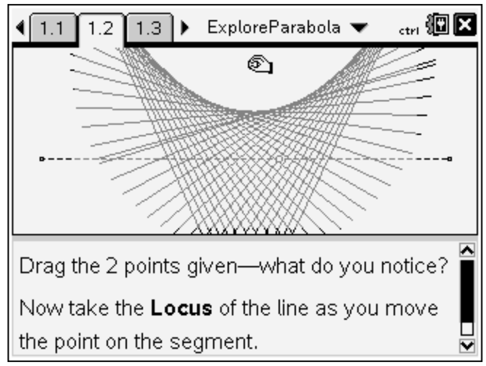
- *This interactive exploration moves between representations, from physical involvement in the building of loci, to geometric construction, to algebraic functions and their graphs. The early work involving popularity contests builds both interest, a sense of fun, and a deeper intuitive understanding of key concepts, including the perpendicular bisector.*
- *It is useful to have students work in pairs, both to encourage verbalization and discussion, and to offer scaffolding when needed. Individuals may work together with their own handhelds, or they may share between partners. Each must keep a record of the process, documenting his/her own observations and any questions that arise. Bringing the class back together for sharing and discussion is very important, particularly to keep some from falling too far behind.*
- *Ideas for optional extensions are provided on page 4 of this document.*

Technical Prerequisites

Students should know how to:

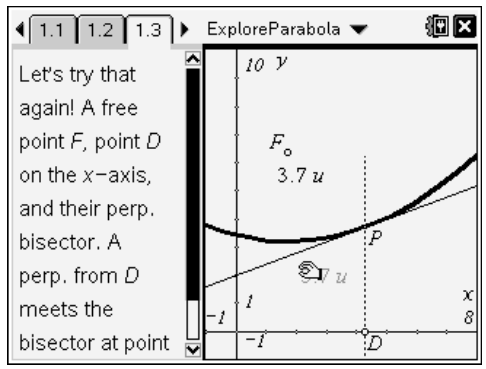
- *Construct simple segment and point models, as well as perpendiculars, perpendicular bisectors, loci, and reflections.*

This activity begins dramatically as students follow very simple instructions leading to the envelope of lines shown here.



At this point, a nice complement to the use of technology involves some simple paper folding. Taking a sheet of paper, students should mark a point somewhere near the center of the page. They can then produce a series of folds from different points along one of the long edges that pass through this point. Tracing along these straight-line folds should produce an envelope similar to the one shown. It is important that students see the connection between the crease in the paper and the perpendicular bisector.

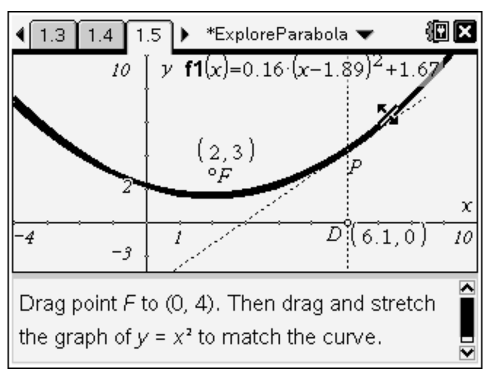
The page-by-page instructions provided in the interactive document for this activity allow students to navigate through the procedures themselves. Nonetheless, it is still recommended that the teacher accompany them on this journey, which allows for class discussion and for teacher comments.



The early "play" is important and students should be encouraged to explore the effects of different parts of the construction: move the point (focus) and the line (directrix), observing each of their roles in the locus construction.

Formalizing the construction involves labeling key points and some discussion as to why point *P* has been chosen. More advanced students may observe that the perpendicular bisector becomes tangent to the parabola at point *P*.

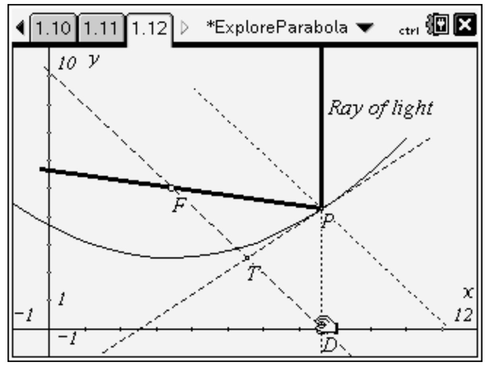
The next phase of the lesson introduces the algebraic notation for the quadratic function, and invites students to engage in the unique "manual regression" which this technology makes possible! As students 'drag' (translate) the fundamental curve $y = x^2$ to match the envelope, it must be highlighted that this process simply acts to model the curve, otherwise students may fall into the same trap as Galileo, misrepresenting a suspended chain (catenary) as a parabola.



Students should be encouraged to explore the relationship between the various components of the algebraic form and the physical features of the curve such as that modeled in the original parabola locus by the students. To build an algebraic understanding, students can use algebra to determine an 'equation' for the curve, based on these properties. This amounts to using the formula for the distance between two points.

Finally, a link to the real world—the defining property of the parabola, which explains so many of the applications of this important curve!

Again, students should be given the time and necessary direction to discover and to realize the implications of this feature of our curve. Physical modeling may again play a role: in addition to the *ray of light* example, consider rolling a ball against a sheet of cardboard bent into a parabolic shape.



Assessment and Evaluation

Primary assessment for this activity should be a detailed report of the investigation, with student comments and observations related to the physical, geometric, and algebraic properties of the parabola.

"Exploring the Parabola" [answers appear in brackets]

1. What is the equation of the parabola with the focus located at (0, 4) and directrix on the *x*-axis? [$y = \frac{1}{8}x^2 + 2$]
2. What is the equation of the parabola with the focus at (2, 3) and directrix $y = 0$? [$y = \frac{1}{6}\left(x-2\right)^2 + \frac{3}{2}$]
3. What is the focal point for the parabola with equation $y = 0.125\left(x-3\right)^2 + 2$? [(3, 4)]
4. What is the significance of the coefficient of *x* in each of these equations? [The coefficient dictates the "speed" of the curve (how quickly it increases), and is given by $\frac{1}{4f}$, where *f* is the *focal length* (half of the distance between the focus and the directrix).]
5. Can you find the general form for the equation of a parabola with its focus located at (*a*, *b*) and directrix on the *x*-axis? [$y = \frac{1}{2b}\left(x-a\right)^2 + \frac{b}{2}$]

Extensions

Extension activities could include:

- The derivation of the general form from its focus and directrix (given above)
- Calculus applications involving tangent, and where the tangent cuts the *x*-axis
- The extension of this construction to other conics (On page 1.13, what would happen if point *T* was not the midpoint of segment *FD*? If it is positioned closer to point *F*, then the curve is an ellipse; if it is closer to *D*, it produces an hyperbola.)