EXPLORATIONS

Activity 2

TryAngle?

Objective

- To investigate properties of triangles and learn how to classify the various types of triangles

Materials

- TI-73 calculator
- 12 6-inch straws with about 40 twist ties
- Ruler, scissors, and marker
- Student Worksheet

In this activity you will:

- discover a test to determine if three line segments can form a triangle
- discover a method to classify triangles as acute, right, or obtuse when the side lengths are known
- find all possible triangles that can be made

You will need to know this math vocabulary:

- scalene, isosceles and equilateral triangles
- acute, right and obtuse triangles

Introduction

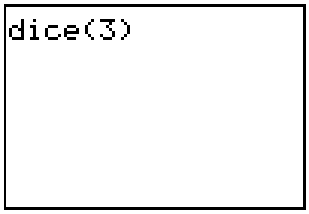
Adrienne has decided to make a mobile of triangles for a geometry project. She will use straws from 1 to 6 inches (whole number lengths only) for the sides and twist ties to connect them. Can she randomly pick straws and make a triangle? If she wants her mobile to include all possible triangles, how many triangles will she need to make?

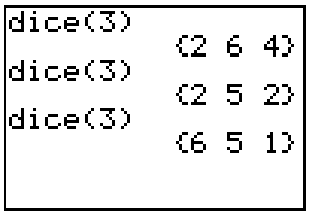
Problem

You will work with a partner to obtain all possible triangles by randomly generating three whole numbers between 1 and 6. Three dice will be rolled to determine the side lengths using the TI-73 dice throw function. You and your partner will take turns throwing the dice.

Activity

1. Go to the Home screen [2nd] [QUIT] and press [CLEAR]. Next press [MATH] [▸] [▸] **PRB** and select **7: dice(**. Type **3** [)] to roll three dice as shown at the right. Press [ENTER] several times to see the random numbers that will come up on the three dice. [CLEAR] your screen to start the game.





a. Press [ENTER] and determine who has the highest sum. This person will go first. Player 1 will press [ENTER] and will try to build a triangle with the numbers rolled on the dice as given side lengths.

Use the ruler and cut straws of the given lengths. Insert the twist ties in the ends of the cut pieces and try to connect to form a triangle. If Player 1 is able to make a triangle with the given lengths, then Player 2 will inspect it and make sure the sides are measured accurately. Player 1 will record his/her results in Table 1. Player 2 will now take a turn. For example, if a player rolls the dice 1, 1, 2 and it is determined that it is not possible to make a triangle, then the result is recorded as 1, 1, 2 in the appropriate player column for "not a triangle." Likewise, if a player rolls the numbers 3, 4, 5 and it is determined possible to make a triangle then these results are recorded in the appropriate player column for "is a triangle."

b. If on a later roll a player rolls 4, 3, 5 when 3, 4, 5 has been rolled earlier, it will still count as "is a triangle." The triangle does not need to be built again since it would be the same triangle.

c. To determine a game winner, find the sum of the columns and then subtract the "not a triangle" column sum from the "is a triangle" column sum. This will become the player's score. The person with the highest score is the winner.

✎ Answer questions 1 through 3 on the Student Worksheet.

2. One way to classify triangles is by their sides. If all sides are congruent (or equal in measure) the triangle is **equilateral**. If two sides are congruent, the triangle is **isosceles**, and if no two sides are congruent, then the triangle is **scalene**.

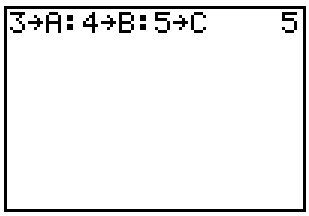
✎ In Table 1 on the Student Worksheet, write an **E** next to any equilateral triangles, an **I** next to any isosceles triangles, and a **S** next to any scalene triangles.

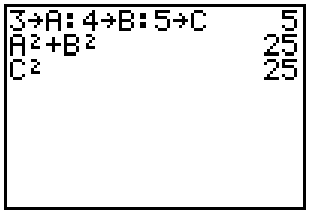
3. Another way to classify triangles is by their angle measures. You may recall that the sum of the angles in any triangle is 180°. A **right triangle** has one right angle (90°) and two acute angles. (An acute angle is less than 90°.) An **acute triangle** has three acute angles and an **obtuse triangle** has one obtuse angle and two acute angles. (An obtuse angle measures greater than 90° and less than 180°.) Inspect the triangles you and your partner have made and classify them according to their angles.

✎ In Table 1, write an **R** next to any right triangles, an **A** next to any acute triangles, and a **O** next to any obtuse triangles. Answer questions 4 and 5 on the Student Worksheet.

4. In a right triangle, the square of the hypotenuse (longest side) is equal to the sum of the squares of the other two sides. This important theorem is called **the Pythagorean Theorem**. A set of numbers that satisfy this equation $a^2 + b^2 = c^2$ is called a Pythagorean triple.

a. Test 3, 4, 5 in this equation to see if it is a Pythagorean triple. Press 2nd[TEXT] 3 STO▸ **A,** select **Done** and press ENTER. Press 2nd [CATALOG] ▲ ▲ ▲(to select :), ENTER. Press 2nd [TEXT] 4 STO▸ **B** then select **Done** ENTER 2nd [CATALOG] ▲ ▲ ▲ (to select :) ENTER. Finally, press 2nd[TEXT] 5 STO▸ **C**, select **Done**, ENTER ENTER. Type in the equation and make sure both sides are equal.





b. Since $3^2 + 4^2 = 5^2$, a triangle with sides of 3, 4, and 5 would be a right triangle. If you do not have a 3, 4, 5 triangle made, make one.

✎ Answer question 6 on the Student Worksheet.

5. Find an acute triangle. If you don't have one already made, make one.

✎ Answer questions 7 and 8 on the Student Worksheet.

6. Use an obtuse triangle you have made. If you don't have one, make three.

✎ Answer questions 9 and 10 on the Student Worksheet.

7. The class will now combine all their data and include any possible missing sets to make an organized list with all possible triangles to be included in the mobile. Remember that 3, 4, 5 is the same as a 5, 3, 4, so don't list twice in the table.

✎ Complete the table with your class and answer question 11 on the Student Worksheet.

Student Worksheet

Name _____

Date _____

Activity 2

TryAngle?

Record your results on the table below. Then answer the questions about the activity.

Part 1

	Player 1:		Player 2:	
Trial	Is not a triangle	Is a triangle	Is not a triangle	Is a triangle
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
Totals				
	Player 1 score:		Player 2 score:	

1. What conclusions can you draw about the lengths of sides of triangles?

   _____

2. Give an example of a set of 3 numbers (different from those in the table above) that could be the sides of a triangle.

   _____

3. Give 3 examples of sets of 3 numbers (different from those above) that could not be the sides of a triangle.

_____

_____

4. Why can't a right triangle have two right angles?

_____

5. An equilateral triangle is said to also be equiangular. What is the measure of each angle in an equilateral triangle?

_____

6. Does the 3, 4, 5 triangle appear to have one right angle and two acute angles?

_____

7. Using an acute triangle, let c=_____(the longest side) and let a = ____(the shortest side) and let b=____(the middle side). Replace the ☐ with <, >, or = to make a true sentence.

$a^2 + b^2 \square c^2$

8. Try 2 other triangles that appear to be acute by substituting in the side lengths into the mathematical sentence in #7 and using the correct symbol. (c= the longest side, a and b will be the shorter sides and it is possible for a=b) Make them if you don't have them.

_____

9. Using an obtuse triangle, let c= (the longest side) and let a = (the shortest side) and let b= (the middle side). It's okay if a=b. Substitute into

$a^2 + b^2 \square c^2$

replacing ☐ with <, >, or = to make three true sentences.

_____

10. Compare your results in **#7** through **#9** above and make a conjecture from your experiment.

_____

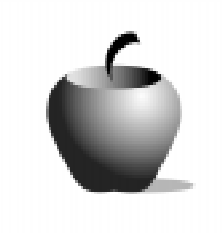
Part 2 - Class Data Table

All triangles with side(s)	List triangles	Total
of 1 inch		
of 2 inches		
of 3 inches		
of 4 inches		
of 5 inches		
of 6 inches		

11. How many triangles would Adrienne need to make for her mobile?

_____

Teacher Notes



Activity 2

TryAngle?

Math Strand

- Algebraic reasoning
- Number sense
- Statistics

Materials

- TI-73 calculators (one per pair or for each student)
- Student Worksheets (page 16)
- 12 six-inch straws for each pair of students
- 40 to 50 twist ties for each pair of students
- A pair of scissors, a marker, and a ruler for each pair of students

Students will investigate properties of triangles by randomly generating possible side lengths for triangles. They will experimentally try to make triangles with the randomly generated numbers and then make conjectures about triangle inequalities.

Vocabulary

equilateral triangle	A triangle with all 3 sides congruent.
isosceles triangle	A triangle with 2 sides congruent.
scalene triangle	A triangle with no 2 sides congruent.
acute triangle	A triangle with 3 acute angles.
right triangle	A triangle with 1 right angle and 2 acute angles.
obtuse triangle	A triangle with 1 obtuse angle and 2 acute angles.

Classroom Management

Students will work with a partner in the investigation part (Part 1) of the activity.

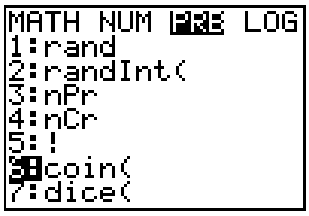
- Demonstrate how to make a triangle with the given materials. Emphasize to the students to measure accurately and use a marker to mark the cut line. Insert the twist ties inside the straws so the straws meet to form vertices. You may need to settle any disagreements students may have on if it is a triangle or not.

In Part 2 of the activity you will lead the class to summarize the data, and the Class Data Table will be completed. The last part is the **Going Further** section and is optional. It could be investigated by the whole class or as a home assignment.

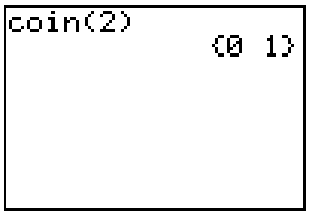
Activity

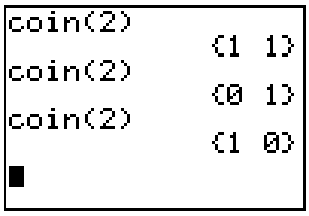
Part 1

1. If the students are not familiar with the random number generators on the calculator, show them the various ways to generate random numbers. Allowing them to play with the coin toss (for any number of coins) and the dice roll (for any number of dice) will help seed the calculators in case these functions have not been used yet. This will avoid the problem of the calculators generating the same numbers.

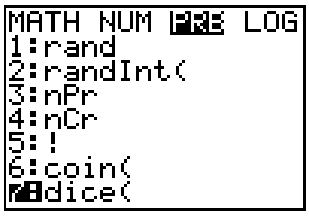


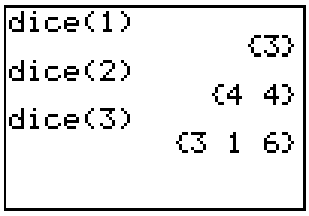
Press MATH ▸▸ **6:coin(2**) (tosses 2 coins) pressing ENTER any number of times. Explain that the numbers **0** and **1** would have to be defined to mean head or tail.





Do the same with the dice roll, changing the number of dice and discussing what it means.

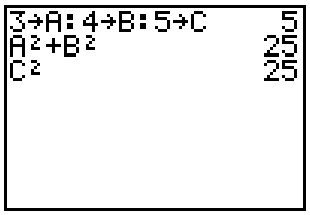




2. Students should have had some prior experience with triangles and classifying by sides.

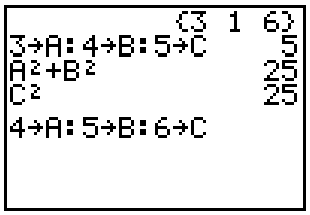
3. Students should be familiar with angle measure and be able to classify angles as acute, right, or obtuse.

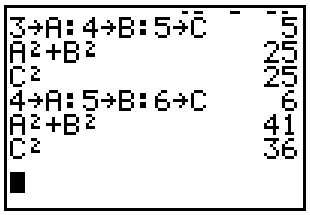
4. You may want to tell the students that after they enter the stored values for **a**, **b**, and **c** and **the Pythagorean Theorem** equation into the calculator once, they don't have to enter it again. They can scroll up, grab and edit if necessary to use again. See screen shots at the right.



5. You may want to re-emphasize that an acute triangle has 3 acute angles.

6. You may want to ask students why an obtuse triangle has only one obtuse angle.

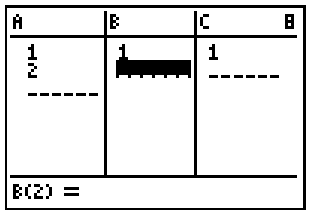




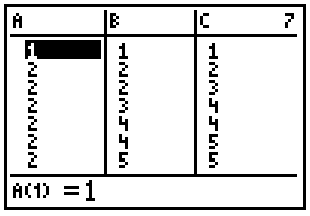
Part 2

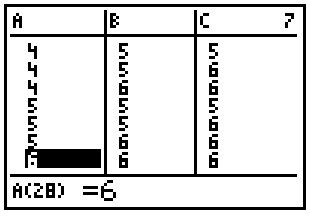
7. When completing the Class Data Table, include missing sets. You may start by asking students to tell the sets that have a 1" side. Write this set down in the second column, second row as 111. Next ask the students to tell all their sets that have a 2" side. Write these sets on board as 3-digit numbers, then go back and order them in ascending order. When you get to the 3" sides, tell students that we can not re-use triangles with 2" sides. Suggest starting with the equilateral triangle 333. You cannot go back to 332 (since it was used in the 2" row) so check out 334, 335, 336, 344, 345, 346, 355, 356, and 366. Continue this same process for 4", 5" and 6".

a. In the final part of this activity, use the calculator list to list all 28 triangles and test to make sure they are triangles using the triangle inequality rule discovered. Name the lists **A**, **B**, and **C** to represent the sides. To access, name, and use formulas in lists, see Appendix A, B, and C respectively.

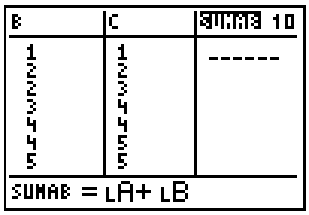


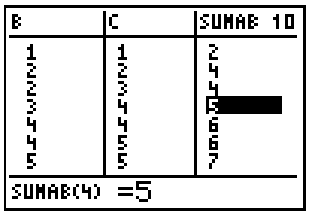
Enter the data going from left to right to reflect the sides of the possible triangles. (It will take longer to enter this way but will probably make more sense to the students.) More experienced students may be able to enter in columns. See the screen shots at the right.



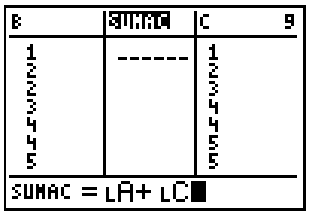


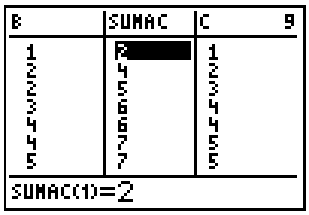
b. Now find the sum of **A** and **B**, scroll down the list and make sure **A + B > C**. Go to the right of **C**. Name the list **SUMAB**. Use the formula **A + B** and then scroll down the list and have students verify that **A + B > C**.





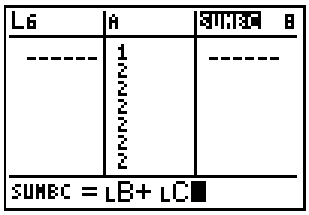
c. To test the second condition of **A + C > B**, insert a list named **SUMAC** to the right of **B** and use the formula **A + C.** Scroll down **SUMAC** list and make sure **A + C > B**.

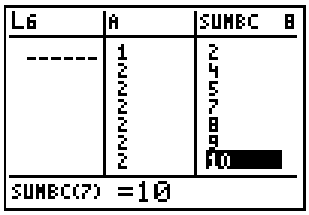




d. Finally, test the third condition of **B + C > A**. Name a list **SUMBC** and insert it to the right of **A**. Scroll down list **SUMBC** and make sure **B + C > A**.

e. Ask students how you can make sure you did not leave any data out. (Make sure the list is organized.) Students can now make a class mobile with all 28 triangles.





Answers to Student Worksheet

1. The sum of two sides has to be greater than the third side.
2. Answers will vary with lots of possibilities such as {3,4,5} {6,6,5}.
3. Answers will vary with even more possibilities such as {1,2,4} {3,2,6}.
4. The sum of 2 angles would equal 180°, meaning the third angle would be 0°.
5. 60°
6. It should.

7. Sample answer a=4 b=5 c=6 $4^2 + 5^2 > 6^2$

8. Answers will vary. $5^2 + 5^2 > 6^2$ $3^2 + 3^2 > 4^2$

9. Answers will vary. $3^2 + 4^2 < 6^2$, $2^2 + 3^2 < 4^2$, $2^2 + 2^2 < 4^2$

10. If $a^2 + b^2 = c^2$, then the triangle is right. If $a^2 + b^2 > c^2$, then the triangle is acute. If $a^2 + b^2 < c^2$, then the triangle is obtuse.

Class Data Table

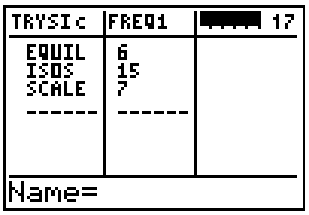
All triangles with side(s)	List triangles	Total
of 1 inch	111 E-A	1
of 2 inches	222 223 234 244 245 255 256 266 E-A I-O S-O I-A S-O I-A S-O I-A	8
of 3 inches	333 334 335 344 345 346 355 356 366 E-A I-A I-O I-A S-R S-O I-A S-O I-A	9
of 4 inches	444 445 446 455 456 466 E-A I-A I-O I-A S-A I-A	6
of 5 inches	555 556 566 E-A I-A I-A	3
of 6 inches	666 E-A	1

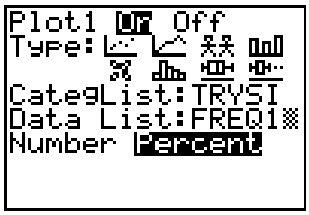
11. 28

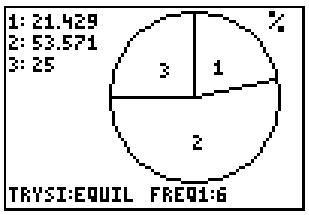
Going Further

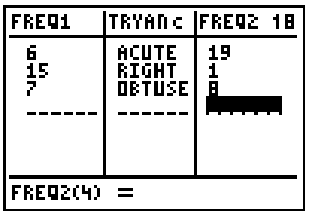
Students could classify all 28 triangles in the Class Data Table by their sides and angles using **E**-Equilateral, **I**- Isosceles, **S**-Scalene and **A**-Acute, **R**-Right, **O**-Obtuse, as done above. Pie charts could be used to compare the number of specific triangles to the whole.

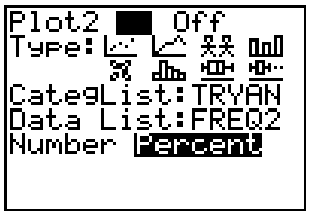
Triangle by sides	Frequency	Triangle by angles	Frequency
Equilateral	6	Acute	19
Isosceles	15	Right	1
Scalene	7	Obtuse	8
Total	28	Total	28

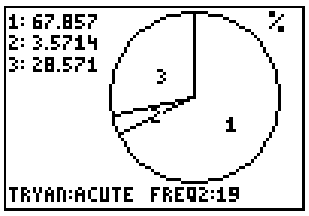






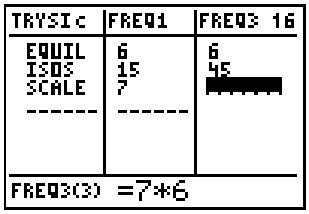






Many different probability questions could be posed, such as:

1. What is the probability of rolling 3 dice and getting a set of numbers that would be a triangle? There are 6^3 or 216 different outcomes. Even though there are 28 triangles, there are more than 28 possible outcomes. For example 345, 354, 435, 453, 534, 543 would be considered one triangle but 6 different outcomes. So if you count the permutations all scalene triangles would have 6 arrangements, Isosceles would have 3, and Equilateral triangles would have only 1 arrangement. Go back to List and multiply the number of equilateral triangles by 1, the number of Isosceles triangles by 3 and the number of Scalene triangles by 6. Sum the List and put that over the total number of possibilities.



Students could each roll dice 72 times and see how many would make triangles.

Compare this experimental probability to the theoretical probability of 31/72.

2. Are you more likely to roll number sets that would or would not make a triangle?

3. How many of your trials in the game produced triangles?

4. What kind of triangle are you most likely to roll?