Exponentially Fast Derivative

ID: 11612

Time Required
40–45 minutes

Activity Overview

In this activity, students will investigate the derivative of exponential functions. For e^x, students will explore the slope of the tangent graphically and use CAS functionality to discover the chain rule for exponential functions. Similarly, a^x is explored graphically and CAS functionality is used to discover the rule.

Topic: Exponential Differentiation

- *Derivative of e^x and the chain rule.*
- *Derivative of $b \cdot a^{u(x)}$*

Teacher Preparation and Notes

- *Students will enter their responses directly into the TI-Nspire handheld or on the accompanying handout. On self-check questions, students can then press (menu) and select **Check Answer** (or press (ctrl) + ▲).*
- *Students will need to know that the derivative of position is velocity, and the derivative of velocity is acceleration. For multiple choice questions at the end, students are expected to know the derivative of trig functions and the product rule.*
- *The activity is designed to be a student-centered discovery and instruction of differential calculus for exponential functions. Teaching the basics before beginning the activity is optional. After completing the activity, students should be more successful with AP* questions like 2003formBAB4, 2002formB AB2&BC2, 1991BC2, and multiple choice questions 1998AB16, and 2003AB9&17.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "11612" in the quick search box.***

Associated Materials

- *ExponentiallyFastDerivative_Student.doc*
- *ExponentiallyFastDerivative.tns*
- *ExponentiallyFastDerivative_Soln.tns*

Suggested Related Activities

- *Exponential Differentiation (TI-Nspire technology) — 8980*
- *The Exponential Derivative (TI-89 Titanium) — 8979*
- *Exponent Rule (TI-84 Plus) — 9730*
- *Derivative Trace (TI-Nspire technology) — 8110*
- *Derivatives of Logarithmic Functions (TI-Nspire CAS technology) — 9093*

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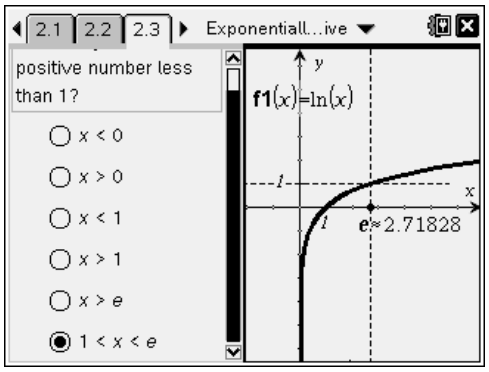
Part 1 – Warm up

On page 1.2 and 1.3, students are to graph exponential functions and consider the range. This knowledge is applicable to many exam questions involving e^x. Next, basic knowledge of the domain and value of the natural logarithm are asked using self-check multiple choice questions. Answers are graphically reinforced. These facts are important in the discovery and derivation of the derivative of a^x.



Further Discussion

- *If needed, review some logarithm properties. For example, ask students for an expression equivalent to ln(e^2). Recognizing e^x and ln(x) are inverses can also be drawn out by asking what $e^{ln(2)}$ is.*

Student Solutions

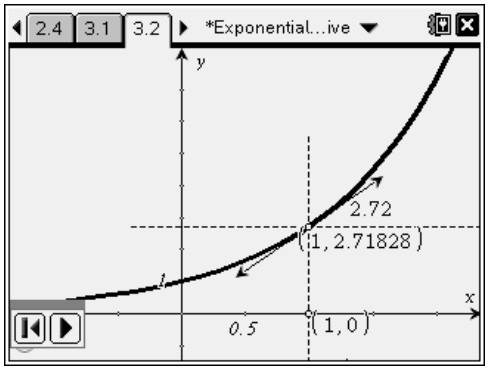
1. $f(x) = e^x$ is always positive and so is $f(x) = e^{-x}$
2. The domain of $y = \ln(x)$ is $x > 0$
3. **a.** $x > 1$

   b. $1 < x < e$

   c. $x > e$

Part 2 – Discover the derivative of e^x

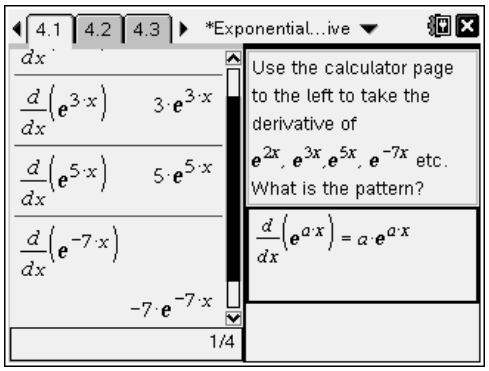
This part of the activity begins on page 3.2 with students putting a tangent line on the marked point on the graph of **f1**(x) = e^x. They also use the **Slope** tool in the Measurement menu. They should realize that the derivative of e^x is unlike the derivative of a polynomial or a power function like x^n.



Discussion Question

- *What is the derivative of a polynomial of degree n? How does that compare with the derivative of e^x?*

Beginning on page 4.1, students use CAS functionality to do a series of problems to observe the pattern of the derivative of $e^{u(x)}$ with respect to x. Students will then apply this rule to other problems. The valuable exercise of reconciling their solution with the answer that TI-Nspire CAS technology gives on page 4.4 is particularly helpful for multiple-choice exam questions.



Student Solutions

4. $\frac{d}{dx}\left(e^x\right) = e^x$

5. $\frac{d}{dx}\left(e^{a \cdot x}\right) = a \cdot e^{a \cdot x}$

6. $\frac{d}{dx}\left(e^{bx^n}\right) = bnx^{n-1}e^{bx^n}$

7. **a.** Since $x^{0.5} = \sqrt{x}$, $\frac{d}{dx}\left(e^{4x^{0.5}}\right) = 4(0.5)x^{-0.5}e^{4x^{0.5}} = \frac{2e^{4\sqrt{x}}}{\sqrt{x}}$

b. Since $e^a e^b = e^{a+b}$, $\frac{d}{dx}\left(e^{e^{3x}}\right) = e^{e^{3x}}3e^{3x} = 3e^{e^{3x}+3x}$

c. $\frac{d}{dx}\left(\sin\left(e^{-2x}\right)\right) = -2e^{-2x}\cos\left(e^{-2x}\right)$

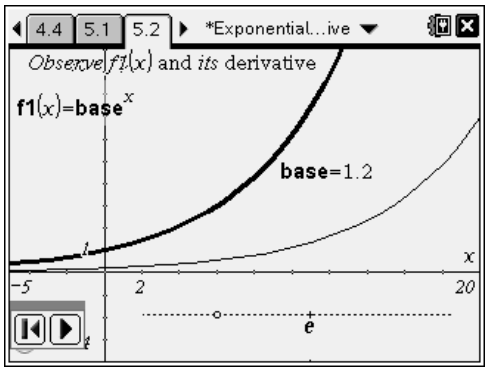
d. $\frac{d}{dx}\left(\cos^3\left(e^x\right)\right) = 3\cos^2\left(e^x\right)\left(-\sin\left(e^x\right)\right)e^x$

e. Since e^x and ln(x) are inverse functions, $e^{\ln(5x)} = 5x$. So the derivative is 5.

Part 3 – Investigate a^x

<u>Discussion Questions</u> *(for page 5.2)*

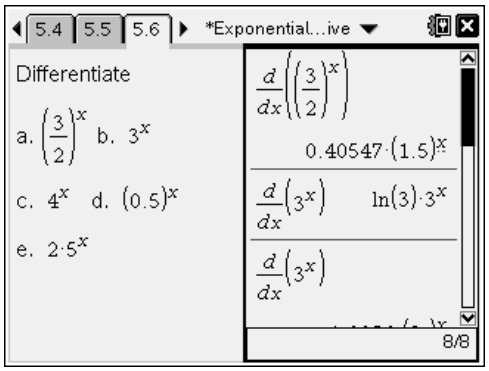
- *For what value of the variable base is the derivative the same as the bold function* ***f1****?*
- *For what values of base is the original function increasing? Concave up?*
- *For what values of base is the derivative increasing? Concave up?*
- *For what values is the derivative less than* ***f1****? Greater than* ***f1****?*



On page 5.3, students review the derivative of a constant times x to set the groundwork for finding the derivative of 2^x. When the constant is not an integer or decimal, students tend to have difficulties.

Page 5.4 shows the steps using implicit differentiation and the derivative of $y = 2^x$.

On page 5.6, students differentiate several exponential functions to find a rule for differentiate exponential functions.



Student Solutions

8. a. ln(2) ≈ 0.69314

b. ln(*e*) = 1

c. ln(4) ≈ 1.38629

9. a. π

b. ln(2)

10. a. $y = \left(\frac{3}{2}\right)^x \Rightarrow y' = 0.40547\left(1.5\right)^x$

b. $y = 3^x \Rightarrow y' = 1.0986\left(3\right)^x$

c. $y = 4^x \Rightarrow y' = 2\ln 2\left(4^x\right) = 1.3863\left(4^x\right)$

d. $y = 0.5^x \Rightarrow y' = \ln\left(\frac{1}{2}\right)0.5^x = -\ln 2\left(0.5^x\right) = -0.69315\left(0.5^x\right)$

e. $y = 2\cdot 5^x \Rightarrow y' = 2\ln 5\left(5^x\right) = 3.2189\left(5^x\right)$

11. $\frac{d}{dx}\left(ba^{u(x)}\right) = ba^{u(x)}\ln(a)\cdot u'(x)$

12. a. $\frac{d}{dx}\left(3^{x^7}\right) = 7x^6 3^{x^7}\ln 3$ There is only a slight difference due to commutative property of multiplication.

b. $\frac{d}{dx}\left(5^{2x+3}\right) = 5^{2x+3}\ln 5\cdot(2) = 2\cdot 5^{2x}5^3\ln 5 = 2\ln(5)\cdot\left(5^2\right)^x 125 = 250\ln\left(5\right)25^x$

c. $\frac{d}{dx}\left(\left(\frac{3}{2}\right)^{2x+2}\right) = \left(\frac{3}{2}\right)^{2x+2}\ln\left(\frac{3}{2}\right)2 = 2\ln\left(\frac{3}{2}\right)\cdot\left(\frac{3}{2}\right)^{2x}\left(\frac{3}{2}\right)^2 = \frac{9}{2}\ln\left(\frac{3}{2}\right)\cdot\left(\left(\frac{3}{2}\right)^2\right)^x = \frac{9\ln\left(\frac{3}{2}\right)\cdot\left(\frac{9}{4}\right)^x}{2}$

d. $\frac{d}{dx}\left(\left(\frac{1}{2}\right)^{3x}\right) = \left(\frac{1}{2}\right)^{3x}\ln\left(\frac{1}{2}\right)\cdot 3 = 3\ln\left(2^{-1}\right)\left(\left(\frac{1}{2}\right)^3\right)^x = -3\ln 2\left(\frac{1}{8}\right)^x$

Part 4 – Extension/Homework: Exam practice questions

These questions are great practice questions for the AP exam. The questions are non-calculator questions. With the use of the CAS functionality the questions become fairly easy. The questions are set up for self-check mode, but with the Teacher Edition of the software, these can be changed to exam mode. With the TI-Nspire Navigator, these questions can be automatically graded and put into students' portfolio.