NUMB3RS Activity: Choosing Contenders
Episode: "Contenders"

Topic: Weighted averages, *z*-scores **Grade Level:** 9 - 12
Objective: Determine ratings using weighted averages of lists of data
Time: 20 - 30 minutes
Materials: TI-83 Plus/TI-84 Plus graphing calculator

Introduction

In "Contenders," the FBI asks Charlie and Amita to look at the official league rankings of each fighter and compare them with their own "rankings" which they created using the skill levels and past performances of each fighter. They find that one fighter has had an easy path to the championship. It seems that someone has arranged the fights so this fighter is always up against opponents who are weaker than he is. In this activity, students use two rankings—the strength of schedule and the power value—to determine a rating for each team in the National Football Conference (NFC) of the National Football League for the 2006 season and use the ratings to predict which teams will contend for the NFC Championship. They can check their predictions with what actually happened.

Discuss with Students

In this activity, students are given the strength of schedule (*SOS*) and the power value (*PV*) for each of the 16 teams in the NFC before the start of the 2006 season. For a given team, the *SOS* is the average of all the opponents' winning percentages in the 2005 season, and the *PV* is a rating that combines such statistics as offensive yards gained, defensive yards allowed, points scored, points allowed, win-loss record, etc. Lower *PV* values indicate a stronger team.

Students will record these data in lists L_1 and L_3 on their calculators. Because the scales in the two lists are different, the first step in constructing the ratings is to find a method to "equalize the variables" in the two lists. Two methods of transforming the data in the lists to equalize the variables are given: (1) find constants r and s so the lists $rL_1 = L_2$ and $sL_3 = L_4$ both have a mean of 1; and (2) find the *z*-scores of the elements in each list so the transformed lists of *z*-scores, by definition, both have a mean of 0 and a standard deviation of 1. The team rating is a weighted average $uL_2 + vL_4$ of the transformed lists using various choices of u and v, including the most natural choice of $u = v = 1$.

You should discuss why each of the two methods "equalizes the variables." Such methods are usually very subjective.

(1) Two variables can be regarded as equivalent if they have a common measure of center, so making the means of both data sets equal to 1 is one way of doing this. For example, multiplying each *SOS* value by the reciprocal of mean(*SOS*) gives a transformed strength of schedule (*TSOS*) with a mean of 1, because multiplying each element of a data set by a positive number k multiplies the mean (and median) by k.

(2) Another way to transform sets of data with different scales is to measure how far the values are from their means in terms of their standard deviations. For example, if set A has a mean of 10 and a standard deviation of 2, and set B has a mean of 50 and a standard deviation of 10, a score of 7 in set A (1.5 standard deviations below the mean) would be relatively higher than a score of 40 in set B (1 standard deviation below the mean). Such transformed scores are called

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z-scores. If *x* is a data value in a set of data *S*, then its *z*-score is $z = \frac{x - \text{mean}(S)}{\text{standard deviation}(S)}$ $= \frac{x - \bar{x}}{\sigma_S}$. The set of *z*-scores has a mean of 0 and a standard deviation of 1.

Student Page Answers:
1. *Opponents with an SOS < 0.5 lost more than half of their games last year.* **2.** *The magnitudes for SOS and PV are very different; PV would dominate the rating.* **3b.** *Multiply by the reciprocal of the mean, or approximately 0.056675* **4b.** *Seattle (1.607); no* **4c.** *San Francisco (2.529); no* **5a.** *Seattle (4.3704); no* **5b.** *New Orleans (5.4492); no.* **6a.** *Since the Bears had the lowest SOS, such weights must exist. One choice is u = 12, v = 1.* **6b.** *A set of weights does exist since both the SOS and PV values for San Francisco were higher than those of Detroit.* **7b.** *Seattle (–2.4895); no* **7c.** *San Francisco (1.7513); no* **8a.** *A set of weights does exist (see 6a); u = 4, v = 1 is one example of suitable weights.* **8b.** *A set of weights does not exist (see 6b).*

Name: _____ Date: _____

NUMB3RS Activity: "Choosing Contenders"

In "Contenders," the FBI asks Charlie and Amita to look at the official league rankings of each fighter and compare them with their own "rankings" which they created using the skill levels and past performances of each fighter. They find that one fighter has had an easy path all the way to the championship. It seems that someone has arranged the fights so this fighter is always up against opponents who are weaker than he is. In this activity, you will use two "rankings"—the strength of schedule and the power value—to determine a rating for each team of the National Football Conference (NFC) of the National Football League for the 2006 season and use this rating to predict which teams will contend for the NFC Championship.

The values of the two variables, the strength of schedule (*SOS*) and the power value (*PV*), are given for each of the sixteen teams in the NFC before the start of the 2006 season. For a given team, the *SOS* is the average of all the opponents' winning percentages in the 2005 season, and the *PV* is a rating that combines such statistics as offense yards gained, defense yards allowed, points scored, points allowed, win-loss record, etc. A lower *PV* indicates a stronger team. How can the values of these two variables be used to predict the NFC champion?

	Carolina	**Seattle**	**Washington**	**NY Giants**
Strength of Schedule (*SOS*)	.504	.457	.516	.543
Power Value (*PV*)	11.78	12.11	13.53	13.83

	Dallas	**Green Bay**	**Atlanta**	**Chicago**
Strength of Schedule (*SOS*)	.504	.449	.508	.445
Power Value (*PV*)	14.51	16.23	17.12	17.19

	Arizona	**Tampa Bay**	**Philadelphia**	**New Orleans**
Strength of Schedule (*SOS*)	.500	.539	.520	.539
Power Value (*PV*)	17.41	17.58	17.60	19.48

	St. Louis	**Minnesota**	**Detroit**	**San Francisco**
Strength of Schedule (*SOS*)	.508	.457	.473	.477
Power Value (*PV*)	20.96	21.80	23.52	27.66

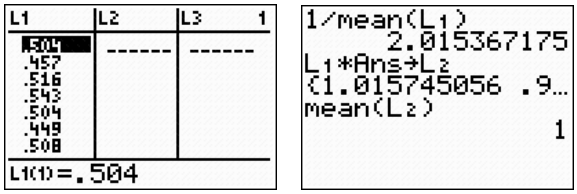
Strength of Schedule source: **http://www.fantasytailgate.com/SOS.html**
Power value source: **http://www.fanbay.net/nfl/projected/2005/power2005-2006.htm**

1. Why are lower values of *SOS* better than higher values?

2. Suppose you decide to add the values of the two variables, *SOS* and *PV*, to determine the team rating. Why is this method of finding a rating likely to be ineffective?

Because these variables have different scales, one way to "equalize the variables" is to transform the data in each set, so the mean of each transformed set of data is 1. Multiplying each *SOS* value by the reciprocal of mean(*SOS*) = 1/mean(*SOS*) gives a transformed strength of schedule (*TSOS*) with a mean of 1.

3. **a.** Using your calculator, enter the original *SOS* data in a list, L_1. Multiply this list by the reciprocal of the mean of L_1 to create another list, L_2, for transformed data set *TSOS*. Verify that the mean of *TSOS* is 1. (To find the command **mean(**, press 2nd [LIST] and go to the **MATH** menu.)



b. By what number can you multiply each *PV* value and create a transformed power value data set (*TPV*) with a mean of 1?

c. Enter the original *PV* data in a list (L_3) and create another list (L_4) for transformed data set *TPV*. Verify that the mean of *TPV* is 1.

For the 2006 season, the Chicago Bears won the NFC Championship and the Detroit Lions had the worst record for the season.

4. **a.** If the team rating *R* is given by *R* = *TSOS* + *TPV*, create a list L_5 of the team ratings.

b. Which team had the best (lowest) rating? Did this team win the NFC championship?

c. Which team had the worst (highest) rating? Did this team have the worst record for the season?

Defining the team rating to be *R* = *TSOS* + *TPV* means that both variables have equal importance. If one variable is considered more important than the other, weights can be assigned to each variable. For example, if *SOS* is thought to be four times as important as PV, then one way to express this rating is *R* = 4 • *TSOS* + *TPV*.

5. **a.** Using this method, which team had the best rating? Did this team win the NFC championship?

b. Which team had the worst rating? Did this team have the worst record for the season?

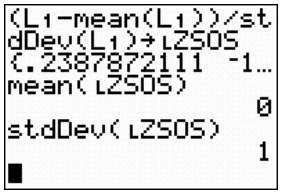
In general, if the weights used are *u* and *v*, then the rating is given by *R* = *u* • *TSOS* + *v* • *TPV*.

6. **a.** Find a set of weights (*u*, *v*) so that the rating for the Chicago Bears is the best (or explain why no such weights exist).

b. Find a set of weights (*u*, *v*) so that the rating for the Detroit Lions is the worst (or explain why no such weights exist).

Another way to transform sets of data with different scales is to measure how far the values are from their means in terms of their standard deviations. For example, if set A has a mean of 10 and a standard deviation of 2, and set B has a mean of 50 and a standard deviation of 10, a score of 7 in set A (1.5 standard deviations below the mean) would be relatively higher than a score of 40 in set B (1 standard deviation below the mean). Such transformed scores are called *z*-scores. If *x* is a data value in a set of data *S*, then its *z*-score is $z = \frac{x - \text{mean}(S)}{\text{standard deviation}(S)}$ $= \frac{x - \bar{x}}{\sigma_S}$. The set of *z*-scores has a mean of 0 and a standard deviation of 1.

7. a. Using your calculator, create a list *ZSOS* for the *z*-score of the original set of data *SOS* as shown below. To name the list, insert **L** before *ZSOS* as follows: press 2nd [LIST], go to the **OPS** menu, and select **B:L** (this will indicate that the variable name is a list). Next, create a list *ZPV* for the *z*-score of the original set of data PV using the same process. (Remember, *SOS* is stored in L_1 and *PV* is stored in L_3.) Create a third list for the team ratings *R* = *ZSOS* + *ZPV*.



b. Using this method, which team had the best rating? Did this team win the NFC championship?

c. Which team had the worst rating? Did this team have the worst record for the season?

8. a. Find a set of weights (*u*, *v*) so that the rating *R* = *u* • *ZSOS* + *v* • *ZPV* for the Chicago Bears is the best (or explain why no such weights exist).

b. Find a set of weights (*u*, *v*) so that the rating *R* = *u* • *ZSOS* + *v* • *ZPV* for the Detroit Lions is the worst (or explain why no such weights exist).

The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

For the Student

1. How could you determine the team ratings using matrix calculations?

2. What could you learn by examining the scatter plot of *SOS* vs. *PV*?

3. The data for the American Football Conference (AFC) is given in the tables below. Repeat Questions 3–8 from the student activity for the AFC data set.

	San Diego	**Denver**	**Indianapolis**	**Pittsburgh**
Strength of Schedule (*SOS*)	.488	.516	.484	.531
Power Value (*PV*)	7.95	8.02	9.48	9.86

	Kansas City	**New England**	**Jacksonville**	**Baltimore**
Strength of Schedule (*SOS*)	.527	.473	.488	.523
Power Value (*PV*)	10.21	11.50	13.01	13.21

	Cincinnati	**Miami**	**NY Jets**	**Oakland**
Strength of Schedule (*SOS*)	.543	.469	.465	.516
Power Value (*PV*)	15.56	16.87	19.00	20.48

	Buffalo	**Cleveland**	**Tennessee**	**Houston**
Strength of Schedule (*SOS*)	.477	.512	.527	.523
Power Value *(PV)*	20.52	22.19	22.52	24.71

Strength of Schedule source: **http://www.fantasytailgate.com/SOS.html**
Power value source: **http://www.fanbay.net/nfl/projected/2005/power2005-2006.htm**

4. How could you create a team rating using three variables? What might be some choices for the third variable?

5. There are many different rating systems for sports teams. Do an Internet search for such ratings for the NBA or WNBA to find some of these systems.

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