Grade level: 9-12
Subject: mathematics
Time required: 45 to 90 minutes

When is Tangent, tangent?

by – Paul W. Gosse

Activity overview

The unit circle (a circle radius 1 and centered at the origin) is a powerful tool for developing an understanding of trigonometry. This activity combines the ideas of unit circle, and a line tangent to the unit circle at 0° (in standard position), to explain how Tangent (the trig. ratio) is related to the concept of tangent to a figure (from geometry). The graph of $y = \tan(x)$ *for* $0° \le x < 90°$ *is also generated.*

The intent is to briefly explore the mathematical history of the trigonometric ratio 'tangent' through an interactive data-gathering construction, simulating an experience that may well mirror how values of trigonometric functions may once have been approximated.

Concepts

Slope, similar triangles, trigonometry, central angle, unit circle, solving angle of elevation problems.

Teacher preparation

Students should have studied the three primary trigonometric ratios in a right triangle setting and have an understanding of similar triangles.

Classroom management tips

The activity is inter-active and exploratory in nature and well-suited for students working in pairs. The activity screens are set up for handheld view and may need re-positioning in computer view. Scaling view to 200% can assist when grabbing points in the activity.

TI-Nspire Applications

Notes (with Q&A), Calculator, L&S, G&G.

Step-by-step directions

There are no precise step-by-step directions for this activity. The screens are self-explanatory. However, there are some comments given below.

The construction shown in 1.5 and used throughout the activity is limited in the sense that angle measures are $0°$ *through* $90°$*. While angles in quadrants one and four produce readable slopes in the activity, the nature of the original construction means angle values disappear in quadrants two and three. Students could be encouraged to determine those values given the existing construction.*

Similarly, the graph of $y = \tan(x)$ *is restricted to* $0° \le x < 90°$ *in terms of the scatterplot on page 2.4.*

Assessment and evaluation

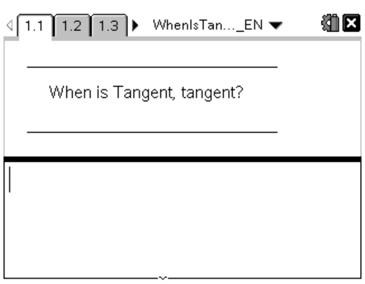
- *Since the activity is historical in nature, assessment could consist of a journal activity inquiring about what was learned about the trigonometric ratio called tangent. Students might even be encouraged to construct a working tangent construction using paper or other materials as a class project.*
- *Answers: [1.2]* $\tan(x) = \dfrac{opposite}{adjacent}$ *; [1.4] Several extensions beyond the given measures are possible; [1.6] The slope of AH appears to be equal to TH and tan(HAT); [1.7] The slope of AC and AH are the same since then related sides involved in the slope are in the same ratio due to similar triangles; [tan (HAT) is TH over AT. TH = tan(HAT). They are equal; [1.13]* $\tan(70°) \approx 2.75$ *; [1.14] They are about the same. [1.15] 2.74748…. Yes, they are about the same; [1.16] This would be the same as our previous calculation but scaled up by 10. So,* $10\tan(70°) \approx 27.5$ *m; [1.20]* $50\tan(70°) \approx 137.4$ *m; [1.21]* $50\tan(50°) \approx 59.6$ *m.*

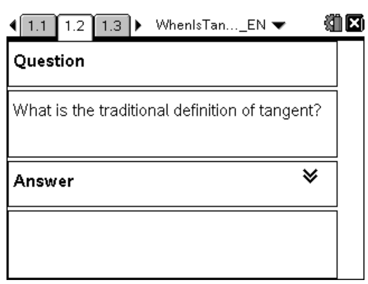
Activity extensions

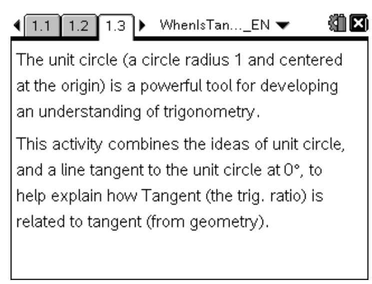
- *Students should explore what values a construction like this one yields if the rotation brings the terminal arm into each of the other quadrants. In particular, the 'ruler' could be dropped to simulate the tangent of second quadrant angles.*
- *Encourage students to discuss what happens when C is rotated to exactly* $90°$ *. (At* $90°$ *extending the terminal arm of the angle results in a parallel to the tangent. Hence the arm never intersects the 'ruler', i.e., the tangent is* $90°$ *does not exist).*
- *Students who have studied the graphs of* $y = \sin(x)$ *and* $y = \cos(x)$ *know that their fundamental period is* $360°$ *. A construction such as the one in this activity could be used to develop reasoning for why the fundamental period of tangent is* $180°$ *, or to justify why the graph of* $y = \tan(x)$ *has odd symmetry.*

Student TI-Nspire Document

WhenIsTangentTangent_EN.tns

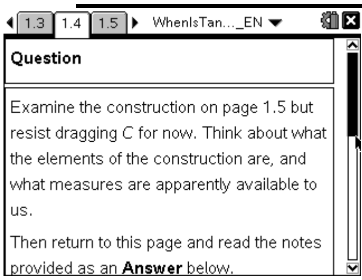


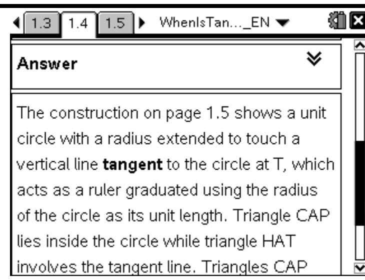


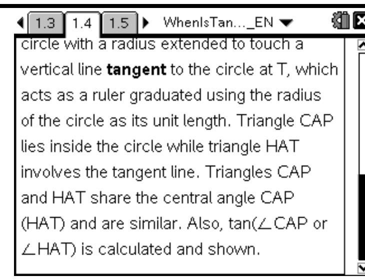


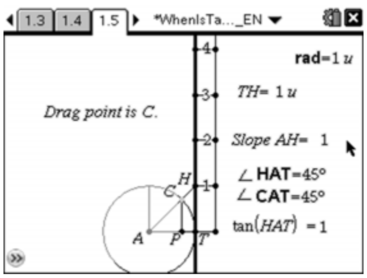
When is Tangent, tangent?
by: Paul W. Gosse
Grade level: secondary
Subject: mathematics
Time required: 45 to 90 minutes

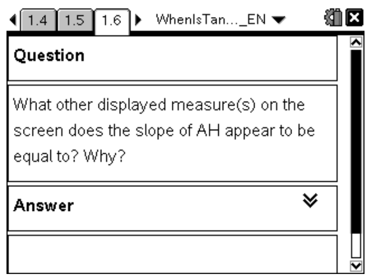
Materials: TI-Nspire

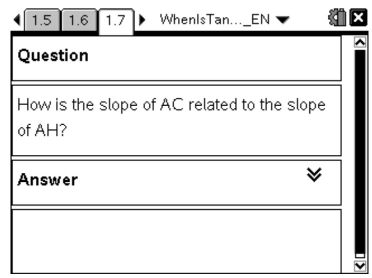


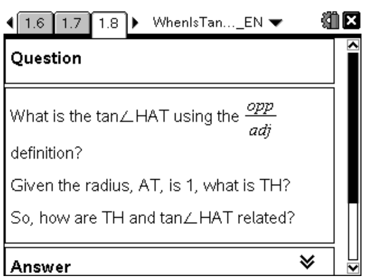


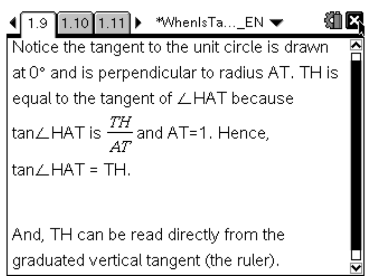


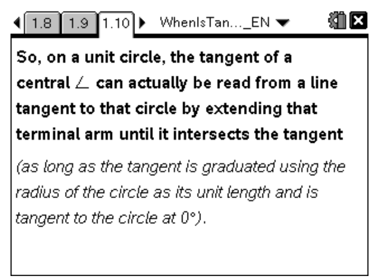


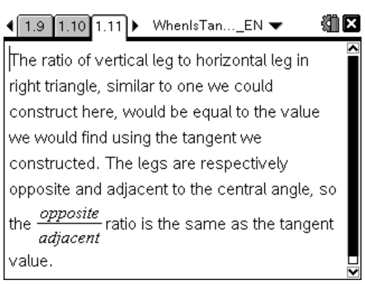


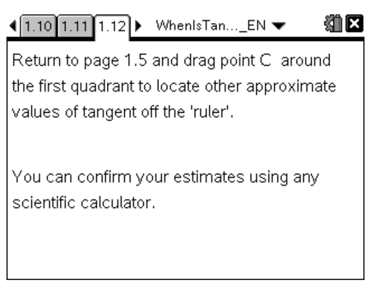


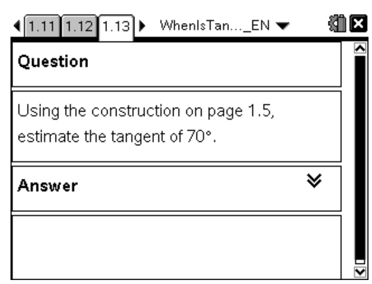












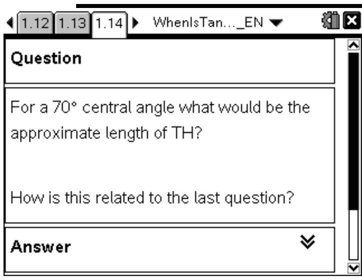
by: Paul W. Gosse

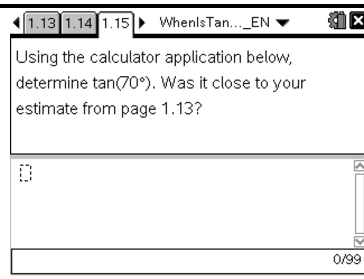
Grade level: secondary

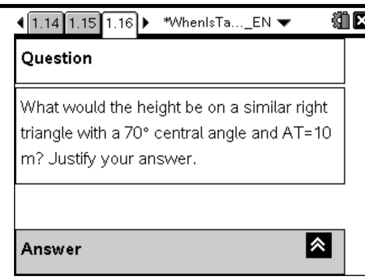
Subject: mathematics

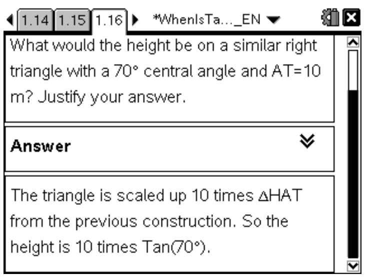
Time required: 45 to 90 minutes

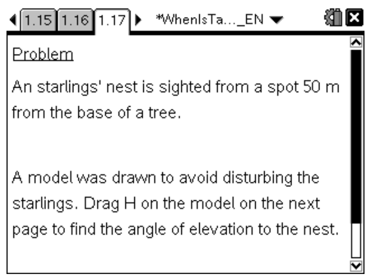
Materials: TI-Nspire

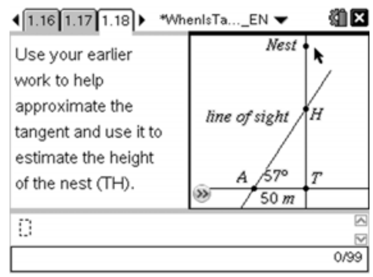


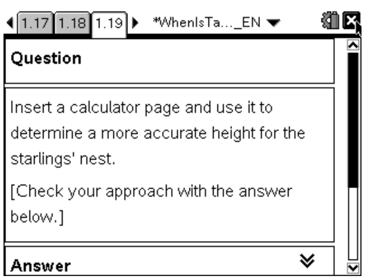


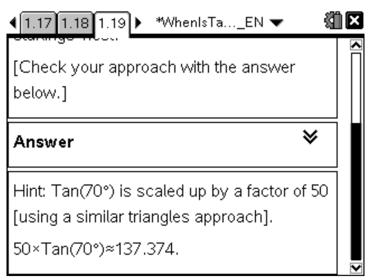


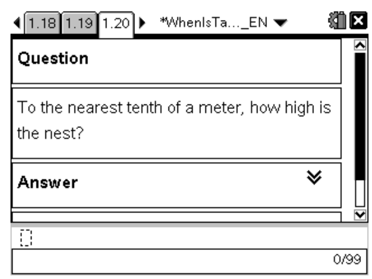


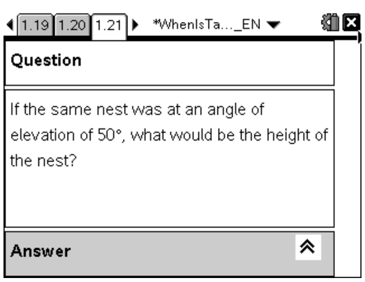


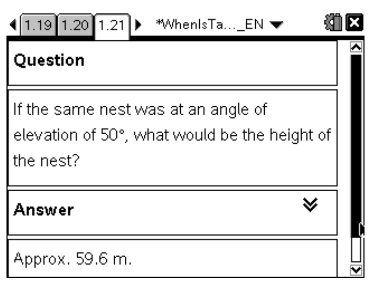


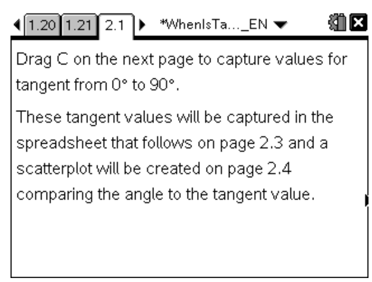












When is Tangent, tangent?

by: Paul W. Gosse

Grade level: secondary
Subject: mathematics
Time required: 45 to 90 minutes

Materials: TI-Nspire