# Factor Game

## Answers

**7 8** 9 10 11 12

TI-84CE Activity Student 90 min

## Equipment

- TI-84CE FactorGame program
- TI-84CE Factors program (optional)
- Counters

## The factor game involves two players. Instructions – How to Play

The factor game involves two players. Player one starts by selecting a number between 1 and 49 inclusive. The selected number is added to player one's score, provided the number has at least one *proper factor*[1] remaining on the board. The selected number is immediately removed from the board. Player two automatically scores the sum of all the remaining proper factors, these factors are then removed from the board. It is now player two's turn to select a number.

Player two selects a number from those remaining on the board. Player two scores this number, provided at least one proper factor remains. The selected number is immediately removed from the board. Player one scores the sum of all the remaining proper factors which are then removed from the board.

If any player selects a number that does not have any proper factors remaining on the board, the selection is deemed invalid. When a player selects an invalid number they score zero points for that turn, so too their opponent, however play is then transferred to the opponent.

The game ends when neither player can make a valid selection.

**Example:**

**Player 1:** The number 44 is selected. Proper factors of 44 are: {1, 2, 4, 11, 22}. As all of these numbers are currently on the board the selection is valid. The number 44 is immediately removed leaving player two to score: 1 + 2 + 4 + 11 + 22 = 40 points. The numbers: 1, 2, 4, 11, 22 and 44 are now all removed from play. It is now player two's turn to select a number.

**Player 2:** The number 33 is selected. Proper factors of 33 are: {1, 3, 11}, however only the number 3 remains on the board. As a proper factor is still in play the number 33 is valid so player two scores 33 points, taking their total to: 40 + 33 = 73 points. Player one scores the sum of the remaining proper factors: 3 bringing their total to: 44 + 3 = 47 points. The numbers now removed from the board include: {1, 2, 3, 4, 11, 22, 33, 44}. It is now player one's turn again.

[1] Proper Factor Example: Factors of 6: {1, 2, 3, 6}. Proper factors of 6: {1, 2, 3}. The original number is ignored when referring to proper factors. In some cases unity (1) is also ignored. For the purposes of this game, the number 1 is included as a proper factor.



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## Calculator & Board

A board has been supplied to play the game with a partner. Use counters to cover up numbers as they are selected from the board.

If you need to verify the factors of any number use the "Factors" program on the calculator.

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| 22 | 23 | 24 | 25 | 26 | 27 | 28 |
| 29 | 30 | 31 | 32 | 33 | 34 | 35 |
| 36 | 37 | 38 | 39 | 40 | 41 | 42 |
| 43 | 44 | 45 | 46 | 47 | 48 | 49 |

## Question & Discussion Points

### Question: 1.

Which number would be better to start with: 46 or 39? Justify your answer and include supporting calculations.

Player 1 selects 46. Score: 46

Player 2 gets sum of factors: Score: 1 + 2 + 23 = 26

Player 1 leads by 20 points

Alternative:

Player 1 selects 39. Score: 39

Player 2 gets sum of factors: Score: 1 + 3 + 13 = 17

Player 1 leads by 22 points

Therefore 39 is the better choice of 39 and 46 (starting).



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**Question: 2.**

Which number is the best starting number? Justify your answer and include supporting calculations.

The number 47 provides the biggest initial lead, however 49 is a better choice.

Player 1 selects 47 and scores 47 points. Player 2 only scores 1 point. When player 2 has their turn they can chose 49 for a total score of 50 points. Player 1 then scores 7 points for a total of 56.

Player 1 leads by 6 points.

Suppose player 1 selects 49 to begin the game, giving them a score of 49 points. Player 2 earns 1 + 7 = 8 points. Now it is player 2's turn, but they can no longer select 47 as there will not be any proper factors remaining on the board. The number 35 is now the next best choice leaving player 2 with a score of 35 + 8 = 43 points. Player 1 now received 5 points, being the only remaining factor for a total score of 49 + 5 = 54 points.

Player 1 leads by 9 points.

So, whilst 47 points will score the best initial score, it leaves 49 in play… it is better to pick 49 first.

**Question: 3.**

Which numbers, if any, would provide a higher score for your opponent if selected first?

Player 1 selects 12. Player 2 scores: 1 + 2 + 3 + 4 + 6 = 16 points.

Other numbers include: 12, 18, 20, 24, 30, 36, 40, 42 and 48.

These numbers are also called 'abundant' numbers.

**Question: 4.**

If you are player two, and player one selected 39 as the first number, what number should you choose? Justify your answer with supporting calculations.

If player 1 selects the number 35 first, player two will automatically score 1 + 5 + 7, so these numbers will be removed from the board. As 1 and 7 have now been removed, 49 is no longer a legitimate number, nor are any other prime numbers. The best number remaining now is 39 leaving the opponent with factors: 3 and 13. Other good numbers include 46, {2, 23} or 33, {3, 11}.

**Question: 5.**

If the number 35 is selected first, what numbers will never be removed from the board?

If the number 35 is selected first its factors {1, 5, 7} will be removed which means that 25 ($5^2$) and 49 ($7^2$) can no longer be removed from the board. Furthermore, since 1 has been removed, any prime numbers greater than 24 will be removed; {29, 31, 37, 41, 43, 47}.



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**Question: 6.**

Will the combined score of player one and two be the same for every game?

From the previous question it can be seen that depending on which numbers are chosen early in the game as to whether or not specific numbers will be removed throughout the course of the game. For example: 47, 43, 41, 37, 31 or 29 can only ever be removed in the very first turn.

## Human vs Computer

The Factor Game is also available on the calculator allowing you to compete against the calculator for the highest score. In this game 'humans' go first, very polite; however you can force the calculator to go first by selecting a '0' for your first number. The game automatically ends when there are no more valid numbers on the board, however the game can be exited prematurely by entering a number greater than 49. The aim is to beat the calculator. Scores are done automatically, however you must keep a record of each turn including factors and the factor sum.

**Question: 7.**

Play the factor game 5 times and record the scoring for each game. Who won the most, calculator or human? Were the combined scores the same each time?

Answers will vary. The combined scores will change each time as they are heavily dependent on which numbers are chosen early in the game.

**Question: 8.**

Thinking just one move in advance; and selecting the best number for each move, what is the ideal sequence of numbers if the number 49 is selected first?

| Player 1 Selection | Factor Score | Progressive Total | Player 2 Selection | Factor Score | Progressive Total | Difference |
|---|---|---|---|---|---|---|
| **49** | | 49 | | {1, 7} | 8 | 41 (P1) |
| | 5 | 54 | **35** | | 43 | 30 (P2) |
| **39** | | 93 | | {3, 13} | 56 | 26 (P1) |
| | 2 | 95 | **26** | | 82 | 24 (P2) |
| **46** | | 141 | | {23} | 105 | 23 (P1) |
| | 11 | 152 | **33** | | 138 | 22 (P2) |
| **45** | | 197 | | {9, 15} | 156 | 27 (P1) |
| | 19 | 216 | **38** | | 194 | 19 (P2) |
| **44** | | 260 | | {4, 22} | 220 | 18 (P1) |
| | 17 | 277 | **34** | | 254 | 17 (P2) |
| **28** | | 305 | | {14} | 268 | 14 (P1) |
| | 6, 21 | 331 | **42** | | 310 | 15 (P2) |



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| Player 1 Selection | Factor Score | Progressive Total | Player 2 Selection | Factor Score | Progressive Total | Difference |
|---|---|---|---|---|---|---|
| **30** | | 361 | | {10} | 320 | 20 (P1) |
| | 8, 20 | 389 | **40** | | 360 | 12 (P2) |
| **32** | | 410 | | {16} | 376 | 16 (P1) |
| | 12 | 422 | **24** | | 400 | 12 (P2) |
| **36** | | 460 | | {18} | 418 | 18 (P1) |

**Question: 9.**

Using an example, show that 'thinking only about one move in advance' is not a sufficient strategy.

The 'best' number to choose first, thinking one move in advance is 47. The player selecting this number obtains a score of 47 and their opponent scores just 1 point providing a lead of 46 points at the completion of this turn. Player 2 however can now select the number 49 giving their opponent a score of just 7 points.

Player 1: 47 + 7 = 54

Player 2: 1 + 49 = 50

Now consider if player 1 selects the number 49 first. Player 2 scores 1 and 7 which is not as good as the previous scenario as this initial move provides a lead of just 41 points. However player 2 cannot select a prime number since 1 has been removed. For player 2 the best number to choose now is 35 transferring the only factor remaining; 5. This produces scores:

Player 1: 49 + 5 = 54

Player 2: 1 + 7 + 35 = 43

Scenario two provides a bigger lead for player one even though the initial selection was not as good.



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