Probability Simulation
The Monty Hall Problem

## Student Activity

7   8   9   10   **11   12**

TI-Nspire™   Activity   Student   40 Minutes

Introduction

The probability puzzle which has come to be known as the *Monty Hall problem* is named after the original host of the American game show *Lets Make a Deal*. A solution to this puzzle was published in *Parade* magazine in 1990, sparking considerable controversy. Some mathematicians refused to believe the solution and wrote letters of complaint to the magazine.
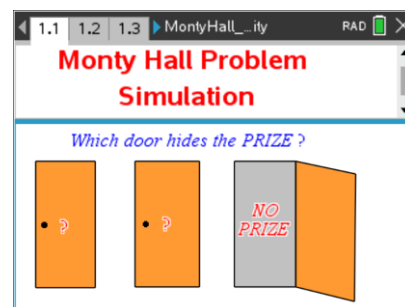
Posing the problem

Suppose that a game show includes a game in which a contestant is shown 3 closed doors.

- A prize is hidden behind one of the doors.
- '**No** prize' behind the other two doors.
- The contestant is invited to choose a door.
- The host then opens a door **not** chosen by the contestant to reveal '**No** Prize'.
- The contestant is invited to either **stay** with the initial choice or **switch** to the other closed door.
- The contestant wins the prize if they choose the door that hides the prize.

What should the contestant do to give themselves the greatest chance of winning the prize?
1. **Switch** their choice of door.
2. **Stay** with their initial choice.
3. 'Staying' or 'switching' makes no difference to the probability of winning the prize.

Question 1

Which of the three options above do you think is correct? Briefly explain why you think that option is correct.

Simulating the problem

In this activity, you will use random number, Lists and Spreadsheet, and Data and Statistics features of TI-Nspire to model the problem and observe graphical displays of the outcomes when the simulated game is repeated many times.

Assumptions in the simulation

- The door with the prize is labelled Door 1 and the doors with 'No prize' labelled Doors 2 and 3 (these are **not** necessarily the first, second and third doors, from left to right).
- If the contestant's initial choice is Door 1, the host makes a random choice between Door 2 or Door 3 to be opened.
- If the contestant's initial choice is Door 2, the host opens Door 3, and vice versa.

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- A single trial in the simulation consists of selecting a door at random (to represent the contestant's initial choice), followed by randomly selecting another door which is neither the initial choice nor Door 1. The output shows the outcome for 'staying' and for 'switching'.
- A favourable outcome consists of the contestant's final selection being the door hiding the prize.
- There are 100 trials in each pass of the simulation. These will initially be looked at one trial at a time. Later in the activity, the results of repeated passes of the simulation will be aggregated to observe the long run outcome for a very large number of trials.

Success fractions for 'stay' and 'switch' strategies

Open the TI-Nspire document '*Monty Hall_simulation*'. Navigate to page 2.3.

Page 2.3 is where the simulation calculations are taking place.

Page 2.4 shows graphically the count of favourable outcomes for the 'stay' and 'switch' strategies.

success fractions for 'stay' and 'switch' strategies, where

$$\text{success fraction} = \frac{\text{favourable outcomes}}{\text{number of trials}}.$$

Use the slider labelled 'game' to observe results one trial (game) at a time on pages 2.4 and 2.5.

To reset, edit the slider value to 'game=0' (zero).

To obtain a new set of 100 trials, increase the 'repeat' slider value by 1.

Observe change in proportion as the number of trials increase.

Question 2

Observe changes in the success fractions on page 2.5 as the number of trials (games) increases from 1 to 100.

a. Comment on the appearance and value of the success fractions as the number of trials increases.

b. Use the 'repeat' slider to select 100 new trials. Comment on comparisons between the success fraction graphs for different sets of 100 trials.

Question 3

Based on your observations in the counts and success fractions for several sets of 100 trials, comment on whether the 'stay, or the 'switch' option is more advantageous, or whether there is no advantage to either of these strategies.

Question 4

On page 2.5, click on each of the green horizontal lines.

a. What are the equations of these lines?

b. What behaviour are the success fractions for the 'stay' and 'switch' strategy showing with respect to these two lines, for any of the sets of 100 trials? What is the significance of this behaviour?
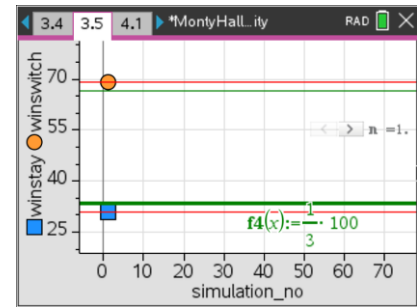
Question 5

On page 2.5, set the slider 'game=100'. The horizontal red lines show the mean (average) of the success fraction for the 100 trials. Comment on the comparison between the red and green horizontal lines for different sets of 100 trials.

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Long run proportion

Long run proportion is the ratio of favourable outcomes to the total number of trials in the experiment after conducting a **very large** number of trials.

On page 3.4 the results of repeated passes of the simulation of sets of 100 trials is captured. The results are aggregated (combined) to observe the behaviour of favourable outcomes for a very large number of trials.

On page 3.5, use the slider labelled 'n' to add new passes of the simulation of 100 trials. The blue dots on the graph represent the number of wins in 100 trials using the 'stay' strategy and the orange dots represent the number of wins in 100 trials using the 'switch' strategy. Observe the changes in the success fraction for a large number of sets of 100 trials.

To RESET: Edit slider to 'n=0'. Then, on page 3.4 select each cell with 'capture' in the formula and press <enter> + <enter>. This is a quick way to clear data.

Question 6

a. What are the equations of the two horizontal green lines on page 3.5?
b. Based on the long run proportion observed in this part of the simulation, what is your best estimate of the theoretical probability of winning the prize for the
   i. 'stay' strategy
   ii. 'switch' strategy
c. In the long run, which of the two strategies will give contestants the greater chance of winning the prize?

Question 7

Imagine that a student gave the following answer to Question 1 above.

'Staying' or 'switching' makes no difference to the probability of winning the prize. The reason is that, suppose the contestant initially chooses the middle door and the host opens the door on the left to reveal 'No prize'. There are two closed doors remaining. Therefore, whether the contestant stays with the middle door or switches to the door on the right, there is a probability of 0.5 that the prize will be behind that door.

a. Do the results of the simulation support the student's prediction that 'stay' or 'switch' are equally likely to result in winning the prize?
b. If not, explain why the prize is **not** equally likely to be behind either door after the host opens a door with 'No prize'?

Extension questions
Conditional probability and Bayes' Theorem

Question 8

The definition of conditional probability for events A and B, where $\Pr(A) \neq 0$ and $\Pr(B) \neq 0$ may be expressed as $\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$ or $\Pr(B|A) = \frac{\Pr(A \cap B)}{\Pr(A)}$.

Use the above equations to show that $\Pr(A|B) = \frac{\Pr(B|A) \times \Pr(A)}{\Pr(B)}$. This is known as Bayes' theorem.

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Question 9

Assume the following.
- The contestant chooses door 1.
- The host then opens door 2 to reveal **no** prize.
- Event A is 'the prize is behind door 1'
- Event B is 'the host opens door 2 to show **no** prize'

a. Determine the following for events A and B.

    i. $\Pr(A)$

    ii. $\Pr(B|A)$ (i.e. $\Pr$(host opens door 2 ... **given** prize is behind door 1))

b. Use the fact that there are 3 doors and that the host will **not** reveal the prize and will **not** open the door chosen by the contestant to explain why

$$\Pr(B) = \left(\frac{1}{3} \times \frac{1}{2}\right) + \left(\frac{1}{3} \times 0\right) + \left(\frac{1}{3} \times 1\right)$$

c. Use Bayes' theorem to determine the probability that the prize is behind door 1 given that the host opens door 2. That is, $\Pr(A|B)$.

d. Hence determine the probability that the contestant wins the prize if they stay with the initial choice (door 1) and the probability of winning the prize if they switch to the other closed door.