Which Note am I playing?

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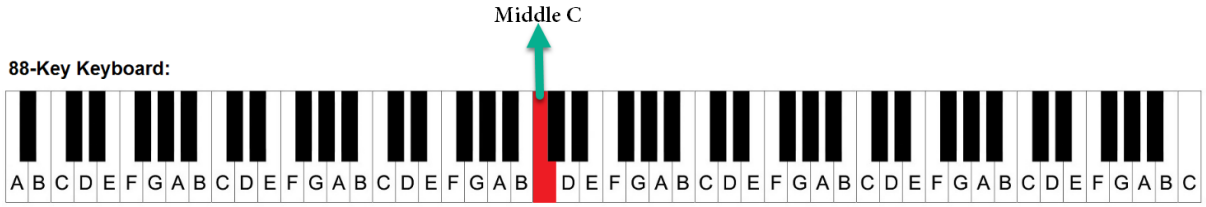
Student Worksheet

7 8 **9** **10** 11 12

TI-Nspire CXII/CXII CAS | Investigation | Student | 50 min

Which Musical Note am I playing?

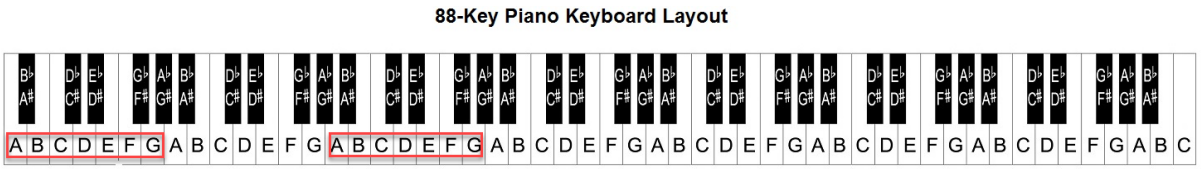
In a full Piano, there are 88 Keys. Each key plays a different note. There is a mathematical pattern associated with the note frequency as we move from left to right. The same happens with any musical instrument.



Part 1

Investigating and analysing the pattern and the note frequency.

Notes are defined by the frequency. On a Piano the key on the extreme left (1st key) plays the lowest note and the last key (extreme right plays the highest note)



Note: The first key on an 88 key piano is the A note and the last keynote is C.

Question 1: On an 88 Key Piano how many A note keys (amongst white keys) do you notice?

Answer: _____

Question 2: On an 88 Key Piano how many C note keys (amongst white keys) do you notice?

Answer: _____

Question 3: On an 88 Key Piano how many D or E or F or G note keys (amongst white keys) do you notice?

Answer: _____

Total number of white and black Keys on a full Piano

8A+8B+8C+7D+7E+7F+7G=52 White Keys

36 Black Keys (Flat and Sharp Notes) **Total 52+36=88 Keys**

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Let's Identify the pattern

The First A note (key 1) has a Note frequency of 27.5 Hz. (Hz=Hertz is the number of cycles /second)

The key numbering now includes the White and the Black keys

Question 4: Complete the Table underneath with

Key No.	Key (Note) Reference	Note Frequency in Hertz	Write a possible Recursive Pattern to determine the frequency
Key 1	1A	27.5	27.5 or $2^0 \times 27.5$
Key 13	2A	55	
Key 25			4×27.5
		220	
	7A		
	8A		

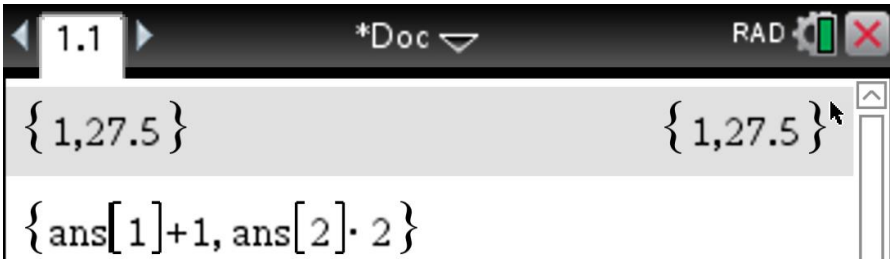
Note: The human audible range is 20Hz-20,000 Hz. The audible range reduces with age. Our audible hearing range typically reduces with age, so it is quite likely that elderly people may not hear frequencies over 12kHz.

By end of this task, we should be able to work out the frequency for the 88th Key

A similar Table can be created for (B or C or D or E or F Notes).

Try this on your TI Nspire

- Enter Line 1 in Curly Brackets (Braces) and Enter
- Enter Line 2 in Curly Brackets. It uses the answer from previous line (line 1 in this case)



Keep hitting the enter key). Do it 7 times.

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- **The First value in the output is "A Note Reference Number" A1, A2 ……..**
- **Second value is the Corresponding frequency for the A Notes 27.5, 55 …….**

Extension Task:

Try obtaining the same pattern on TI-Nspire

a. using the List and Spread sheet Application
b. Generate a sequence (using the sequence command)
c. You may try to obtain the pattern in TI-Nspire using
    i. TI-Basic
    ii. Python.

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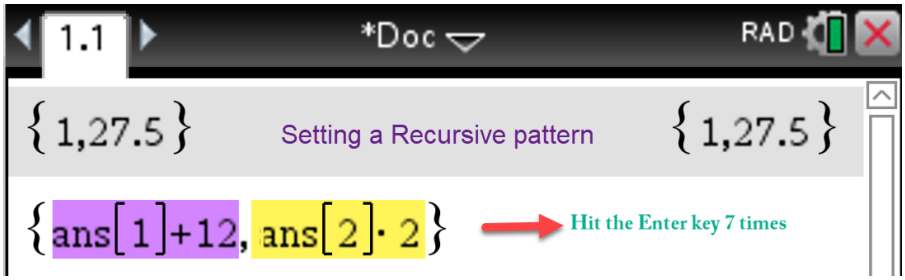
Part 2: What is Exponential Growth and what is the Exponential Pattern for Music Notes.

Introduction to Exponential Equations and Exponential Regression



This is the result

{1,27.5}	{1,27.5}
{{1,27.5}[1]+12,{1,27.5}[2]· 2}	{13,55.}
{{13,55.}[1]+12,{13,55.}[2]· 2}	{25,110.}
{{25,110.}[1]+12,{25,110.}[2]· 2}	{37,220.}
{{37,220.}[1]+12,{37,220.}[2]· 2}	{49,440.}
{{49,440.}[1]+12,{49,440.}[2]· 2}	{61,880.}
{{61,880.}[1]+12,{61,880.}[2]· 2}	{73,1760.}
{{73,1760.}[1]+12,{73,1760.}[2]· 2}	{85,3520.}

Question 1: What possibly is represented by the first of the two values in the output in each line for the 8 rows?

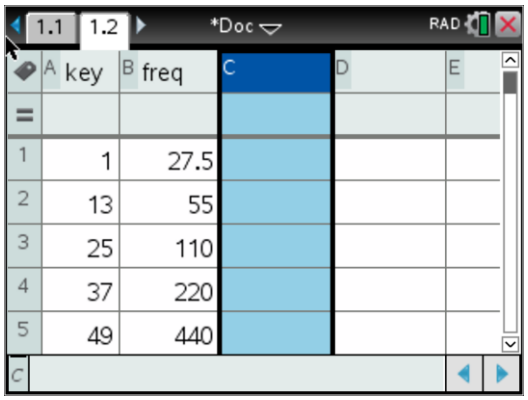
Ans:

Using the List & Spread Sheet and Data & Statistics Applications on TI-Nspire

Step 1: Enter the Values as shown below in a **List&Spreadsheet Application**

Col A: Key {0,12,24,36,48,60,72,84}

Col B: freq {27.5,55,110,220,440,880,1760,3520}



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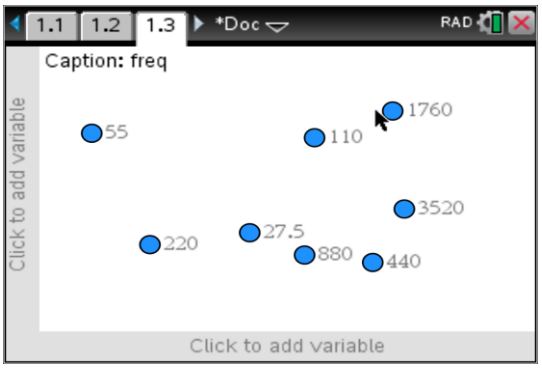
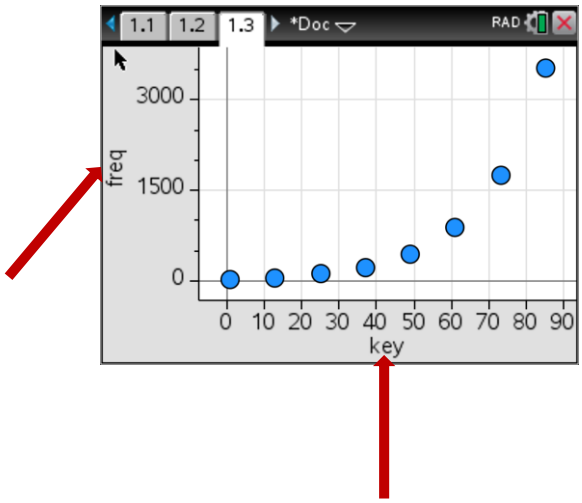
Step 2: Open the Data&Statistics Application

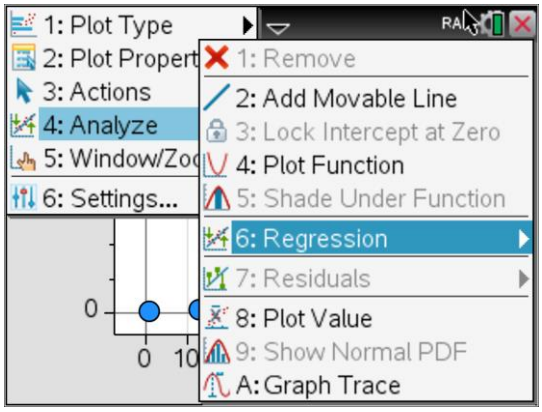


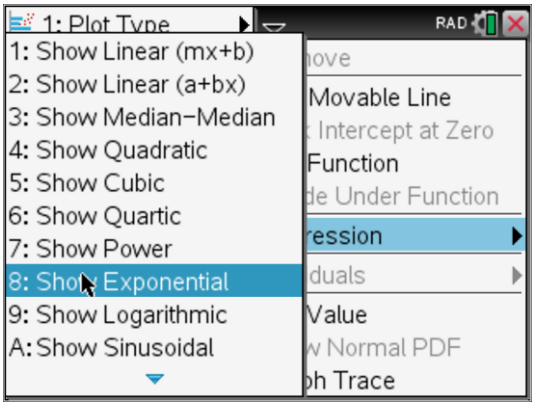
Next Click on add variable on x-axis and select key for x-axis

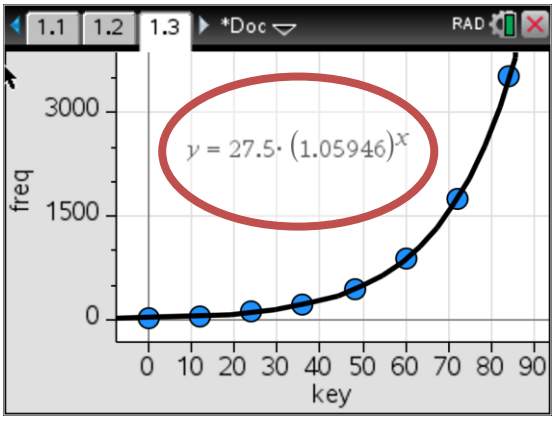
Repeat the same to add freq as variable on y-axis



Step 3: Obtaining a regression equation (Menu+Analyse) and follow the steps as under







Question 2:

i. What is the value **27.5** in the regression equation?

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ii. What will; x *input value represent* $in\ y = 27.5 \times 1.05946^x$?

Hint: we started with zero and not 1 for keys,

X= 0 represents Key 1, x=1 represents Key number 2, x=87 represents key number 88

So, the A Notes are on keys {1,13,25……85} which correspond to x= {0,12,24…….84}

iii. What information will; y *output value represent* $in\ y = 27.5 \times 1.05946^x$

when x Є {0,1,2,3…….87}

iv. Do you want to guess what the value 1.0594 may be??? You'll find the answer at the end of this worksheet.

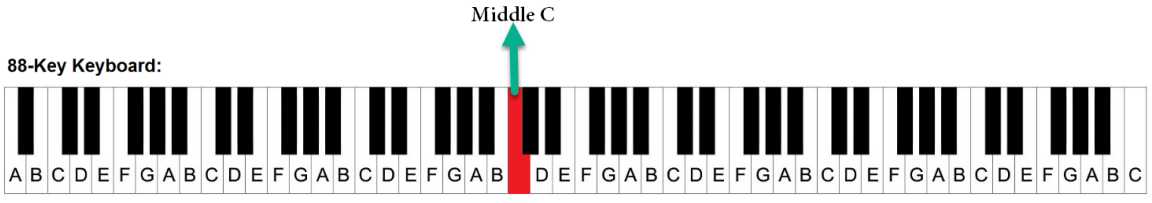
Question 3: Using the Equation $y = 27.5 \times 1.05946^x$

For this question express your answers to 2 decimal places.

a. Find the Note frequency for the 14th Key (Hint: This key is not an A Note)

b. Find the Note Frequency for the 88th Key (Last key on the Piano). Note this is a C Note

c. For Musicians Middle C is an important note. On an 88 key Piano it is the 40th Key (including white and black keys). Determine the Note frequency for the Middle C Note



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Question 4: Using the Equation $y = 27.5 \times 1.05946^{x}$

a. Complete this table for the first 12 keys (This includes the white and the Black Keys)

The lowest note on the 88 Piano key is 27.5 Hz and corresponds to A_1 (key number 1)

The table on the next page is for the first 12 keys of the Piano. You need to complete it for Keys 6-12

The table underneath is for the first 12 keys of the Piano

Note	A	A#	B	C	C#	D	D#	E	F	F#	G	G#
Key (n)	1	2	3	4	5	6	7	8	9	10	11	12
Freq in Hz	27.500	29.135	30.867	32.703	34.647							
Key Colour												

b. Using the table values state the ratio for the following to two decimal places

$\frac{Key2}{Key1} =$ ☐ $\frac{Key4}{Key3} =$ ☐ $\frac{Key12}{Key11} =$ ☐

c. Hence using the ratio value, develop a recursive pattern for two consecutive keys to obtain the frequency of Key_{n+1} in terms of Key_n (Key_n is the preceding key to Key_{n+1})

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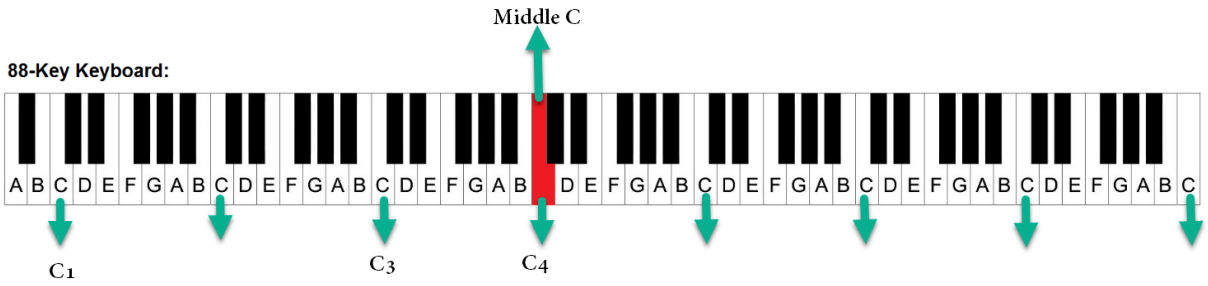
Part 3: This section is Meant for students in the Year 10 Advanced Mathematics course

Understanding Octaves and Exponential Equations



Question 1:

a. On an 88 key Piano, how many keys can play the C note?

b. What would be a quick way to Identify the C note key on a Piano in relation to the black keys?

c. Ignoring the first black key, what pattern do you observe with the black keys?

d. What will be a quick way to identify the B note on a Piano keyboard?

Octave: An octave includes 12 keys between two musical notes that have the same letter Note.

It is called an 'octave' because there are eight notes in a scale ('octo' is Latin for 'eight')

The white keys unnatural notes are assigned letters A to G. The Black Keys are assigned the letter symbol followed by a sharp (symbol) or a flat (symbol) so if we are moving from left, the black key to the right of C would be C sharp and the same black key which is also to the left of D can be classified as D flat so C sharp and D flat will have the same frequency and are the same key

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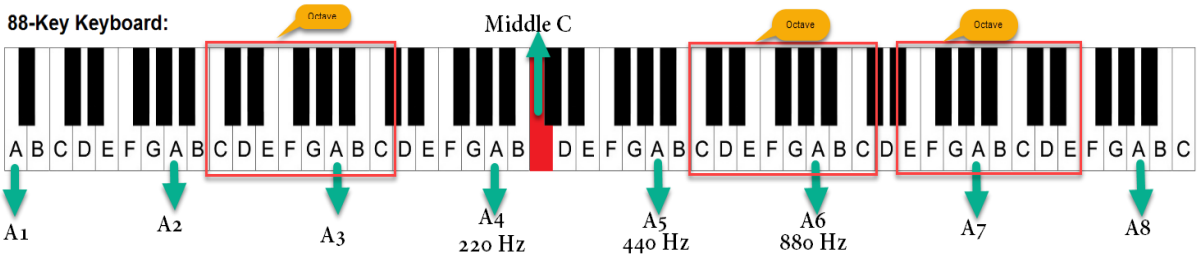
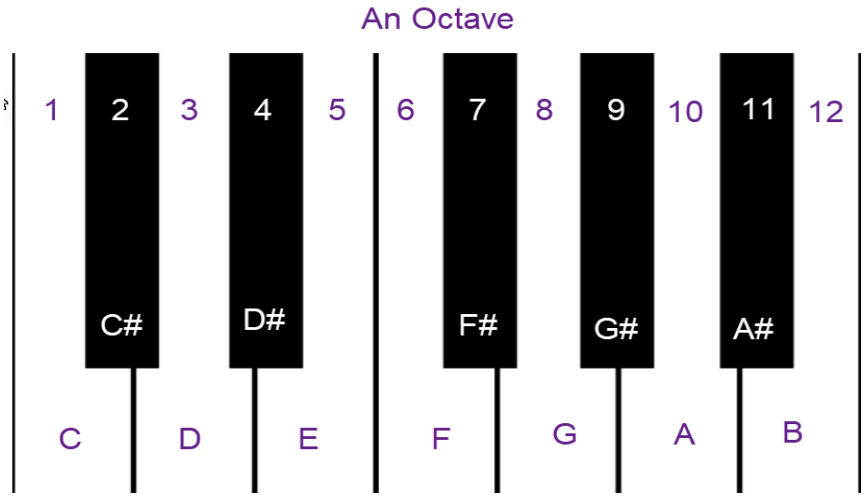
Likewise, the next black key (in the group of Black keys) will be D sharp and E Flat





Question 2: A Mathematical rule to determine the frequency of the A notes is

$A(n) = 27.5 \times 2^{n-1}; n \in Z \quad \text{and } 1 \le n \le 8$

a. Explain how this rule may have been obtained

b. Using the same Mathematical logic state a rule to obtain the frequency for all the Eight C Notes on the Piano in the form $C(n) = F \times 2^{n-1}; n \in Z \quad \text{and } 1 \le n \le 8$. You may need to obtain data values from the table you completed in the previous section

Considering the fact that there are 12 keys in an octave, we will modify the rule

$$A(n) = 27.5 \times 2^{n-1}; n \in Z \quad \text{and } 1 \le n \le 8$$

to obtain the frequency for the first 12 keys

Question3:

a. Write your rule in the form $F(n) = 27.5 \times 2^{\frac{n-1}{b}}; n \in Z$ and $1 \le n \le 12$

by assigning a numeric value to b. Explain how you obtained the value of b?

b. Modify your rule to obtain the Note frequency for all the 88 Keys on a Piano

Conclusion: Not all Keyboards have 88 Keys; hence the first key will not always be 27.5 Hz, therefore the Mathematical rule obtained by you is modified to make 440 as the principal frequency.

$$f(n) = 440\left(2^{\frac{n-49}{12}}\right) \quad ; 1 \le n \le 88$$

And the answer to the guessing question is

$2^{\frac{1}{12}}$ 1.05946

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