by: Dwight Stead

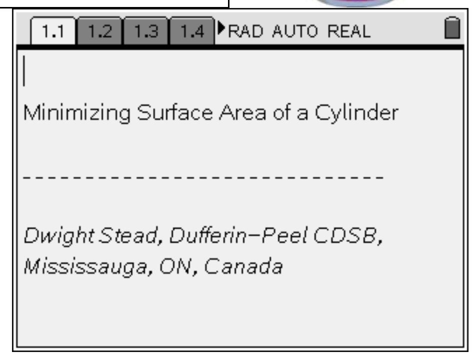
Grade level: Secondary
Subject: Mathematics
Time required: 75 minutes

Materials: Cylinder Area – Student.tns

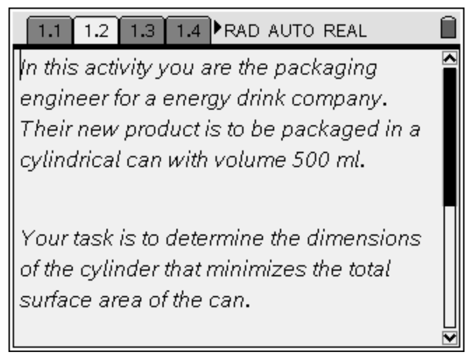
In this activity you and a partner are packaging engineers for a beverage company. Your task is to determine the dimensions of a cylindrical can for the company's new 500ml energy drink that minimizes the total surface area. You will collect data from a geometric model, estimate the minimum radius from a scatter plot and use calculus and CAS to determine the actual dimensions.

Turn on the Nspire unit.

From the **Home** screen choose **My Documents** and open the file: *Cylinder Area – Student.tns*

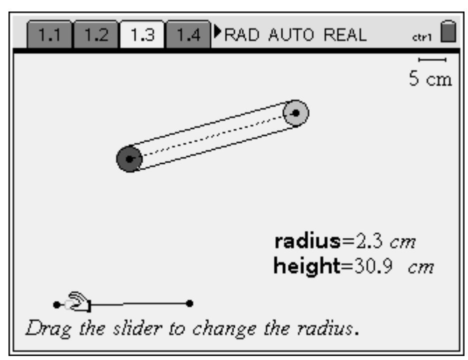


Go to page 1.2 and review the instructions for the activity.



Go to page 1.3. This construction is a geometric model of a cylinder with volume 500 ml. The slider is used to change the radius. The height of the cylinder will automatically be adjusted to keep the volume 500 ml.

Move the cursor over the white dot on the slider, the dot will pulse and the cursor changes to an open hand.



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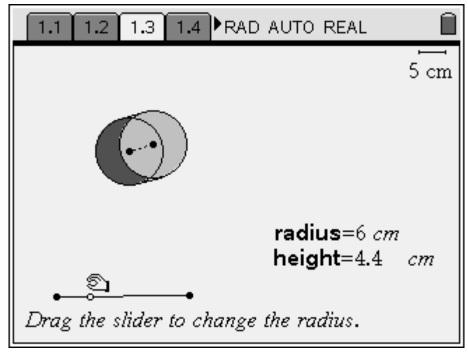
To grasp the white dot to move it, press the **NavPad** button and hold it until the hand closes.

Move the white dot slowly back and forth until you have all the possible values for the radius. The radius and corresponding height will be displayed. The geometric model will also be updated.

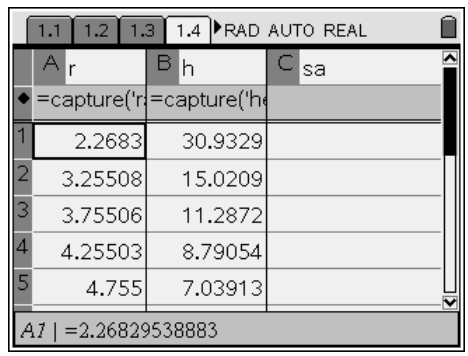
How are the radius and height related?



Go to page 1.4. As you were changing the radius a list of the radius and height pairs were collected in this spreadsheet.

You must determine a formula for the **surface area (sa)** in terms of only the **radius (r)**.

First, you need an equation for **height (h)** in terms of **radius (r)**. You know the volume is 500 ml so using the formula for volume of a cylinder, write an equation for **h** in terms of **r**.



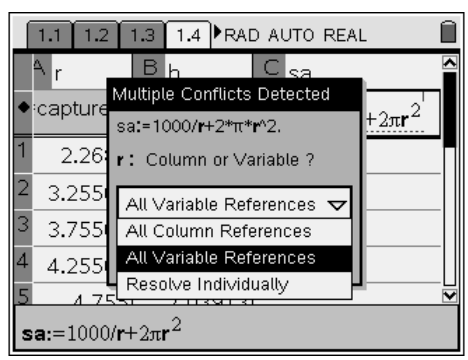
Next, write the formula for the total surface area of a cylinder and then substitute your previous equation to give an equation for **sa** in terms of **r**.

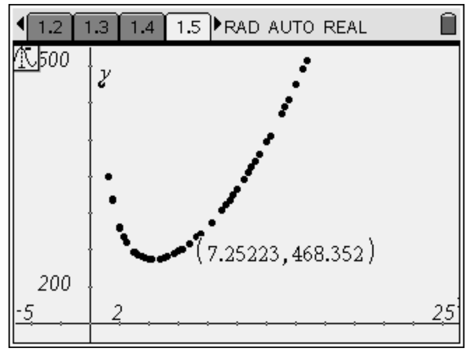
Record your equation:

Move the cursor to the cell in column C directly below the label **sa**. Enter the right side of your equation beginning with **=**.

After you press enter select **All Variable References** from the dialog box that appears by using ▼on the **NavPad**. Press enter to select **OK**.

The surface area for each pair of radius and height you gathered will be calculated and displayed in column C





Move to page 1.5 and a scatter plot of radius versus surface area appears.

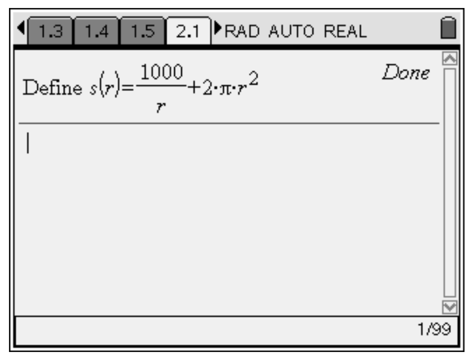
You need to estimate the radius that minimizes the surface area.

Press menu **5 1** to select *Graph Trace*. The coordinates of one of the points will be displayed. By using ◄ and ► on the **NavPad** you can move from point to point.

Determine the point that appears to minimize the surface area.

Radius = ___________________________

Surface Area = ______________________

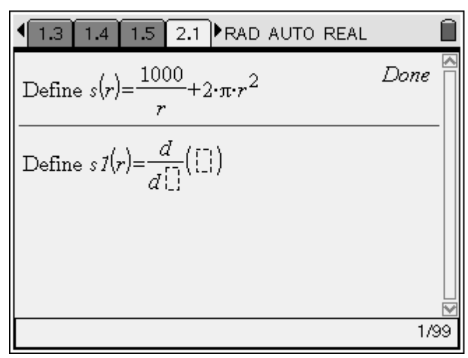


Move to page 2.1. To be sure you have the actual radius, you will use calculus and the CAS features of Nspire.

Define a function for the surface area (**s**) in terms of **r** using the equation you used earlier in the spreadsheet..

Press menu **1 1** and enter **s(r)=** followed by your equation from the spreadsheet

The function is stored.



To find the minimum surface area you must find the first derivative.

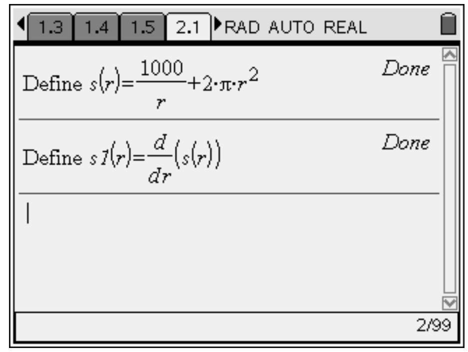
Define another function **s1(r)** to be the first derivative.

Press menu **1 1** and enter **s1(r)=**

Press menu **5 1** to insert a derivative expression**.**

In the denominator type **r** press tab and type **s(r)** and press enter.

The derivative function is stored.



To calculate the radius that minimizes the surface area you solve the derivative function for zero.

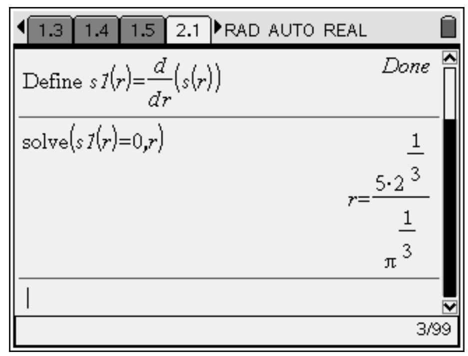
Press menu **4 1** and type **s1(r)=0, r** and press enter.

The exact radius is displayed.

To see a decimal equivalent press ctrl and then enter.

The dimensions of the cylinder that minimizes surface area are:

Radius = ___________ Height = ____________________
(*Hint*: use an earlier equation)



To calculate the minimum surface area you must substitute the radius into the surface area formula.

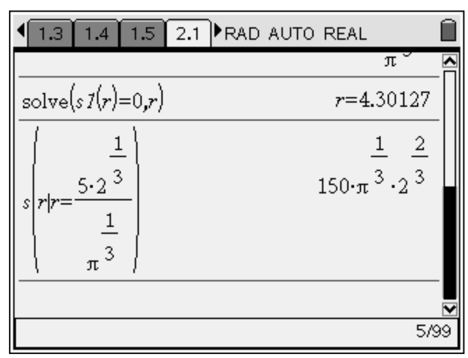
Type **s(r** | The vertical line symbol means "such that" and is a grey key found in the top row of the Nspire keyboard.

The value of **r** is on the screen from a previous calculation. To retrieve it without having to retype it, use ▲ on the **NavPad** to return to previous calculations. When you move the cursor to the equation giving the value of **r**, press enter to paste it into the expression.

Finally, press enter to evaluate the exact minimum surface area.

To see a decimal equivalent press ctrl and then enter.

The minimum surface area is: ______________________



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Reflection:

The dimensions of the can with minimum surface area have nearly equal diameter and height. Drink cans usually have a larger height than diameter. Why would companies use these dimensions even though they cost more to create?

Extension:

If the cost of materials per cm^2 for the top and bottom of the can were twice the cost per cm^2 for the lateral surface how would the dimensions change for the cylinder with minimum surface area?

Repeat the process for this new problem beginning on page 3.1.