

Law of Cosines

by – Vicki Carter

Activity overview

Students are introduced to the concept of the Law of Cosines. They will explore the concept graphically, numerically, and algebraically. They will discover the Law of Cosines at the conclusion of the activity using TI-Nspire™ CAS.

Concepts

Relationships of angles and sides in triangles

Data collection – representation and interpretation using scatter plots

Expanding trigonometric and algebraic expressions

Teacher preparation

This investigation is an introduction to the Law of Cosines. Students should be familiar with right triangle trigonometry. In the rectangular plane, they should also be able to represent coordinates in terms of trigonometric functions.

- *Download the Law of cosines.tns file.*

Classroom management tips

*The first part (Problem 1) of this activity is intended to be **student-centered** with the teacher acting as a facilitator while students work cooperatively. Students will answer the question posed on the Q&A Notes page.*

*The second part (Problem 2) of this activity is intended to be **teacher-led**. Students will follow along using their handhelds. Students will answer the questions posed on the Q&A Notes pages.*

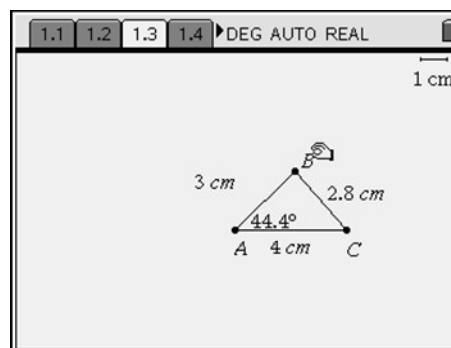
- *As all questions are posed in the .tns file, the intent of this activity is for the teacher to collect the document from the students at the conclusion of the activity. As an alternative, you may wish to have the class record their answers on a separate sheet of paper or simply use the questions posed to engage the students in a class discussion.*

TI-Nspire Applications

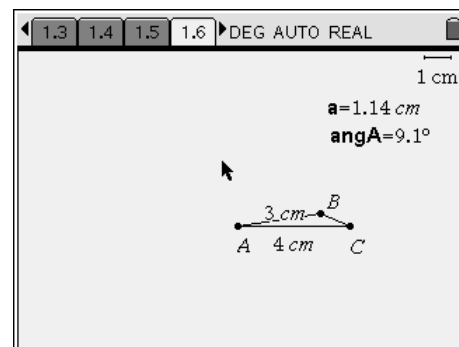
Graphs & Geometry, Lists & Spreadsheet, Notes, Notes with Q&A templates, Calculator

Step-by-step directions
Problem 1 – Investigating the Law of Cosines graphically and numerically

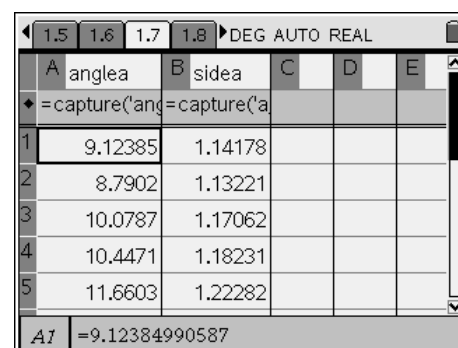
Step 1: Students should grab point B and observe the changes in angle A and side a. You may wish to discuss the types of triangles observed: acute, right, and obtuse. You might want to talk about the Pythagorean Triple: 3-4-5. Students should answer the question on page 1.4. The intended observation is that as angle A increases side a also increases.



Step 2: Students are instructed to again drag point B on the diagram on page 1.6. The figure on page 1.6 starts with a small value for angle A. Students should not try to find smaller angles. They will move point B to increase the size of angle A. Students are instructed to stay within 180° .



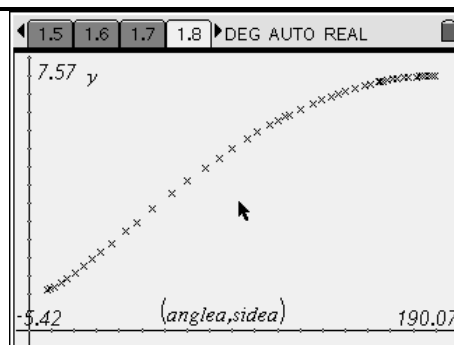
Step 3: As the students move point B, a table of values for angle A and side a are captured automatically. Here is an example of what their table should look like. At this point, you might want to look through the list of values. Although an angle of 90° might not be captured in the table, most students should have a value near 90° and they can observe that the measure of side a is close to 5. You again have the opportunity to talk about the Pythagorean Triple: 3-4-5. There is also an opportunity to observe the values of side a as angle A approaches 180° . Hopefully the students will observe that side a is getting close to 7. You might want to pose the question: "Could side a = 7?" This is an opportunity to discuss the theorem about the sum of the lengths of two sides of a triangle must be greater than the length of the third side.



| A | anglea | B | sidea | C | D | E |
|---|---------|---|---------|---|---|---|
| 1 | 9.12385 | | 1.14178 | | | |
| 2 | 8.7902 | | 1.13221 | | | |
| 3 | 10.0787 | | 1.17062 | | | |
| 4 | 10.4471 | | 1.18231 | | | |
| 5 | 11.6603 | | 1.22282 | | | |

A1 = 9.12384990587

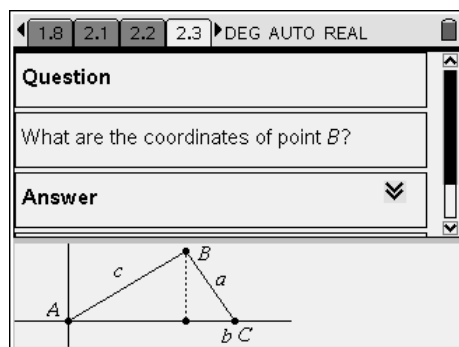
Step 4: The resulting scatter plot is produced on page 1.8. You may want to discuss the type of function that produces this graph. Later in the document, the student will actually find the equation for side a as a function of angle A.



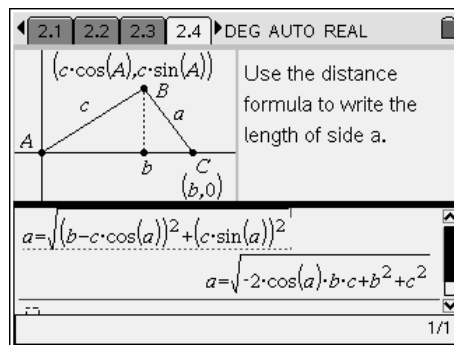
Problem 2 – Investigating the Law of Cosines algebraically

The second part (Problem 2) of this activity is intended to be **teacher-led**.

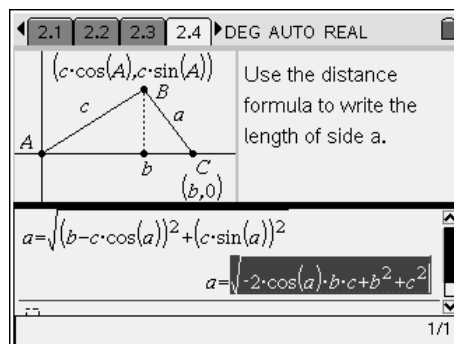
Step 5: With your directions, students should discuss the mathematics they need to use to formulate the coordinates of point B. Some review of right triangle trigonometry may be necessary. Page 2.3 gives the students a hint toward the use of right triangle trigonometry with the altitude from point B to the base of triangle ABC. Coordinates for B are $(c \cdot \cos(A), c \cdot \sin(A))$.



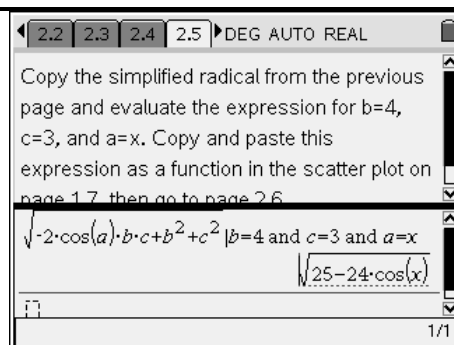
Step 6: Students are now directed to use the distance formula to write an expression for the length of side a. When the students press $\frac{\square}{\text{enter}}$, the expression is simplified. Care must be taken in placing parentheses around the expressions to be squared, to use a multiplication sign between c and the trig function, and to write the trig expressions in function form. Even if the student writes the angle as a capital A, the TI-Nspire™ CAS converts it to a lower case a. Caution students that the a in the cosine expression is the angle.



Step 7: On page 2.5, students are instructed to return to page 2.4 and to copy the expression for side a. They should not copy the equation just the radical.

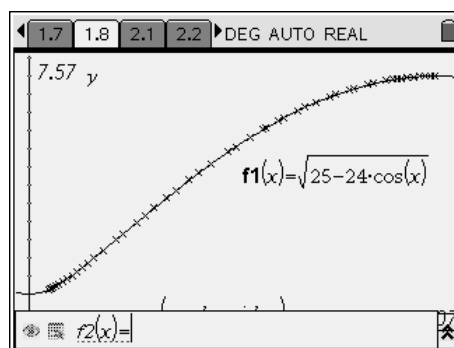


Step 8: On page 2.5, they should paste the radical expression and then evaluate it for $b=4$, $c=3$, and $a=x$. See the figure to the right for the proper notation. The students now have the function to use on the scatter plot page (page 1.7) to compare to their collected data points. Students may copy the equation and paste it in the

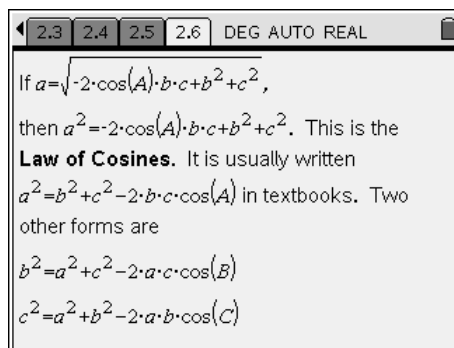


TI-84 Plus calculator screen showing the expression $\sqrt{-2 \cdot \cos(A) \cdot b \cdot c + b^2 + c^2}$ with $b=4$ and $c=3$ and $a=x$. The simplified radical expression $\sqrt{25 - 24 \cdot \cos(x)}$ is displayed below.

Step 9: On page 1.7, students will need to display the Entry Line. **MENU > 2:View > 6:Show Entry Line** or use **Ctrl+G**. You may want to discuss the closed domain, $[0, 180]$ of this function in this problem situation.



Step 10: On page 2.6, the Law of Cosines is written in the form seen in most textbooks.



TI-84 Plus calculator screen showing the Law of Cosines formula and its variations:

$$\text{If } a = \sqrt{-2 \cdot \cos(A) \cdot b \cdot c + b^2 + c^2},$$

then $a^2 = -2 \cdot \cos(A) \cdot b \cdot c + b^2 + c^2$. This is the **Law of Cosines**. It is usually written $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos(A)$ in textbooks. Two other forms are

$$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos(B)$$

$$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos(C)$$

Assessment and evaluation

The teacher could collect the document from the students at the conclusion of the activity to check for understanding. As an alternative, you may wish to have the class record their answers on a separate sheet of paper or simply use the questions posed to engage the students in a class discussion.

Student TI-Nspire Document

Law of cosines.tns

1.1 1.2 1.3 1.4 ▸ DEG AUTO REAL

The Law of Cosines

An investigation

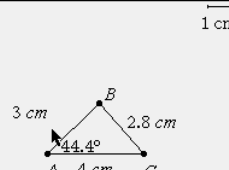
By Vicki Carter

1.1 1.2 1.3 1.4 ▸ DEG AUTO REAL

Given the lengths of two sides and the included angle of a triangle, how do we find the third side?

Let's look at some triangles. On the next page, triangle ABC has sides of length 4 cm and 3 cm. Drag point B to investigate the changes in the measured angle and the corresponding side.

1.1 1.2 1.3 1.4 ▸ DEG AUTO REAL



1.1 1.2 1.3 1.4 ▸ DEG AUTO REAL

Question

What do you notice about the length of side BC as angle A increases?

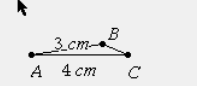
Answer

1.2 1.3 1.4 1.5 ▸ DEG AUTO REAL

On the next page, we will again drag point B. As you drag B, data will be collected. Do not produce an angle greater than 180°.

Go to page 1.7 to investigate the spreadsheet. Page 1.8 contains a scatter plot of side a with respect to angle A.

1.3 1.4 1.5 1.6 ▸ DEG AUTO REAL




1.4 1.5 1.6 1.7 ▸ DEG AUTO REAL

| A | angleA | B | sidea | C | D | E |
|---|---------|---|---------|---|---|---|
| 1 | 9.12385 | | 1.14178 | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |

A7 =9.12384990587

1.5 1.6 1.7 1.8 ▸ DEG AUTO REAL

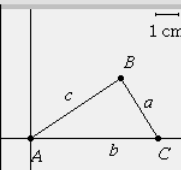


1.6 1.7 1.8 2.1 ▸ DEG AUTO REAL

Let's find an equation to find side a using the measure of its corresponding angle. On the next page, triangle ABC is placed in the coordinate plane.

1.7 1.8 2.1 2.2 ▸ DEG AUTO REAL

The coordinates of C are (b,0). Let's find the coordinates of point B in terms of angle A and side c.

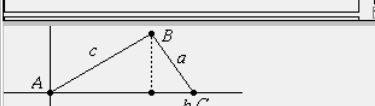


1.8 2.1 2.2 2.3 ▸ DEG AUTO REAL

Question

What are the coordinates of point B?

Answer



2.1 2.2 2.3 2.4 ▸ DEG AUTO REAL

$(c \cdot \cos(A), c \cdot \sin(A))$

Use the distance formula to write the length of side a.

a =

0.99

2.2 2.3 2.4 2.5 DEG AUTO REAL

Copy the simplified radical from the previous page and evaluate the expression for $b=4$, $c=3$, and $a=x$. Copy and paste this expression as a function in the scatter plot on page 1.7, then go to page 2.6

0/99

2.3 2.4 2.5 2.6 DEG AUTO REAL

If $a = \sqrt{-2 \cdot \cos(A) \cdot b \cdot c + b^2 + c^2}$,
 then $a^2 = -2 \cdot \cos(A) \cdot b \cdot c + b^2 + c^2$. This is the
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