Perpendicular Slopes

ID: 8246

Time required
45 minutes

Activity Overview

In this activity, students investigate the “negative reciprocal” relationship between the slopes of perpendicular lines. The final phase of the activity is appropriate for more advanced students as they are led through an algebraic proof of the relationship. Optional geometric activities use the result to verify that (1) the radius of a circle and its tangent line are perpendicular and (2) a triangle inscribed in a circle with the diameter as one side is a right triangle.

Topic: Linear Functions

- *Graph lines whose slopes are negative reciprocals and measure the angles to verify they are perpendicular.*

Teacher Preparation and Notes

This activity is appropriate for students in Algebra 1. It is assumed that students have recently been introduced to the notion of slope and perhaps the fact that two lines are parallel if and only if they have the same slope.

- *This activity is designed to have students explore* ***individually and in pairs****. However, an alternate approach would be to use the activity in a whole-class format. By using the computer software and the questions found on the student worksheet, you can lead an interactive class discussion on the slope of perpendicular lines.*
- *Some pages in the .tns file are vertically split to show both Graphs and Lists & Spreadsheet applications. Calculations that drive the implementations are hidden in the spreadsheet; and the “split” is designed to expose only Column A (or sometimes, the worksheet is completely hidden). Caution students to leave the hidden areas of the spreadsheet alone.*
- *Information for an optional extension is provided at the end of this activity; information for students is provided in Problems 5 and 6 of the student TI-Nspire document. Should you not wish students to complete the extension, you may delete these problems from the TI-Nspire document.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter “8246” in the keyword search box.***

Associated Materials

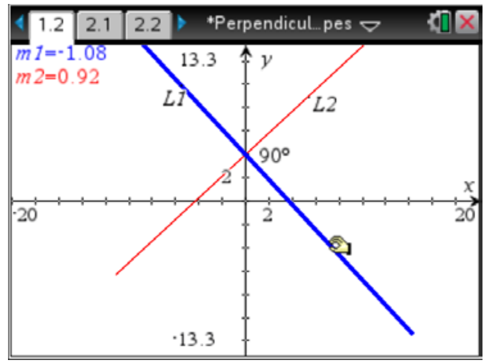
- *PerpendicularSlopes.tns*
- *PerpendicularSlopes_Student.doc*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Application of Slopes (TI-Nspire™ technology) — 13443*
- *Finding the Slope of a Line (TI-Nspire™ technology) — 13747*
- *Weight by the Foot (TI-Nspire™ technology) — 13762*

Problem 1 – An Initial Investigation



Students will first examine the relationship between the slopes of perpendicular lines by rotating line *L1* and comparing the slopes *m1* and *m2*.

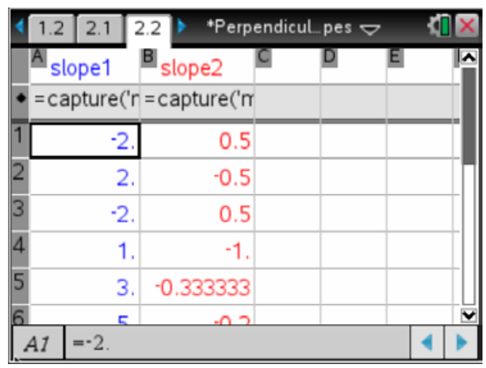
Anticipated conclusions

- If either *m1* or *m2* is equal to 0, the other is undefined.
- If either *m1* or *m2* is equal to 1, the other is –1.
- *m1* and *m2* are always opposite in sign.
- As *m1* gets larger in magnitude, *m2* gets smaller (and vice versa).

TI-Nspire™ Navigator™ Opportunity: *Live Presenter*

See Note 1 at the end of this lesson.

Problem 2 – A Closer Examination



In this setting, students can refine their conclusion by inputting specific values for *m1* and viewing the corresponding values of *m2*.

First, allow students to specify convenient slopes of their choosing (e.g., 1, 2, $\frac{1}{2}$, $\frac{1}{3}$, etc.) by enter each value into cell A1 on page 2.1.

After choosing several different values, students can view the results in the spreadsheet on page 2.2. As they are asked to conjecture an algebraic formula that relates *m1* and *m2*, encourage them to use this data as a "history" to help them visualize the relationship. Students should test their conjecture by entering their formula (in terms of the variables/ column names ***slope1*** and ***slope2***) into the gray formula cell in Column C (syntax shown in parentheses below).

Possible anticipated conjectures

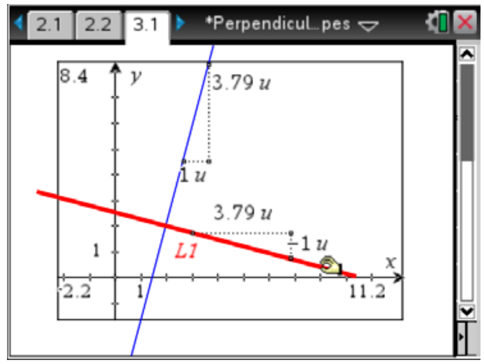
- $m1 = -\dfrac{1}{m2}$ (= –1/slope2)
- $m2 = -\dfrac{1}{m1}$ (= –1/slope1)
- $m1 \cdot m2 = -1$ (= slope1 • slope2)

TI-Nspire™ Navigator™ Opportunity: *Quick Poll*

See Note 2 at the end of this lesson.

Problem 3 – A Geometric Look

Students are provided another look at the relationship between the slopes of two perpendicular lines, represented by triangles with a hypotenuse on each line and one leg of length 1 (corresponding to the definition of slope as "rise over run").



Expected observations

- The rise/run triangles are congruent in size.

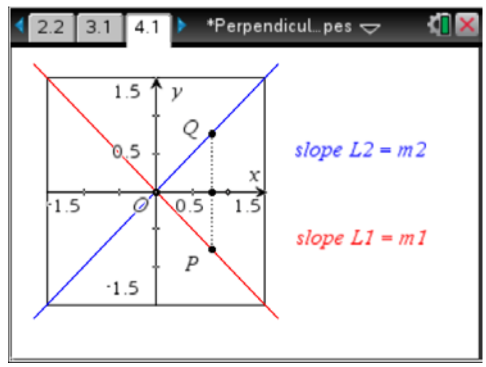
TI-Nspire™ Navigator™ Opportunity: *Class Capture*

See Note 3 at the end of this lesson.

Problem 4 – The Analytic Proof

At this point, students should have conjectured the relationship is something equivalent to $m1 \cdot m2 = -1$ (assuming neither $m1$ nor $m2$ is equal to zero).

Page 4.1 is a static diagram of two perpendicular lines passing through the origin, and the student worksheet guides them through the steps of the proof.



Solutions

11. L1: $y = (m1) \cdot x$; **L2:** $y = (m2) \cdot x$

12. $P = (1, m1)$; $Q = (1, m2)$

13. $\overline{OP} = \sqrt{1 + (m1)^2}$; $\overline{OQ} = \sqrt{1 + (m2)^2}$; $\overline{PQ} = \sqrt{(m1 - m2)^2}$

14. $\overline{PQ}$ is the hypotenuse, and all of the radicals cancel, so:

$$\left[1 + (m1)^2\right] + \left[1 + (m2)^2\right] = (m1 - m2)^2$$

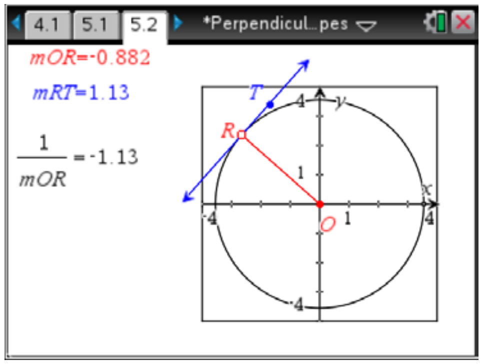
$$2 + \cancel{(m1)^2} + \cancel{(m2)^2} = \cancel{(m1)^2} - 2(m1)(m2) + \cancel{(m2)^2}$$

$$2 = -2(m1)(m2)$$

$$-1 = m1 \cdot m2$$

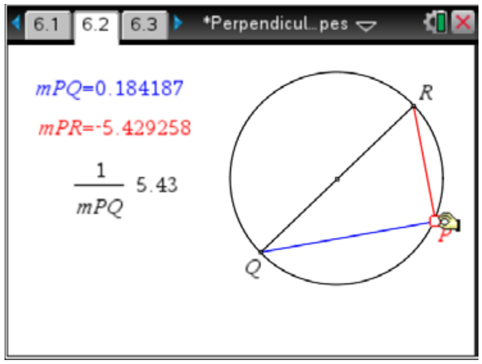
Problem 5 – Extension Activity #1

In this problem, students use the relationship between slopes of perpendicular lines to observe that the radius of a circle and the corresponding tangent line are perpendicular.



Problem 6 – Extension Activity #2

In this problem, students use the relationship between slopes of perpendicular lines to observe that a triangle inscribed in a circle such that one side of the triangle is the diameter of the circle is always a *right* triangle.



TI-Nspire™ Navigator™ Opportunities

Note 1
Question 1, *Live Presenter*
Use Live Presenter to aide in the discussion of comparing the two slopes of the perpendicular lines focusing in on the ideas of the slopes being opposite signs and if one is zero the other is undefined.

Note 2
Question 2, Quick Poll
Use Quick Poll to determine if students are developing the concept the slopes of perpendicular lines having opposite reciprocals. For Example, send students Quick polls asking for the slope of a line that is perpendicular to another line with a slope of $\frac{2}{3}$, $\frac{5}{2}$, $-\frac{6}{7}$, etc.

Note 3
Question 3, *Class Capture*
Use Class Capture to facilitate the discussion of how the rise/run triangles are reciprocals of each other with the graphed perpendicular lines.