Volume of a Football

Introduction

In this activity you will determine the volume of an AFL™ football using calculus.

Modelling the ball

Open the TI-Nspire document: Sherrin™.

The document contains a picture of an official AFL football. The axis scale has been set to accurately represent the dimensions of the ball where 1 unit on the scale corresponds to 1cm.

Two points have been plotted to help you identify the size of the ball.

Drag the yellow and blue dots to the respective edges of the ball to help determine the size.

Change the graph entry type to: Relation
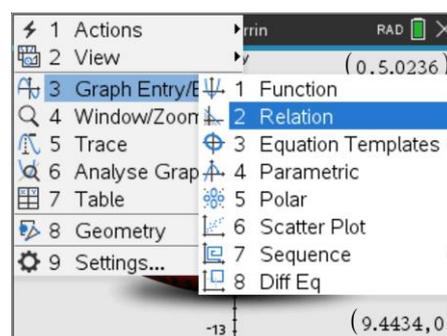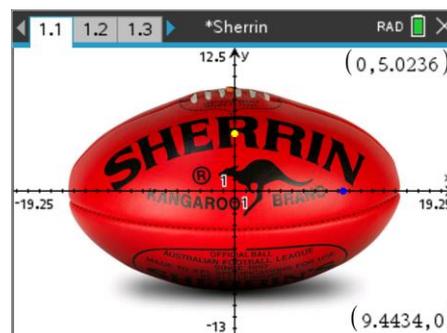
Question: 1

Use your measurements from the points to help determine an appropriate equation for the shape of the ball.

Question: 2

Use calculus to determine the volume of the ball: $V = \pi \int y^2 dx$

Question: 3

Use a practical method to measure the actual volume of a regulation size AFL football and compare the results to those obtained using calculus.

Question: 4

Express your equation for the AFL football in parametric form:

$x(t) = a \cdot \cos(t)$ and $y(t) = b \cdot \sin(t)$

Graph your equations using parametric mode to ensure they match the ball.

Author: P. Fox

TEXAS INSTRUMENTS

Question: 5

The equation for the solid of revolution in Cartesian form can easily be changed to suit parametric equations:

$$V = \pi \int_{x_1}^{x_2} y^2 \, dx = \pi \int_{t_1}^{t_2} y^2 \frac{dx}{dt} \, dt$$

a) Show that the volume is equal to: $-ab^2 \pi \int_{t_1}^{t_2} \sin(t)^3 \, dt$

b) Determine the values of t_1 and t_2.

c) Determine the volume and compare the result to the previous calculation using the Cartesian equation.

Sherrin is the official football of the AFL, they have a sizing chart on their website: https://www.sherrin.com.au/size-chart

The dimensions of a size 5 football (Game Ball) is given by two circumference measurements. The smaller circumference (55cm) is for the circular cross-section and the larger (72.5cm) for the elliptical cross section.

and the j - k plane is an ellipse.

Question: 6

Determine the size of the semi-minor axis given the small circumference is 55cm.
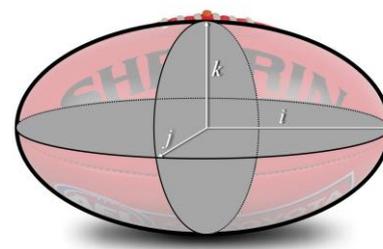
The i - j plane is an ellipse.
The j - k plane is a circle.

A "formula" does not exist for the circumference of an ellipse; however it is possible to use calculus to determine the arc length.

Cartesian form: $L = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$

Parametric form: $L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$

We know that the long circumference (arc length) for a match football is 72.5cm, what we need to determine is a more accurate measurement for the semi-major axis length.

Question: 7

Determine a more precise measurement for the semi-major axis length using the Cartesian model and arc-length formula, combined with the semi-minor axis measurement obtained in question 6.

Question: 8

Determine a more precise measurement for the semi-major axis length using the parametric model and arc-length formula, combined with the semi-minor axis measurement obtained in question 6.

Author: P. Fox

TEXAS INSTRUMENTS

3D Modelling

Insert another Graph application into your document.

Change the view to 3D.

> menu > **View** > **3D Graphing**

Once you have entered the 3D graphing mode, change to parametric.

> menu > **3D Graph Entry/Edit** > **Parametric**
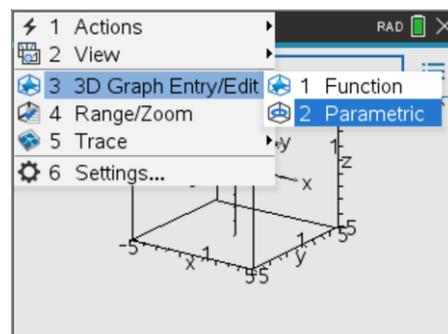
Insert a slider into your 3D graph:

> menu > **Actions** > **Insert Slider**

Set the slider variable to: **n**

Match the settings shown opposite. Also check the box for 'minimised' and uncheck the box for show variable and show scale.
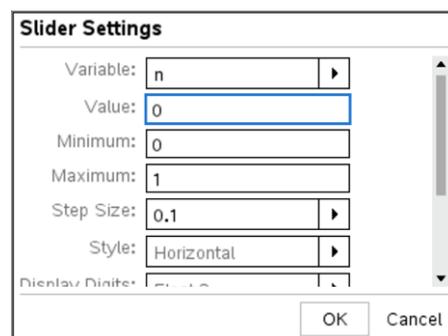
It's time to enter the equation for the graph. For this step your equation must have already been produced in parametric form in the 2D graphing page. (Same problem)

Once these expressions have been entered, the parameters need to be set. Press **TAB** to enter the parameters.

$T_{min} = 0$ $\qquad$ $T_{max} = \pi$

$U_{min} = 0$ $\qquad$ $U_{max} = 2\pi$
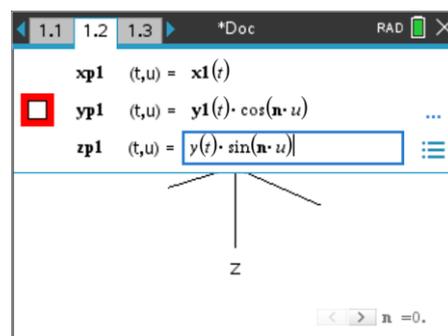
To make the view 'clearer', axis values and the 3D box can be hidden.

> menu > **View** > **Hide Box**

Similarly, with end values:

> menu > **View** > **Hide Box End Values**

The arrow keys can be used to rotate the view. Press X, Y or Z to view along a specific axis or A to auto-rotate.

Once you are happy with the display, click on the slider to see the relation rotate around the x axis, then try rotating the view!
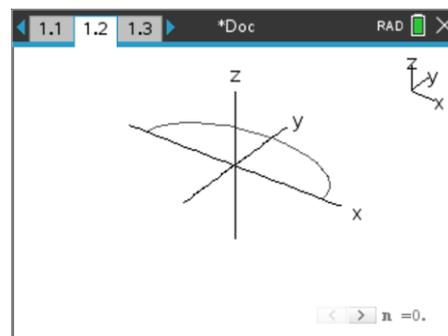
Question: 9

Sherrin have a range of football sizes on their website. Select one of the smaller sizes and determine its volume, based on the dimensions provided on the website.

TEXAS INSTRUMENTS