



# MATH AND SCIENCE @ WORK

AP\* PHYSICS Educator Edition



## LUNAR SURFACE INSTRUMENTATION

### Instructional Objectives

Students will

- add, subtract, and resolve displacement and velocity vectors to determine components of a vector along two specified, mutually perpendicular axes; and
- determine the net displacement of a particle or the location of a particle relative to another.

### Degree of Difficulty

For the average AP Physics student, this problem is at a moderate level of difficulty. The problem is a straightforward application of vector concepts.

### Class Time Required

This problem requires 40-50 minutes.

- Introduction: 5-10 minutes
  - Read and discuss the background section with the class before students work on the problem. This background is identical to *Training for a New Spacecraft: Moment of Inertia*.
- Student Work Time: 30 minutes
- Post Discussion: 5-10 minutes

### Background

*This problem is part of a series of problems that apply math and science principles to human space exploration at NASA.*

Exploration provides the foundation of our knowledge, technology, resources, and inspiration. It seeks answers to fundamental questions about our existence, responds to recent discoveries and puts in place revolutionary techniques and capabilities to inspire our nation, the world, and the next generation. Through NASA, we touch the unknown, we learn and we understand. As we take our first steps toward sustaining a human presence in the solar system, we can look forward to far-off visions of the past becoming realities of the future.

**Grade Level**  
11-12

**Key Topic**  
Vector Addition

**Degree of Difficulty**  
Moderate

**Teacher Prep Time**  
5 minutes

**Class Time Required**  
40-50 minutes

**Technology**  
TI-Nspire™ Handhelds

**Learning Objectives for AP Physics**  
Newtonian Mechanics:  
-Kinematics

**NSES Science Standards**  
- Science and Technology  
- History and Nature of Science

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Outpost concepts are now being designed and studied by engineers, scientists, and sociologists to facilitate long-duration human missions to the surface of the Moon or other planetary bodies (Figure 1). Such outposts will include habitat modules, laboratory modules, power systems, transportation, life support systems, protection from the environment, communications for planetary surface operations, and communications back to Earth.

During past and current space missions, astronaut activity outside of the vehicle (e.g. space shuttle) is referred to as an extravehicular activity, or EVA. In a similar way, extrahabitat activities, or EHA, will be performed during a mission to accomplish exploration work. One EHA may be to place environmental sensors and instruments within the proximity of an outpost (Figure 2).

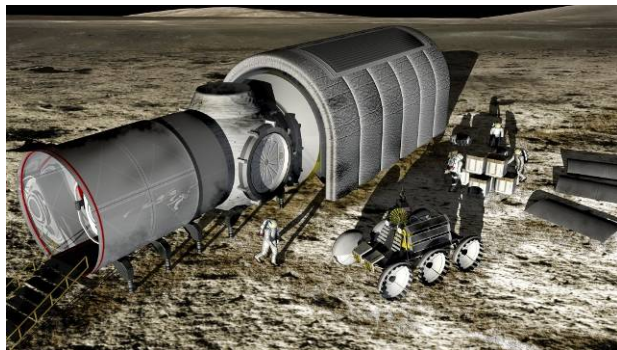


Figure 1: Habitat, airlock, and vehicles (NASA concept)



Figure 2: An astronaut services a surface instrument (NASA concept)

Such instruments may measure the radiation received from solar flares or characterize micrometeorites impacting the surface. Telescopes may also be set up for observations of Earth, other planets, and stars.

## Learning Objectives for AP Physics

### Newtonian Mechanics

- Kinematics
  - Vectors, coordinate systems
  - Motions in two dimensions

## NSES Science Standards

### Science and Technology

- Abilities of technological design

### History and Nature of Science

- Science as a Human Endeavour

## Problem and Solution Key (One Approach)

Students are given the following problem information within the TI-Nspire™ document, *Lunar Surface Instrumentation.tns*. The questions are embedded within the TI-Nspire™ document.

An astronaut services three instruments on the relatively flat lunar surface around an equatorial lunar outpost. She starts at the lunar habitat airlock and walks 180 meters southwest to replace the sample cell in the first instrument. She then walks 140 meters due north to add a lens to a second instrument. She finishes the task by walking 100 meters 30 degrees north of east where she resets the pointing of a third instrument. The astronaut walks directly back to the same habitat airlock and reenters the



habitat module. Using a Cartesian coordinate system with the x-axis pointed east and y-axis pointed north, determine the following information for her EHA. Round all answers to one decimal place.

- A. Determine the astronaut's displacement vector (distance and direction) from the airlock when she is standing at each instrument. Include a sketch of the path taken by the astronaut.

**Step 1:** Sketch the path taken by the astronaut.

Place the coordinate system origin at the airlock door.

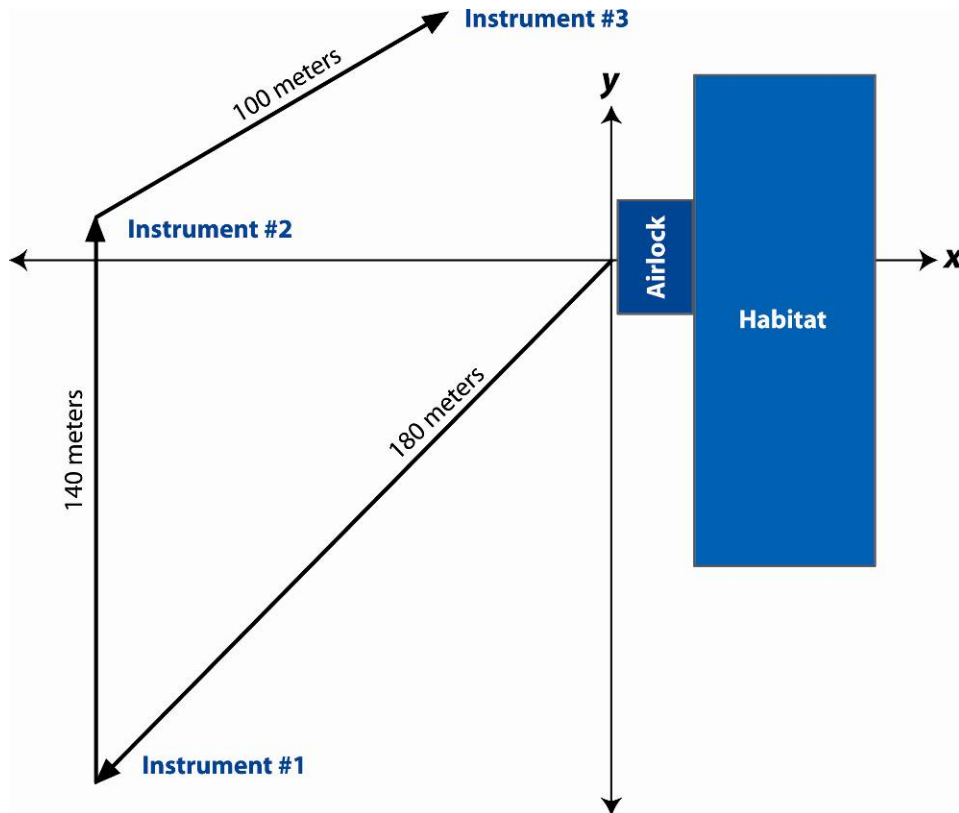
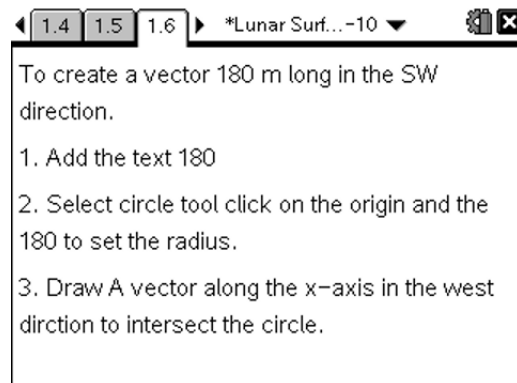
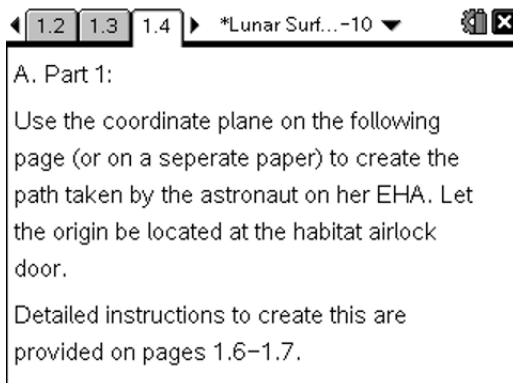


Figure 3: Locations of the lunar surface instruments

Students are also given instructions on creating this path within the TI-Nspire™ document.

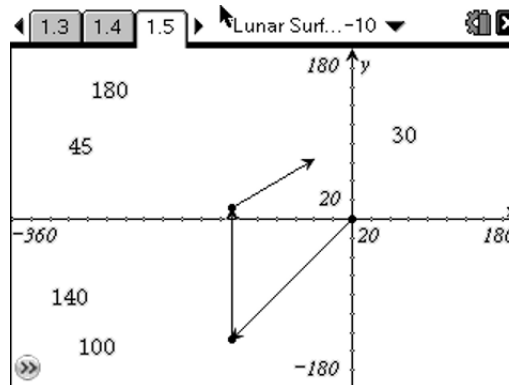




1.5 1.6 1.7 \*Lunar Surf...-10

Rotate the vector to the appropriate position

- Select the text tool and enter the angle, in this case 45.
- Select the transfer tool and rotation.
- Click on the vector you would like rotated, then the point to rotate around and then the angle of rotation.



**Step 2:** Determine the (x, y) components of each leg of the trip.

First Leg:

$$x = 180 \cos(225^\circ) = -127.3 \text{ m}$$

$$y = 180 \sin(225^\circ) = -127.3 \text{ m}$$

Second Leg:

$$x = 140 \cos(90^\circ) = 0.0 \text{ m}$$

$$y = 140 \sin(90^\circ) = 140.0 \text{ m}$$

Third Leg:

$$x = 100 \cos(30^\circ) = 86.6 \text{ m}$$

$$y = 100 \sin(30^\circ) = 50.0 \text{ m}$$

1.6 1.7 1.8 \*Lunar Surf...-10

$x1 = 180 \cdot \cos(225)$   
-127.279

$y1 = 180 \cdot \sin(225)$   
-127.279

$x2 = 140 \cdot \cos(90)$   
0.

$y2 = 140 \cdot \sin(90)$   
140.

13/13

instrument 1  
x = -127.3 m  
y = -127.3 m

instrument 2  
x = -127.3 m  
y = 12.7 m

instrument 3  
x = -40.7 m  
y = 62.7 m

Suggested Response:

**Step 3:** Determine the (x, y) position of each instrument.

Add successive legs of the trip as vector components.

First leg from the origin to instrument #1:

$$x = 0.0 \text{ m} + (-127.3 \text{ m}) = -127.3 \text{ m}$$

$$y = 0.0 \text{ m} + (-127.3 \text{ m}) = -127.3 \text{ m}$$

Instrument #1 location: (-127.3, -127.3) m

Second leg from instrument #1 to instrument #2:

$$x = -127.3 \text{ m} + 0.0 \text{ m} = -127.3 \text{ m}$$

$$y = -127.3 \text{ m} + 140.0 \text{ m} = 12.7 \text{ m}$$

Instrument #2 location: (-127.3, 12.7) m

1.6 1.7 1.8 \*Lunar Surf...-10

$x3 = 100 \cdot \cos(30)$   
86.6025

$y3 = 100 \cdot \sin(30)$   
50

$i2x = x1 + x2$   
-127.279

$i2y = y1 + y2$   
-127.279

13/13

instrument 1  
x = -127.3 m  
y = -127.3 m

instrument 2  
x = -127.3 m  
y = 12.7 m

instrument 3  
x = -40.7 m  
y = 62.7 m

Suggested Response:



Third leg from instrument #2 to instrument #3:

$$x = -127.3 \text{ m} + 86.6 \text{ m} = -40.7 \text{ m}$$

$$y = 12.7 \text{ m} + 50.0 \text{ m} = 62.7 \text{ m}$$

Instrument #3 location:  $(-40.7, 62.7) \text{ m}$

The screenshot shows a window titled '\*Lunar Surf...-10'. On the left, there are three rows of calculations:  $i2x = x1 + x2$  with a result of -127.279,  $i2y = y1 + y2$  with a result of 12.7208, and  $i3x = x1 + x2 + x3$  with a result of -40.6767. Below these is  $i3y = y1 + y2 + y3$  with a result of 62.7208. On the right, a list shows the coordinates for three instruments: instrument 1 (x=-127.3 m, y=-127.3 m), instrument 2 (x=-127.3 m, y=12.7 m), and instrument 3 (x=-40.7 m, y=62.7 m). The bottom of the window shows 'Suggested Response:'.

- B. Determine the astronaut's displacement (using unit-vector notation) from the airlock when she is standing at each instrument.

Since the three vectors are from the origin to the three instruments, convert the position  $(x,y)$  values to vector notation using unit vectors.

Instrument #1:  $(-127.3i - 127.3j) \text{ m}$

Instrument #2:  $(-127.3i + 12.7j) \text{ m}$

Instrument #3:  $(-40.7i + 62.7j) \text{ m}$

The screenshot shows a window titled '\*Lunar Surf...-10'. The main text area contains the question: 'B. Determine the astronaut's displacement (using unit-vector notation) from the airlock when she is standing at each instrument.' Below this, three vector expressions are entered:  $(-127.3 i + -127.3 j)m$ ,  $(-127.3 i + 12.7 j)m$ , and  $(-40.7 i + 62.7 j)m$ . The bottom of the window shows 'Suggested Response:' and a text input field with the placeholder 'Type suggested response here (optional)'.

- C. Determine the astronaut's displacement from the first instrument and the third instrument.

**Step 1:** Find the distance between instruments #1 and #3 using the distance formula.

$$d = \sqrt{(y_3 - y_1)^2 + (x_3 - x_1)^2}$$

$$d = \sqrt{(62.7 - (-127.3))^2 + ((-40.7) - (-127.3))^2}$$

$$d = 208.8 \text{ m}$$

The screenshot shows a window titled '\*Lunar Surf...-10'. On the left, calculations show  $dx = -40.7 - -127.3 = 86.6$ ,  $dy = 62.7 - -127.3 = 190$ , and  $d = \sqrt{(86.6)^2 + 190^2} = 208.805$ . Below this, the angle is calculated as  $\theta = \tan^{-1} \left( \frac{i3y - i1y}{i3x - i1x} \right)$ . On the right, the question text is visible: 'C. Determine the astronaut's displacement from the first instrument and the third instrument.' Below the question, the answers  $d = 208.8 \text{ m}$  and  $\theta = 24.5^\circ$  are entered. The bottom of the window shows 'Suggested Response:'.



**Step 2:** Find the angle between instruments #1 and #3 using the inverse tangent function.

$$\angle \theta = \tan^{-1} \left( \frac{x_3 - x_1}{y_3 - y_1} \right)$$

$$\angle \theta = \tan^{-1} \left( \frac{-40.7 - (-127.3)}{62.7 - (-127.3)} \right) = 24.5^\circ$$

Instrument #3 is 208.8 m and 24.5° east of north from instrument #1.

The screenshot shows a problem window with the following content:

- Problem text: "C. Determine the astronaut's displacement from the first instrument and the third instrument."
- Input field:  $d = \sqrt{(86.6)^2 + 190^2}$
- Result:  $d = 208.805$
- Input field:  $\theta = \tan^{-1} \left( \frac{i3x - i1x}{i3y - i1y} \right)$
- Result:  $\theta = 24.5036$
- Answer field:  $d = 208.8 \text{ m}$
- Answer field:  $\theta = 24.5^\circ$
- Suggested Response: (empty)

D. Determine the distance she walked from the third instrument to the habitat airlock.

$$d = \sqrt{(y_0 - y_3)^2 + (x_0 - x_3)^2}$$

$$d = \sqrt{(0 - 62.7)^2 + (0 - (-40.7))^2}$$

$$d = 74.8 \text{ m}$$

The screenshot shows a problem window with the following content:

- Problem text: "D. Determine the distance she walked from the third instrument to the habitat airlock."
- Input field:  $d_{air} = \sqrt{(0 - i3x)^2 + (0 - i3y)^2}$
- Result:  $74.7562$
- Answer field:  $74.8 \text{ m}$
- Suggested Response: Type suggested

E. Determine the total distance she traveled on her EHA.

First Leg: 180.0 m

Second Leg: 140.0 m

Third Leg: 100.0 m

Fourth Leg: 74.8 m

Sum of the four individual leg lengths:

$$d = 180.0 + 140.0 + 100.0 + 74.8 = 494.8 \text{ m}$$

The screenshot shows a problem window with the following content:

- Problem text: "E. Determine the total distance she traveled on her EHA."
- Input field:  $d_{total} = 180 + 140 + 100 + 74.8$
- Result:  $494.8$
- Answer field:  $494.8 \text{ m}$
- Suggested Response: Type suggested response here (optional)

F. Why is it important to use vector analysis for this solution?

Answers will vary, but should include that vector analysis is an optimum way to graphically display the location of the instruments and their positions in respect to the airlock.



## Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school AP Physics instructors.

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