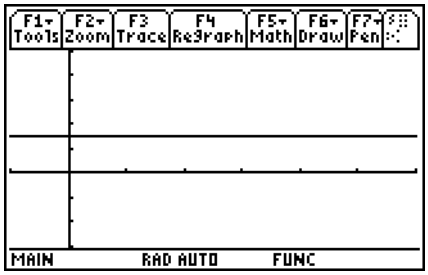
Introduction to the Fundamental Theorem

Name _____

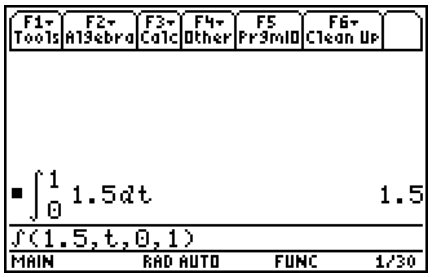
Class _____

Problem 1 – Constant Integrand

Suppose you have the function $y = 1.5$ as seen at the right. How will the area under the curve change as we go from 0 to x? Find the area of the by evaluating the definite integral $\int_0^x 1.5dt$.



For each value of x, you are looking at a rectangle with x for the length and 1.5 for the height.



1. Use the **Integrate** command (**HOME > F3:Calc > 2:Integrate**) to complete the table.

x	$\int_0^x 1.5dt$
1	
2	
3	
4	
5	

2. If $x = 0$, what is $\int_0^x 1.5dt$? Why?

3. For every 1 unit that x changes, how much does $\int_0^x 1.5dt$ change?

4. If you were to graph the ordered pairs $\left(x, \int_0^x 1.5dt\right)$, what would the graph look like?

Use the **Stats/List Editor** to enter the data in the table above into *list1* and *list2*. Then plot the data.

5. What does your graph look like? Was this graph what you predicted in Question 4?

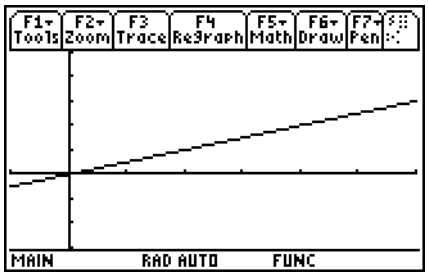
6. If you changed the integrand from 1.5 to 0.5, what would the graph of $\left(x, \int_0^x 0.5dt\right)$ look like?

Problem 2 – Non-Constant Integrand

Suppose you have the function $y = \frac{x}{2}$ as seen below. How will the area under the curve change as you go from 0 to *x*? Find the area of the triangle by hand or by evaluating the definite integral $\int_0^x \frac{t}{2} dt$.

7. Complete the table.

x	$\int_0^x \frac{t}{2} dt$
1	
2	
3	
4	
5	



8. If $x = 0$, what is $\int_0^x \frac{t}{2} dt$? Why?

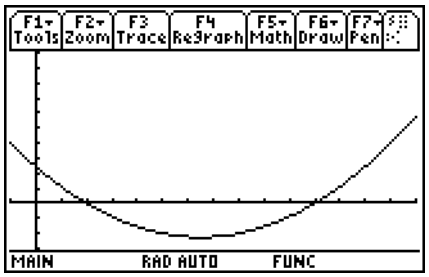
9. Explain why, when *x* increases by 1, the value of $\int_0^x \frac{t}{2} dt$ does not increase by the same amount every time?

10. Is the graph of $\left(x, \int_0^x \frac{t}{2} dt \right)$ linear? Explain.

Problem 3 – An Integrand That Changes Sign

In the previous exercises, the function was positive over the interval. This time you are going to examine a function which changes sign, $y = \frac{x^2 - 13x + 22}{9}$. How will the area under the curve change as we go from 0 to *x*? Find the area of the by evaluating the definite integral $\int_0^x \frac{t^2 - 13t + 22}{9} dt$. Complete the table.

x	$\int_0^x \frac{t^2 - 13t + 22}{9} dt$
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	



11. At what value of *x* does the integral's value begin to decrease?

12. a. What are all the values of *x* for which the definite integral's value is decreasing?

b. What is true at these values of *x*?

13. a. What are all the values of *x* for which the integral's value is increasing?

b. What is true of the integrand at these values of *x*?

14. a. What is the smallest value of the integral, and at what value of *x* is this reached?

b. What happens with the integrand at this value of *x*?

15. Is the connection between the location of the minimum value of $\int_0^x \frac{t^2 - 13t + 22}{9} dt$ and the sign change of the integrand from negative to positive one you that you have seen before? If so, in what context?