EXPLORATIONS

Activity 9

Pick and Choose! Using Pick's Theorem!

Objective

- To find the area of any geoboard polygon using Pick's Theorem

Materials

- TI-73
- Student Activity pages (pp. 97 – 106)

In this activity you will

- Discover Pick's Theorem.
- Use Pick's Theorem to find the area of a variety of geoboard shapes.

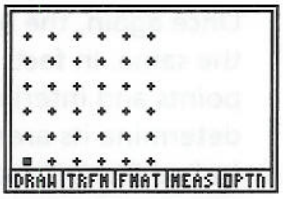
Introduction

Pick's Theorem is a quick, straightforward method for finding the area of any polygon with vertices on a grid. This method can even be used to find areas of lakes and land regions by using a scaled map and grid paper.

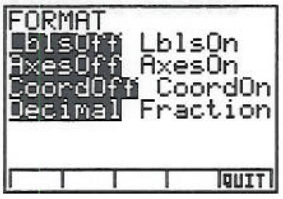
Investigation

This investigation will help you use Pick's Theorem to find the area of polygons.

1. From the main Geoboard menu, select **2:6x6**.



2. To format the geoboard, select **FMAT** and make sure that the following settings are selected:

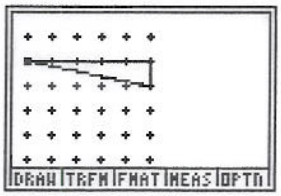


LblsOff (Labels are off)
AxesOff (Axes are off)
CoordOff (Coordinates are off)
Decimal (Measurement is in decimal form)

Select **QUIT** to exit the FORMAT menu.

3. Every geoboard point on the perimeter of a shape is called a boundary point. If the shape were made with one rubber band on a geoboard, every peg touching the rubber band would be a boundary point. An interior point is a point on the inside of the shape that does not touch the boundary.

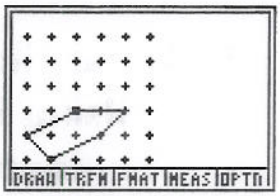
   Make the following shape on your geoboard.

   Determine the number of boundary points. _____

   Determine the number of interior points. _____

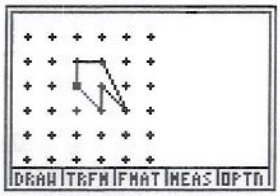
4. Make the following shape on your geoboard.

   Determine the number of boundary points. _____

   Determine the number of interior points. _____

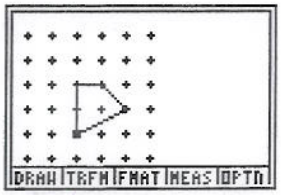
5. The polygon at the right has six boundary points and no interior points. Find its area. _____

   Construct two additional, different polygons, each with six boundary points and no interior points. Find the area of each. _____ _____

   Everyone should get the same area for each of their shapes. This seems a bit surprising since some could have made triangles, others quadrilaterals, others pentagons, and still others hexagons.

6. The polygon at the right has five boundary points and one interior point. Find its area. _____

   Construct two additional, different polygons, each with five boundary points and one interior point. Find the area of each. _____ _____

   Once again, the area of everyone's shapes should be the same. In fact, knowing the number of boundary points and interior points for a shape is sufficient to determine its area. You will discover the formula (called Pick's Theorem) in *Activity 9.1: Pick and Choose, Discovering Pick's Theorem.*

Student Activity

Name _____

Date _____

Activity 9.1: Pick and Choose! Discovering Pick's Theorem

In this activity, you will discover a formula (called Pick's Theorem) for finding the area of any geoboard polygon knowing only its number of boundary points and interior points.

1. Complete the table for shapes with the indicated number of boundary points and no interior points. Use your geoboard to construct each shape and find its area.

Number of boundary points	Number of interior points	Area
3	0	
4	0	
5	0	
6	0	
7	0	

2. If a shape with no interior points has 20 boundary points, what is its area?

_____

3. If a shape with no interior points has *b* boundary points (where *b* represents any number of boundary points), what is its area?

_____

4. Complete the table for shapes with the indicated number of boundary points and one interior point. Use your geoboard to construct each shape and finds its area.

Number of boundary points	Number of interior points	Area
3	1	
4	1	
5	1	
6	1	
7	1	

How do these values differ from those in #1?

5. If a shape with one interior point has 20 boundary points, what is its area?

6. Complete the table for shapes with the indicated number of boundary points and two interior points. Use the geoboard to construct each shape and find its area.

Number of boundary points	Number of interior points	Area
3	2	
4	2	
5	2	
6	2	
7	2	

How do these values differ from those in #4? In #1?

7. If a shape with two interior points has 20 boundary points, what is its area?

8. If a shape has b boundary points (where b represents any number of boundary points) and i interior points (where i is any number of interior points), what is the shape's area?

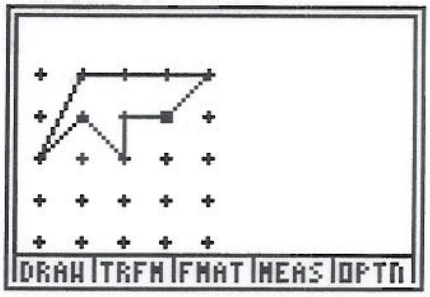
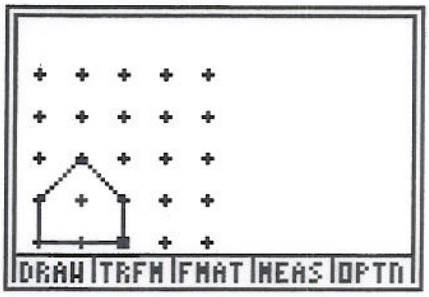
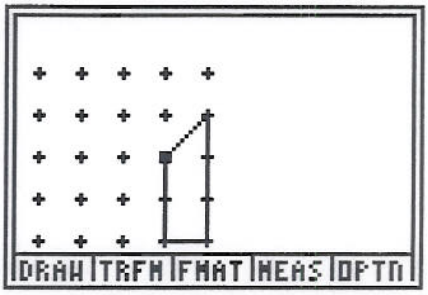
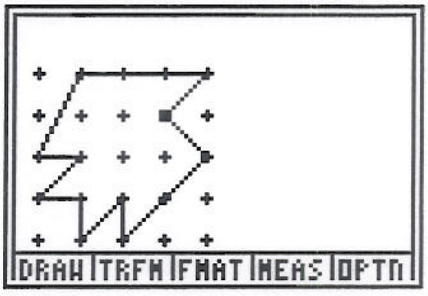
Student Activity

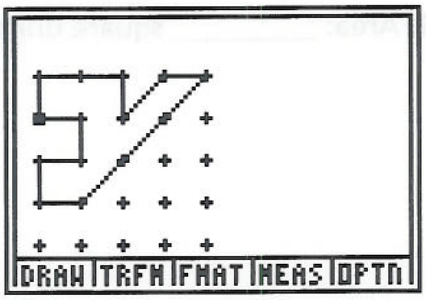
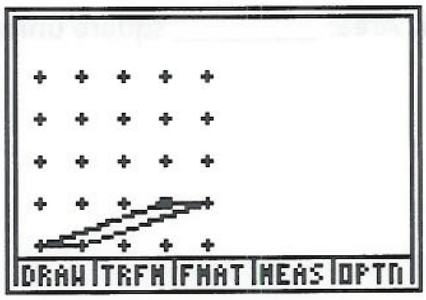
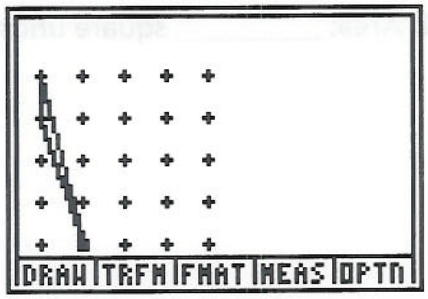
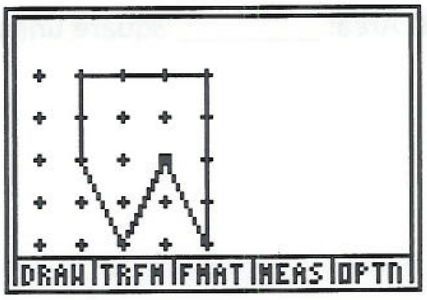
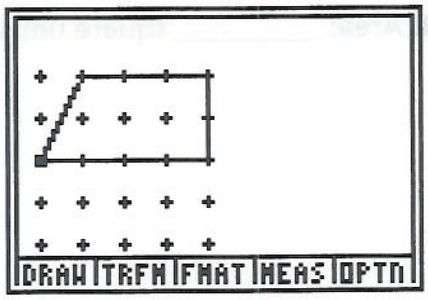
Name _____

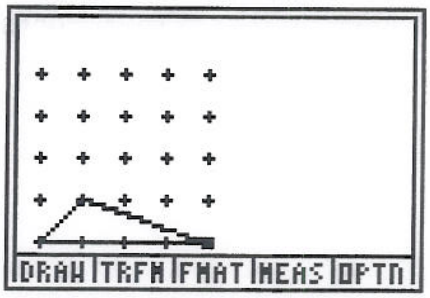
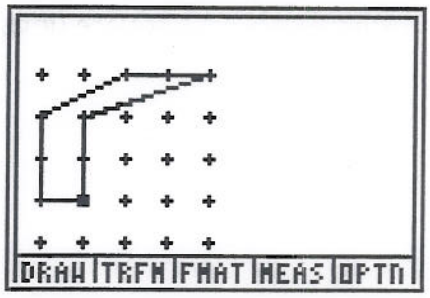
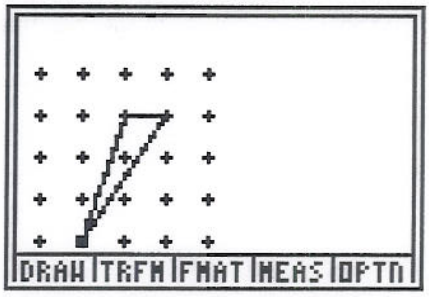
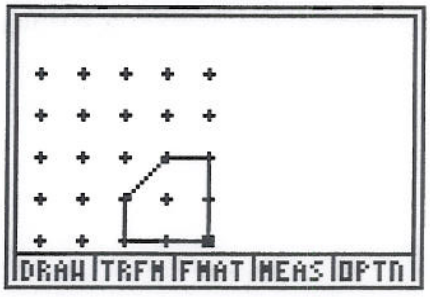
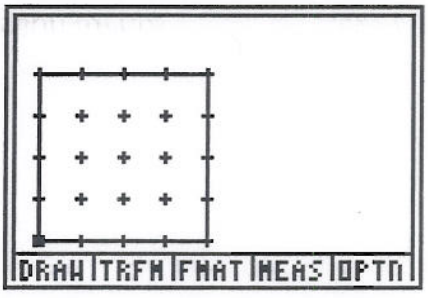
Date _____

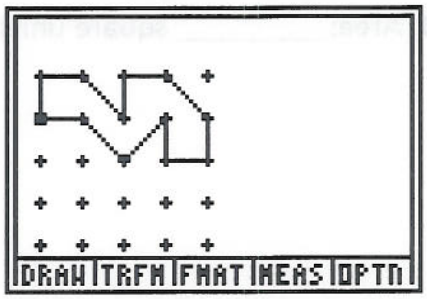
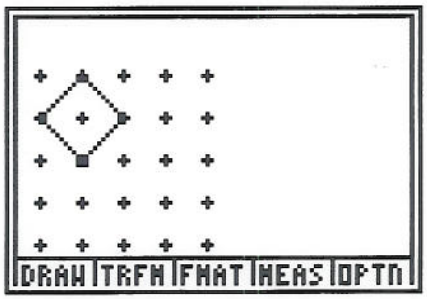
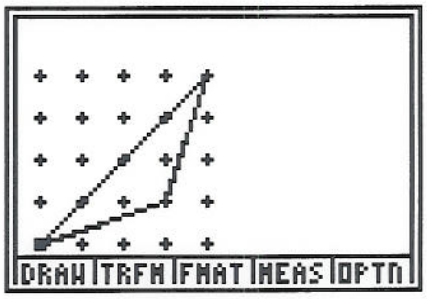
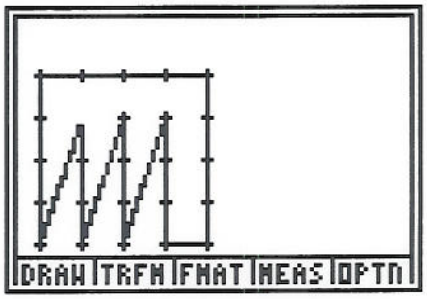
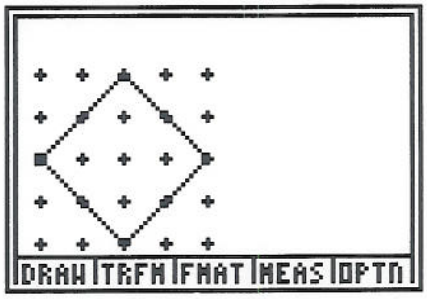
Activity 9.2: Finding areas using Pick's Theorem

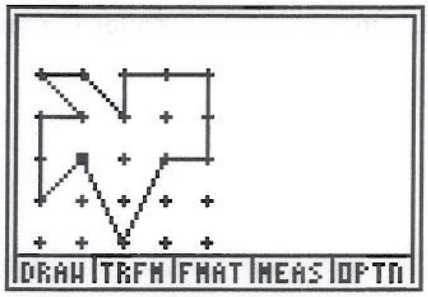
Make each shape on your geoboard and find its area using Pick's Theorem. Check your answers using the TI-73.

1. Area: _____ square units	
2. Area: _____ square units	
3. Area: _____ square units	
4. Area: _____ square units	

5. Area: _____ square units	
6. Area: _____ square units	
7. Area: _____ square units	
8. Area: _____ square units	
9. Area: _____ square units	

10. Area: _____ square units	
11. Area: _____ square units	
12. Area: _____ square units	
13. Area: _____ square units	
14. Area: _____ square units	

15. Area: _____ square units	
16. Area: _____ square units	
17. Area: _____ square units	
18. Area: _____ square units	
19. Area: _____ square units	

20. Area: _____ square units	

Student Activity

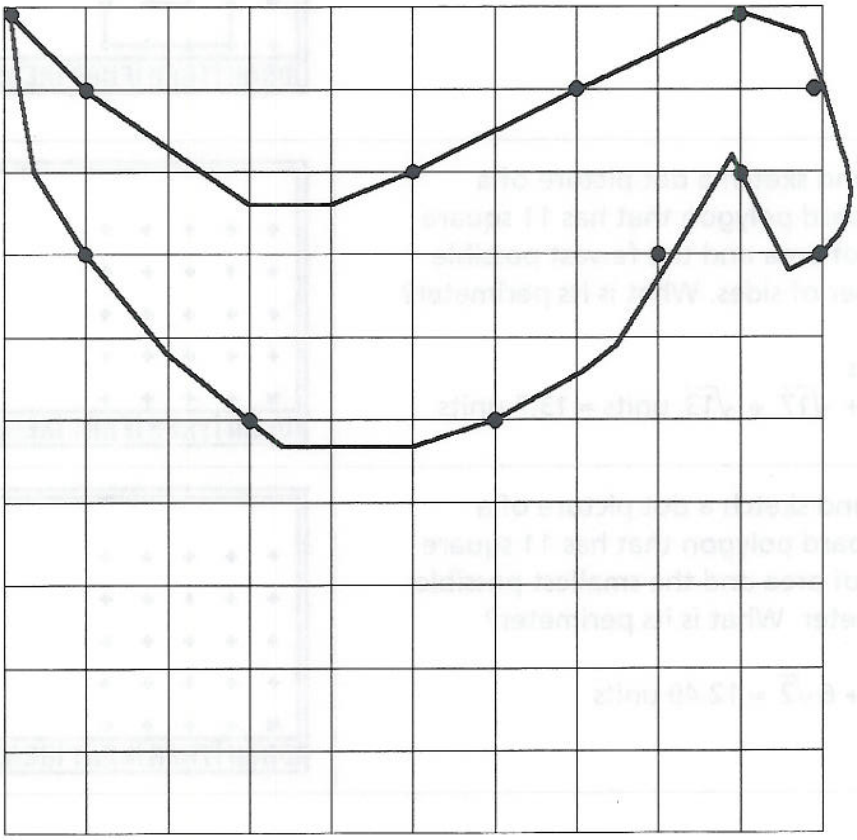
Name _____

Date _____

Activity 9.3: Pick's Theorem—Aerial Surveyors

A variety of water and land regions have irregular shapes. In order to estimate the areas of such regions, aerial surveyors can use grids of known sizes to help them find these areas.

An aerial photograph of Dino Lake is shown with its overlapping grid. The scale is 1 unit = ¼ mile.



1. Use Pick's Theorem to estimate the area of Dino Lake in square units, then convert this estimate into square miles.

   _____

2. Choose another method for finding the area of Dino Lake. Describe your method and compare the results with those from question 1.

   _____

   _____

   _____

Student Activity

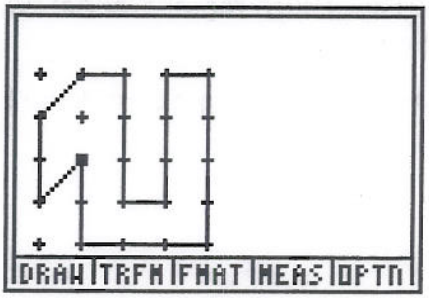
Name _____

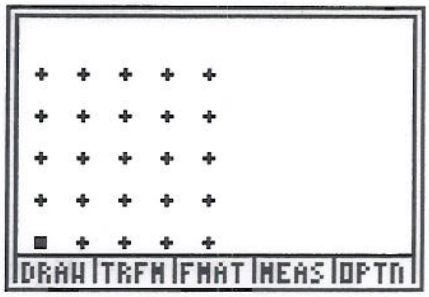
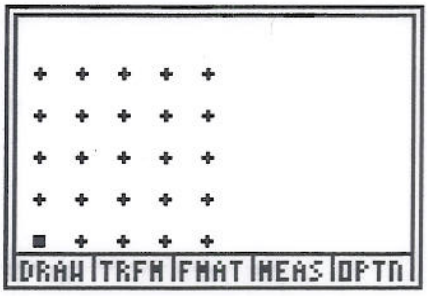
Date _____

Activity 9.4: Pick's Theorem—Smallest and Largest Perimeters

Use a 5×5 geoboard for each of the following problems.

The geoboard polygon shown at the right has an area of 11 square units and a perimeter of $20 + 2\sqrt{2}$, or about 22.8 square units.



1. Find and sketch a dot picture of a geoboard polygon that has 11 square units of area and the fewest possible number of sides. What is its perimeter? 4 sides $P = 6 + \sqrt{17} + \sqrt{13}$ units $\approx$ 13.7 units	
2. Find and sketch a dot picture of a geoboard polygon that has 11 square units of area and the smallest possible perimeter. What is its perimeter? $P = 4 + 6\sqrt{2} \approx 12.49$ units	

⑨ The area is 11 square units 	⑨ The shape has 9 interior points 
⑨ All 3 pairs of opposite sides are parallel 	⑨ All 3 pairs of opposite sides are congruent 
❾ There are 6 sides 	❾ There are 6 boundary points 

Teacher Notes

Activity 9

Pick and Choose! Using Pick's Theorem!

Objective

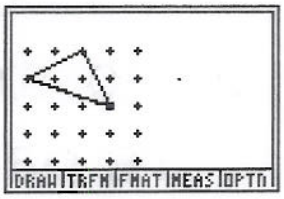
- To find the area of any geoboard polygon using Pick's Theorem

NCTM Standards

- Select and apply techniques and tools to accurately find... area...to appropriate levels of precision
- Solve problems that arise in mathematics and in other contexts
- Use symbolic algebra to represent situations and to solve problems

Investigation

Knowing Pick's Theorem can help students construct shapes with certain properties. For example, if asked to make a triangle with area 2½ square units, students might work backwards using the Theorem:



$$A = \frac{b}{2} + i - 1, \qquad 2\frac{1}{2} = \frac{b}{2} + i - 1$$

Some possibilities that fit are b = 5 and i = 1 or b = 3 and i = 2. The second combination is a solution.

Answers to Student Activity pages

Activity 9.1: Pick and Choose! Discovering Pick's Theorem

1.

Number of boundary points	Number of interior points	Area
3	0	$\frac{1}{2}$
4	0	1
5	0	$1\frac{1}{2}$
6	0	2
7	0	$2\frac{1}{2}$

2. 9 square units

3. $\frac{b}{2} - 1$

4. Each area is one square unit larger

Number of boundary points	Number of interior points	Area
3	1	$1\frac{1}{2}$
4	1	2
5	1	$2\frac{1}{2}$
6	1	3
7	1	$3\frac{1}{2}$

5. 10 square units

6. Each area is 1 square unit larger than that in Table 2 and 2 units larger than that in Table 1.

Number of boundary points	Number of interior points	Area
3	2	$2\frac{1}{2}$
4	2	3
5	2	$3\frac{1}{2}$
6	2	4
7	2	$4\frac{1}{2}$

7. 11 square units

8. $A = \frac{1}{2}b + i - 1$ or $A = \frac{b}{2} + i - 1$ or $A = \frac{(b - 2)}{2} + i$

Activity 9.2: Finding areas using Pick's Theorem

1. $3\frac{1}{2}$ square units

2. 3 square units

3. $2\frac{1}{2}$ square units
4. 10 square units
5. 6 square units
6. 1 square unit
7. $\frac{1}{2}$ square unit
8. 9 square units
9. 7 square units
10. 2 square units
11. $3\frac{1}{2}$ square units
12. $1\frac{1}{2}$ square units
13. $3\frac{1}{2}$ square units
14. 16 square units
15. 5 square units
16. 2 square units
17. 4 square units
18. $11\frac{1}{2}$ square units
19. 8 square units
20. $9\frac{1}{2}$ square units

Activity 9.3: Pick's Theorem—Aerial Surveyors

1. Area = $\frac{b}{2}$ + i - 1

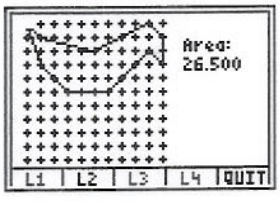
   Area = $\frac{12}{2}$ + 22 - 1

   Area = 27 square units

   The area in square miles is $\frac{27}{16}$ = .6875 (1.69)

2. The area is 28 square units by counting squares and half-or-more squares. The area is 26.5 square units by using the TI-73.

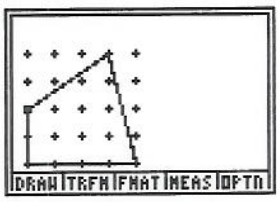
   The results are quite close using these two methods.



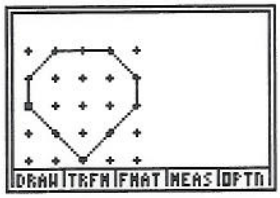
Activity 9.4: Pick's Theorem—Smallest and Largest Perimeters

1. 4 sides

   $P = 6 + \sqrt{17} + \sqrt{13}$ units ≈ 13.7 units



2. $P = 4 + 6\sqrt{2} \approx 12.49$ units



Group Problem Solving: The area of polygons

The Group Problem Solving cards are challenge problems that can be used alone or with the individual sections of this book. The problems are designed to be used in groups of four (five or six in a group are possibilities using the additional cards) with each person having one of the first four clues. Students can read the information on their cards to others in the group but all should keep their own cards and not let one person take all the cards and do the work.

The numbers at the top of the cards indicate the lesson with which the card set is associated. The fifth and sixth clues (the optional clues) have the lesson number shown in a black circle.

The group problems can be solved using the first four clues. The fifth and sixth clues can be used as checks for the group's solution or they can be used as additional clues if a group gets stuck. Some problems have more than one solution. Any shape that fits all the clues should be accepted as correct.

One solution for this problem solving exercise: