

USING DERIVE

1. CURVE SKETCHING

Derive 5 - [Algebra 1 E:\curve sketching.dfw]

File Edit Insert Author Simplify Solve Calculus Declare Options Window Help

Step 1: Define the function

#1: $f(x) := 4 \cdot x^3 - 7 \cdot x^2 - 5 \cdot x + 6$

Step 2: X-intercepts - solve $f(x)=0$.
Highlight $f(x)$. Solve>Expression(Real). Click "SOLVE" button. "v" in DERIVE means "or".

#2: SOLVE($f(x)$, x , Real)

#3: $x = \frac{3}{4} \vee x = 2 \vee x = -1$

Step 3: Y-intercepts - $x=0$. Type $f(0)$, then [Enter] then [=] toolbar icon. Answer is $y=6$.

#4: $f(0)$

#5: 6

Step 4: Stationary points - $f'(x)=0$. Type $f'(x)=0$. Then Solve>Expression(Real). Click "SOLVE".

#6: $f'(x) = 0$

#7: SOLVE($f'(x) = 0$, x , Real)

#8: $x = \frac{7}{12} - \frac{\sqrt{109}}{12} \vee x = \frac{\sqrt{109}}{12} + \frac{7}{12}$

Click the approximate button to get a rational answer

#9: $x = 1.453358875 \vee x = -0.2866922090$

Step 5: y-value of Stationary points.
Type $f(1.453358875)$ then [Enter] then [approximate]. Repeat for $f(-0.2866922090)$. Use copy/paste.

#10: $f(1.453358875)$

#11: -3.773117636

#12: $f(-0.286692209)$

#13: 6.763858377

HINT: Shortcut to SOLVE > Expression

Label all points on the graph

Derive 5

File Edit Insert Author Simplify Solve Calculus Declare Options Window Help

2D-plot 1:1

Algebra 1 E:\curve sketching.dfw

Step 3: Y-intercepts - $x=0$. Type $f(0)$, then [Enter] then [=] toolbar icon. Answer is $y=6$.

#4: $f(0)$

#5: 6

Step 4: Stationary points - $f'(x)=0$. Type $f'(x)=0$. Then Solve>Expression(Real). Click "SOLVE".

#6: $f'(x) = 0$

#7: SOLVE($f'(x) = 0$, x , Real)

#8: $x = \frac{7}{12} - \frac{\sqrt{109}}{12} \vee x = \frac{\sqrt{109}}{12} + \frac{7}{12}$

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#10: $f(1.453358875)$

#11: -3.773117636

#12: $f(-0.286692209)$

#13: 6.763858377

2. ABSOLUTE VALUE FUNCTION

The screenshot shows the Derive 5 software interface with two algebra windows. The left window, titled 'Algebra 2', shows the definition of a function $f(x) := x^2 - 4$ and its graph, which is a parabola opening upwards with its vertex at (0, -4). The right window, titled 'Algebra 1', contains the following text:

For the absolute value, type:
 $f(x) := \text{abs}(x^2 - 4)$ [Enter]

#1: $f(x) := |x^2 - 4|$

The absolute value function reflects the negative values of $f(x)$ in the x -axis.

Below this text is a graph of the absolute value function $f(x) := |x^2 - 4|$, which is a W-shaped curve with its vertices at (-2, 0) and (2, 0), and a local maximum at (0, 4). The x-axis ranges from -4 to 4, and the y-axis ranges from -6 to 6.

At the bottom of the interface, the command line shows $f(x) := \text{ABS}(x^2 - 4)$ and a toolbar with various mathematical symbols.

3. TRIGONOMETRY

(a) Solving Trig equations in either radians or degrees

The screenshot shows the Derive 5 software interface. On the left, a window titled 'Algebra 1' contains text about radians and degrees, and a 'Simplification Settings' dialog box. The dialog box has the following settings: Transformation Direction (Exponential: Auto, Trigonometry: Auto, Logarithm: Auto, Trig Powers: Auto), Angular unit (Radian selected, Degree and Radian also visible), Precision (Mode: Exact, Digits: 10), and Branch (Principal). On the right, a window titled 'Algebra 2 trig.dfw' contains text and mathematical examples for solving trigonometric equations.

RADIANS & DEGREES
 The factory default setting is in radians.
 To change between radians and degrees,
DECLARE > Simplification Settings
 Change the "angular unit" in the dialog box

SOLVING TRIG EQUATIONS
 Derive gives 3 solutions over $-\pi \leq x \leq \pi$
Examples 1:
 find x if $\sin 2x = \sqrt{3}/2$, $-\pi \leq x \leq \pi$

Step 1: Type $\sin(2x)=\sqrt{3}/2$ [Enter]
Step 2: SOLVE > Expression(Algebraic & Real)

#1: $\text{SIN}(2 \cdot x) = \frac{\sqrt{3}}{2}$

#2: $\text{SOLVE}\left(\text{SIN}(2 \cdot x) = \frac{\sqrt{3}}{2}, x, \text{Real}\right)$

#3: $x = \frac{\pi}{6} \vee x = -\frac{2 \cdot \pi}{3} \vee x = \frac{\pi}{3}$

Use a unit circle to see that the fourth solution is $-5\pi/6$

Example 2:
 find x if $\sin 2x = \sqrt{3}/2$, $0 \leq x \leq 2\pi$

Use a unit circle to see that
 $-5\pi/6 = 7\pi/6$ and $-2\pi/3 = 4\pi/3$

Solutions are $\pi/6, \pi/3, 7\pi/6, 4\pi/3$

Press F1 for Help

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(b) Simplifying & Expanding Trig expressions (Double angle formulas)

The screenshot shows the Derive 5 software interface with two main windows: Algebra 1 and Algebra 2 trig2.dfw.

Algebra 1 Window:

EXPANDING & COLLECTING (simplifying)
 Set DERIVE to either EXPAND or COLLECT terms in a trig expression

DECLARE > Simplification Settings

Select the setting in "TRIGONOMETRY"

Simplification Settings Dialog:

- Transformation Direction:
 - Exponential: Auto
 - Logarithm: Auto
 - Trigonometry: Expand
 - Trig Powers: Auto, Collect, Expand
- Angular unit: Radian
- Precision:
 - Mode: Exact
 - Digits: 10
- Branch: Principal

Buttons: OK, Cancel, Reset

Algebra 2 trig2.dfw Window:

Example 1. Expand
 (a) $\cos 2\theta$
 (b) $\sin(x + y)$

STEP 1: *DECLARE >Simplification >Trig >Expand*

#1: *Trigonometry := Expand*

STEP 2: *cos(2θ) [Enter] [=], sin(x+y) [Enter] [=]*

#2: $\cos(2 \cdot \theta)$

#3: $2 \cdot \cos(\theta)^2 - 1$

#4: $\sin(x + y)$

#5: $\cos(x) \cdot \sin(y) + \sin(x) \cdot \cos(y)$

Example 2. Simplify (collect)
 (a) $2\cos\theta \sin\theta$
 (b) $\cos x \cos y - \sin x \sin y$

STEP 1: *DECLARE >Simplification >Trig >Collect*

#6: *Trigonometry := Collect*

STEP 2: *2cos(θ)sin(θ) [Enter] [=], etc.*

#7: $2 \cdot \cos(\theta) \cdot \sin(\theta)$

#8: $\sin(2 \cdot \theta)$

#9: $\cos(x) \cdot \cos(y) - \sin(x) \cdot \sin(y)$

#10: $\cos(x + y)$

When finished, return to "TRIGONOMETRY >Auto"

Press F1 for Help

Bottom toolbar: $\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta, \theta, \iota, \kappa, \lambda, \mu, \nu, \xi, \omicron, \pi, \rho, \sigma, \tau, \upsilon, \phi, \chi, \psi, \omega$

4. Exponentials

NOTE: e^{-2x} is entered as $\hat{e}^(-2x)$ in DERIVE

(a) Simplifying

$$\frac{6^{2-x} \times 2^x}{3^{1-x}}$$

Example: Simplify

(b) Solving indicial equations

Example: Solve to 3 decimal places
 $4e^x + 6e^{-x} - 11 = 0$

The screenshot shows the Derive 5 software interface with two algebra windows open.

Algebra 2 Simplify indices.dfw:

SIMPLIFYING INDICES
Example: Simplify

$$\frac{6^{2-x} \times 2^x}{3^{1-x}}$$

Type
 $(6^{(2-x)} * 2^x) / (3^{(1-x)})$ [Enter] [=]

#1: $\frac{6^{2-x} \cdot 2^x}{3^{1-x}}$

#2: 12

The expression simplifies to 12

Algebra 1:

SOLVING INDICIAL EQUATIONS
Example: Solve to 3 decimal places
 $4e^x + 6e^{-x} - 11 = 0$

Step 1: $4\hat{e}^x + 6\hat{e}^{-x} - 11 = 0$ [Enter]

#1: $4 \cdot \hat{e}^x + 6 \cdot \hat{e}^{-x} - 11 = 0$

Step 2: SOLVE >Expression (Algebraic, Real)

#2: SOLVE($4 \cdot \hat{e}^x + 6 \cdot \hat{e}^{-x} - 11 = 0, x, \text{Real}$)

#3: $x = \text{LN}\left(\frac{3}{4}\right) \vee x = \text{LN}(2)$

To get an approx. answer, click [≈]

#4: $x = -0.2876820724 \vee x = 0.6931471805$

Answer: $x = -0.288$ or $x = 0.693$

5. Logarithms

NOTE: $\log_3 x$ is entered as $\log(x,3)$ in DERIVE
 $\log_e x$ is entered as $\ln(x)$ in DERIVE, or as $\log(x,\hat{e})$

(a) Simplifying

Example: Simplify
 $\log_4(x-2)^3 - \log_4(x-2)$

(b) Solving log equations

Example: Solve
 $\log_e(x+1) - \log_e(2x-1) = \log_e 5$

The screenshot shows the DERIVE software interface with two algebra windows open.

Algebra 1: log simplify.dfw

Example:
 Simplify $3\log_2 7 + \log_2 7^3 - \log_2 7^6$

Type
 $3\log(7,2) + \log(7^3,2) - \log(7^6,2)$
 Then [Enter] [=]

#1: $3 \cdot \text{LOG}(7, 2) + \text{LOG}(7^3, 2) - \text{LOG}(7^6, 2)$

#2: 0

The expression simplifies to zero

Algebra 2

Example: Find x such that
 $\log_e(x+1) - \log_e(2x-1) = \log_e 5$

Step 1: Type
 $\ln(x+1) - \ln(2x-1) = \ln(5)$
 Then [Enter]

#1: $\text{LN}(x + 1) - \text{LN}(2 \cdot x - 1) = \text{LN}(5)$

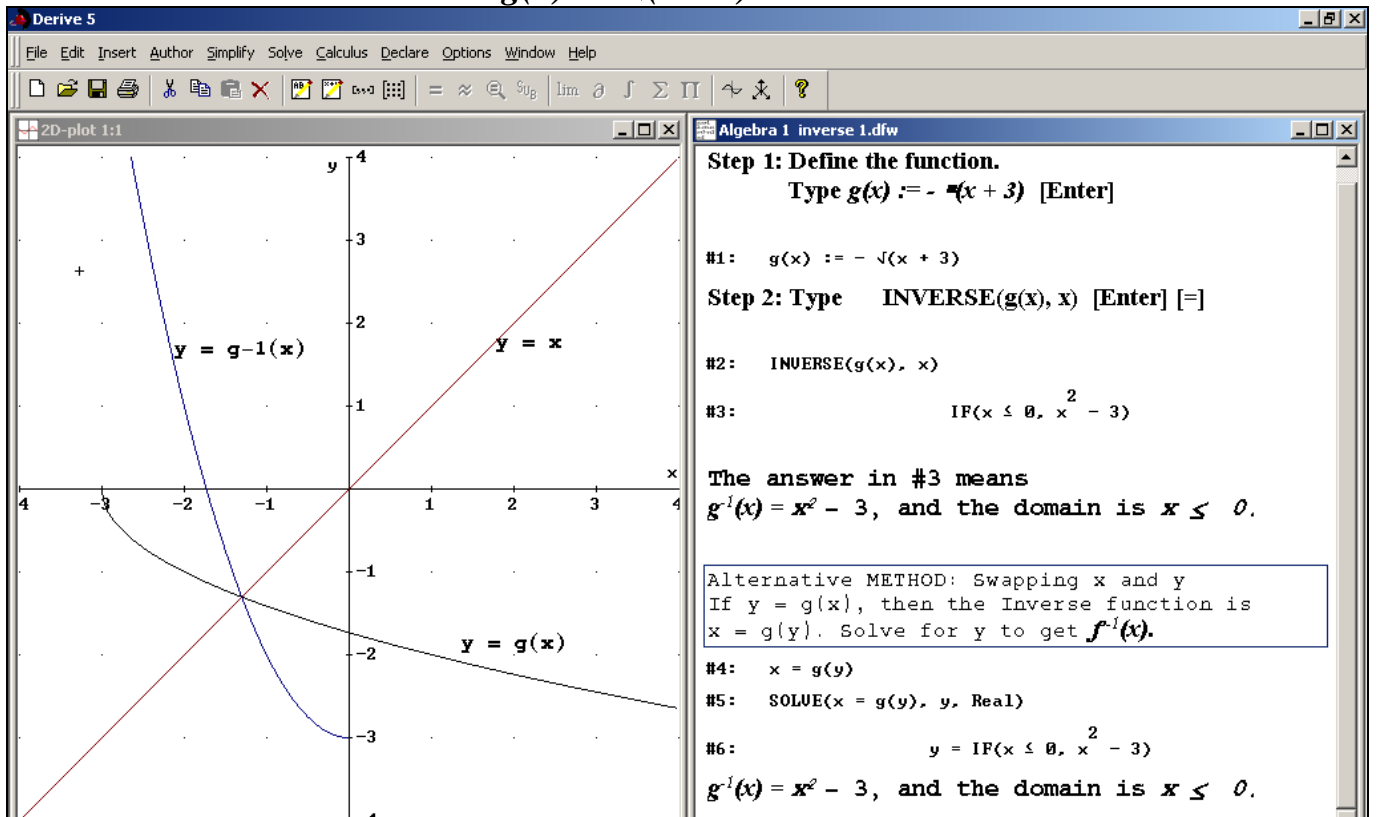
Step 2: SOLVE >Expression (Algebraic, Real)

#2: $\text{SOLVE}(\text{LN}(x + 1) - \text{LN}(2 \cdot x - 1) = \text{LN}(5), x, \text{Real})$

#3: $x = \frac{2}{3}$

6. Inverse functions

- Find the inverse function of $g(x) = -\sqrt{x+3}$.



Restricted Domain

Example:

- Find the inverse function of $f(x) = x^2 - 1$, domain $x \geq 0$
- Draw graphs of $f(x)$, $f^{-1}(x)$ and $y = x$, on the same set of axes

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The screenshot shows the Derive 5 interface. On the left, a 2D plot shows a green curve $y = f(x)$ and a yellow curve $y = f^{-1}(x)$ intersecting at the point (1, 1). A red line $y = x$ is also shown. On the right, the Algebra 1 window contains the following steps:

Step 1: Define the function.
Type $f(x) = x^2 - 1$ [Enter].

#1: $f(x) := x^2 - 1$

Step 2: to graph $f(x) = x^2 - 1$, domain $x \geq 0$
Type $\text{if}(x \geq 0, f(x))$ and graph #2

#2: $\text{IF}(x \geq 0, f(x))$

Step 3: Inverse($f(x), x$) [Enter] [=]
to find $f^{-1}(x)$

#3: $\text{INVERSE}(f(x), x)$

#4: $\sqrt{x + 1}$

Alternative METHOD: Swapping x and y
If $y = f(x)$, then the Inverse function is $x = f(y)$. Solve for y to get $f^{-1}(x)$.

#5: $x = f(y)$

#6: $\text{SOLVE}(x = f(y), y)$

#7: $y = -\sqrt{x + 1} \vee y = \sqrt{x + 1}$

Since $x \geq 0$, we want $y = \sqrt{x + 1}$.

7. Tangents & Normals

Method 1: Using $y - y_1 = m(x - x_1)$

EXAMPLE

Find the equations of the tangent and normal to $f(x) = 0.2x^2 - 4$ at $x = 1$

The screenshot shows the Derive 5 interface with two algebra windows. The left window, titled 'Algebra 1', shows the steps for finding the tangent line:

Equation of TANGENT at $x = 1$, $m = f'(1)$

#1: $f(x) := 0.2 \cdot x^2 - 4$

$y - y_1 = m(x - x_1)$, with $x_1 = 1$, becomes $y - f(1) = f'(1)(x - 1)$

#2: $y - f(1) = f'(1) \cdot (x - 1)$

Solve for y to get the equation of TANGENT

#3: $\text{SOLVE}(y - f(1) = f'(1) \cdot (x - 1), y, \text{Real})$

#4: $y = \frac{2 \cdot x - 21}{5}$

To get the decimal coefficients, click [≈]

#5: $y = 0.2 \cdot (2 \cdot x - 21)$

The right window, titled 'Algebra 2 Tangent&Normal 2.dfw', shows the steps for finding the normal line:

Equation of NORMAL at $x = 1$, $m = -1/f'(1)$

#1: $f(x) := 0.2 \cdot x^2 - 4$

$y - y_1 = m(x - x_1)$, with $x_1 = 1$, becomes $y - f(1) = -1/f'(1)(x - 1)$

#2: $y - f(1) = \frac{1}{f'(1)} \cdot (x - 1)$

Solve for y to get the equation of NORMAL

#3: $\text{SOLVE}\left(y - f(1) = \frac{1}{f'(1)} \cdot (x - 1), y, \text{Real}\right)$

#4: $y = \frac{25 \cdot x - 63}{10}$

To get the decimal coefficients, click [≈]

#5: $y = 0.1 \cdot (25 \cdot x - 63)$

Method 2: Using $\text{TANGENT}(f(x), x, 1)$ and

$\text{PERPENDICULAR}(f(x), x, 1)$

Find the equations of the tangent and normal to $f(x) = 0.2x^2 - 4$ at $x = 1$

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The screenshot shows the Derive 5 software interface. On the left, the 'Algebra 1' window displays the following steps:

Find the equation of the tangent at $x = 1$
 Step 1: Define $f(x) := 0.2x^2 - 4$

#1: $f(x) := 0.2 \cdot x^2 - 4$

Step 2: *Tangent*($f(x)$, x , 1) [Enter] [=] or [↵]
 #2: TANGENT($f(x)$, x , 1)

#3:
$$\frac{2 \cdot x - 21}{5}$$

#4: $0.4 \cdot x - 4.2$

To find equation of normal at $x = 1$
 Step 3: PERPENDICULAR($f(x)$, x , 1) [Enter] [=]

#5: PERPENDICULAR($f(x)$, x , 1)

#6:
$$-\frac{25 \cdot x + 13}{10}$$

#7: $-2.5 \cdot x - 1.3$

On the right, the '2D-plot 1:1' window shows a graph of the function $y = f(x)$ (green parabola) on a coordinate plane. The x-axis ranges from -4 to 4, and the y-axis ranges from -6 to 8. A red line represents the tangent to the curve at $x = 1$, with the equation $\text{Tangent } y = 0.4x - 4.2$. A yellow line represents the normal to the curve at $x = 1$, with the equation $\text{Normal } y = -2.3x - 1.3$.

NOTE: The equation of the tangent must NOT be written as: $0.4x - 4.2$.
 The equation MUST be written as: $y = 0.4x - 4.2$

8. Integration

(a) Indefinite integral

Find $\int \left(\frac{1}{x} + 3 \cos \frac{x}{2} \right) dx$

(b) Definite integral

Evaluate $\int_1^{\pi} \left(\frac{1}{x} + 3 \cos \frac{x}{2} \right) dx$

The screenshot shows the Derive software interface with two algebra windows and a dialog box.
Algebra 2 (Left): Shows the process of defining the function $f(x) := \frac{1}{x} + 3 \cdot \cos\left(\frac{x}{2}\right)$ and then performing an indefinite integral. The result shown is $\ln(x) + 6 \cdot \sin\left(\frac{x}{2}\right)$. Annotations include "Choose 'Indefinite'" pointing to the radio button and "Enter 'Constant'" pointing to the constant field set to 0.
Algebra 1 (Right): Shows the same function definition, followed by performing a definite integral from 1 to π . The result shown is $\ln(\pi) - 6 \cdot \sin\left(\frac{1}{2}\right) + 6$. Annotations include "Choose 'Definite'" pointing to the radio button and "Upper = π Lower = 1" pointing to the limit fields.
Calculus Integrate #1' (Dialog): Shows the configuration for the definite integral with the variable set to x, the integral type set to Definite, and the upper and lower limits set to π and 1 respectively.

Further Notes on Indefinite Integral

- (1) #3 (left window) gives the *exact* answer to $\int \left(\frac{1}{x} + 3 \cos \frac{x}{2} \right) dx = \log_e x + 6 \sin(x/2) + c$
- (2) You can set the "constant of integration" as c , 0 , or any other value.

Further Notes on Definite Integral

- (3) #3 (right window) gives the *exact* answer to $\int_1^{\pi} \left(\frac{1}{x} + 3 \cos \frac{x}{2} \right) dx = \log_e \pi + 6 \sin(1/2) + 6$.
- (4) To obtain a rational (decimal) approximation, highlight #3 and click [≈].

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USING DERIVE : 9. Signed region

(a) Draw the graph of the region bounded by $f(x) = x^3 + 2x^2 - x - 2$ and the x-axis.

#1: $f(x) := x^3 + 2x^2 - x - 2$

Step 2: Find the x-intercept: solve $f(x) = 0$

#2: $f(x) = 0$

#3: SOLVE($f(x) = 0$, x , Real)

#4: $x = -2 \vee x = -1 \vee x = 1$

The intercepts range from -2 to 1, so plot region: PlotInt($f(x)$, x , -2, 1) [Enter [=]

#5: PlotInt($f(x)$, x , -2, 1)

#6: $\left[\begin{array}{l} x^3 + 2x^2 - x - 2, y < x^3 + 2x^2 - x - 2 \\ -2 < y < 0 \wedge -2 \leq x \leq 1, x^3 + 2x^2 - x - 2 < y < 0 \wedge -2 \leq x \leq 1 \end{array} \right]$

Do a 2-D plot of #6

(b) Area of signed region

Find the area of the region bounded by $f(x) = x^3 + 2x^2 - x - 2$ and the x-axis.

NOTE: From the graph, Area = $\int_{-2}^{-1} f(x) dx - \int_{-1}^1 f(x) dx$

#1: $f(x) := x^3 + 2x^2 - x - 2$

Step 2: Highlight $f(x)$, click [∫] and enter the integrals

Calculus Integrate #1' dialog box: Variable: x , Definite integral, Upper Limit: -1, Lower Limit: -2

#2: $\int_{-2}^{-1} f(x) dx - \int_{-1}^1 f(x) dx$

Step 3: Click [=] to obtain exact answer. Click [≈] for the decimal approximation

#3: $\frac{37}{12}$

#4: 3.083333333

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(c) Area between two curves

NOTE: Area = $\int_a^b [f(x) - g(x)] dx$, if $f(x) > g(x)$ in the interval $[a, b]$

Example

Find the **area** of the region bounded by $f(x) = x + 1$ and $g(x) = x^2 - 1$.

(i) Find the points of intersection (From #4 below, $x = -1, x = 2$)

Algebra 1

```
#1: f(x) := x + 1
#2: g(x) := x^2 - 1
Step 2: Find the points of intersection
SOLVE >System. y=x+1 and y=x^2-1 in dialog
box: Solve 2 equation(s)
1 y=x+1
2 y=x^2-1
-Solution Variables
x
y
OK Solve Cancel
#3: SOLVE([y = x + 1, y = x^2 - 1], [x, y])
#4: [x = -1 ^ y = 0, x = 2 ^ y = 3]
```

Graph window: Derive 5 - [20:44:31]

Graph showing the intersection of $f(x) = x + 1$ and $g(x) = x^2 - 1$. The curves intersect at $(-1, 0)$ and $(2, 3)$.

(ii) Find the area

Derive 5 - [Algebra 3 Area between.dfw]

Step 3: Type $f(x) - g(x)$ [Enter]

```
#5: f(x) - g(x)
```

Step 4: With #5 highlighted, click []
Choose DEFINITE integral and enter upper and lower limits

Calculus Integrate - Area between.dfw #5

Variable: x

Integral: Definite Indefinite

Definite integral: Upper Limit: 2, Lower Limit: -1

Indefinite integral: Constant: 0

OK Simplify Cancel

```
#6: ∫_{-1}^2 (f(x) - g(x)) dx
#7: 9/2
#8: 4.5
```

Area between is **9/2 or 4.5 sq units**

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10. Binomial Distribution

- Binomial coefficients, ${}^n C_x$, evaluated in DERIVE with "**comb(n,x)**"
- $\Pr(X = x) = {}^n C_x p^x (1-p)^{n-x}$, evaluated in DERIVE with "**binomial_density(x,n,p)**"
- $\Pr(X \leq a)$, evaluated in DERIVE with $\sum_{x=0}^a \text{binomial_density}(x,n,p)$
- $\Pr(X \geq a)$ evaluated in DERIVE with $\sum_{x=a}^n \text{binomial_density}(x,n,p)$
- $\Pr(a \leq X \leq b)$ evaluated in DERIVE with $\sum_{x=a}^b \text{binomial_density}(x,n,p)$

Example:

Jo rolls a die 4 times ($n = 4$). She defines "success" as rolling an even number ($p = 0.5$). The random variable, X , denotes "rolling an even number".

- What is the probability of rolling exactly 2 even numbers ($\Pr(X = 2)$)?
- What is the probability of rolling no more than 2 even numbers ($\Pr(X \leq 2)$)?
- What is the probability of rolling at least 3 even numbers ($\Pr(X \geq 3)$)?
- What is the probability of rolling between 1 and 3 even numbers ($\Pr(1 \leq X \leq 3)$)?

The screenshot shows the DERIVE software interface with the following content:

```
Algebra 1 Binomial.dfw
Jo rolls a die 4 times (n = 4). "Success" is rolling an even number (p = 0.5).
(a)What is the probability of rolling exactly 2 even numbers ( Pr(X = 2) )?
#1:  BINOMIAL_DENSITY(2, 4, 0.5)
#2:                                     0.375

(b)What is the probability of rolling no more than 2 even numbers Pr(X ≤ 2)
#3:  BINOMIAL_DENSITY(x, 4, 0.5)

With #3 highlighted, click 'Σ' button.
Enter 'Lower limit'=0, 'Upper limit'=2

#4:  Σ_{x=0}^2 BINOMIAL_DENSITY(x, 4, 0.5)
#5:                                     0.6875

(c)What is the probability of rolling at least 3 even numbers ( Pr(X ≥ 3) )
Copy and paste #4, but change the limits to " x,3,4 "

#6:  Σ_{x=3}^4 BINOMIAL_DENSITY(x, 4, 0.5)
#7:                                     0.3125

What is the probability of rolling between 1 and 3 even numbers ( Pr(1 ≤ X ≤ 3) )
Copy and paste #4, but change the limits to " x,1,3 "

#8:  Σ_{x=1}^3 BINOMIAL_DENSITY(x, 4, 0.5)
#9:                                     0.875
```

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11. Hypergeometric Distribution

- Binomial coefficients, ${}^n C_x = \binom{n}{x}$, evaluated in DERIVE with " **comb(n,x)**
- $\Pr(X = x) = \frac{\binom{D}{x} \binom{N-D}{n-x}}{\binom{N}{n}}$, evaluated in DERIVE with **hypergeometric_density(x,n,D,N)**
- $\Pr(X \leq a)$, evaluated in DERIVE with $\sum_{x=0}^a \text{hypergeometric_density}(x,n,D,N)$
- $\Pr(X \geq a)$ evaluated in DERIVE with $\sum_{x=a}^n \text{hypergeometric_density}(x,n,D,N)$
- $\Pr(a \leq X \leq b)$ evaluated in DERIVE with $\sum_{x=a}^b \text{hypergeometric_density}(x,n,D,N)$

Example:

Eggs are sold in cartons of 12 (**N = 12**). A carton contains 5 brown eggs (**D = 5**). Alex selects 3 eggs at random (without replacement) and breaks them to make an omelet (**n = 3**).

- (a) What is the probability of selecting exactly 2 brown eggs ($\Pr(X = 2)$)?
- (b) What is the probability of selecting no more than 2 brown eggs, $\Pr(x \leq 2)$?
- (c) What is the probability of selecting at least 2 brown eggs, $\Pr(x \geq 2)$?
- (d) What is the probability of selecting between 1 and 3 brown eggs, $\Pr(1 \leq x \leq 3)$?

Eggs are sold in cartons of 12 (**N = 12**). A carton contains 5 brown eggs (**D = 5**). Alex selects 3 eggs at random and breaks them to make an omelet (**n = 3**).

(a) What is the probability of selecting exactly 2 brown eggs (**Pr(X = 2)**)?

#1: **HYPERGEOMETRIC_DENSITY(2, 3, 5, 12)**

#2: $\frac{7}{22}$

(b) What is the probability of selecting no more than 2 brown eggs, $\Pr(X \leq 2)$?

#3: **HYPERGEOMETRIC_DENSITY(x, 3, 5, 12)**

With #3 highlighted, click 'Σ' button.
Enter 'Lower limit'=0, 'Upper limit'=2

#4: $\sum_{x=0}^2 \text{HYPERGEOMETRIC_DENSITY}(x, 3, 5, 12)$

#5: $\frac{21}{22}$

(c) What is the probability of selecting at least 2 brown eggs, $\Pr(X \geq 2)$?

Copy and paste #4, but change the limits to " x,2,3 "

#6: $\sum_{x=2}^3 \text{HYPERGEOMETRIC_DENSITY}(x, 3, 5, 12)$

#7: $\frac{4}{11}$

(d) What is the probability of selecting between 1 and 3 brown eggs , $\Pr(1 \leq X \leq 3)$?

Copy and paste #4, but change the limits to " x,1,3 "

#8: $\sum_{x=1}^3 \text{HYPERGEOMETRIC_DENSITY}(x, 3, 5, 12)$

#9: $\frac{37}{44}$

USING DERIVE

USING DERIVE

12. Normal Distribution

- $\Pr(X < a)$, evaluated in DERIVE with “**normal(a, μ , σ)**”
- $\Pr(X > a) = 1 - \Pr(X < a)$ evaluated in DERIVE with “ $1 - \text{normal}(a, \mu, \sigma)$ ”
- $\Pr(a < X < b) = \Pr(X < b) - \Pr(X < a)$
evaluated in DERIVE with “ $\text{normal}(b, \mu, \sigma) - \text{normal}(a, \mu, \sigma)$ ”.

Example: Assume X = “VCE study score” is normally distributed.

VCE Chemistry has a mean study score of 30 ($\mu = 30$) and a standard deviation of 7 ($\mu = 7$).

- (a) What is the probability that a randomly chosen student has a score less than 27 ($\Pr(X < 27)$)?
 (b) What is the probability that a randomly chosen student has a score above 40
 ($\Pr(X > 40) = 1 - \Pr(X < 40)$)?
 (c) What is the probability that a randomly chosen student has a score between 35 and 40
 ($\Pr(35 < X < 40) = \Pr(X < 40) - \Pr(X < 35)$)?

Derive 5 - [Algebra 2 Normal dist.dfw]

File Edit Insert Author Simplify Solve Calculus Declare Options Window Help

VCE Chemistry has a mean study score of 30 ($\mu = 30$) and a standard deviation of 7 ($\mu = 7$).
 (a) What is the probability that a randomly chosen student has a score less than 27 ($\Pr(X < 27)$)?

#1: NORMAL(27, 30, 7)

#2: 0.3341175708

Approx. 33% of students score less than 27

(b) What is the probability that a randomly chosen student has a score above 40 ($\Pr(X > 40) = 1 - \Pr(X < 40)$)?

#3: 1 - NORMAL(40, 30, 7)

#4: 0.07656372550

Approx. 7.7% of students score above 40

(c) What is the probability that a randomly chosen student has a score between 35 and 40 ($\Pr(35 < X < 40) = \Pr(X < 40) - \Pr(X < 35)$)?

#5: NORMAL(40, 30, 7) - NORMAL(35, 30, 7)

#6: 0.1609615365

Approx. 16% of students score less between 35 and 40.

E:\12 CAS taskbook\Normal dist.dfw saved

normal(40,30,7)-normal(35,30,7)

α β γ δ ε ζ η θ ι κ λ μ ν ξ ο π ρ σ τ υ φ χ ψ ω

USING DERIVE

Inverse Normal Distribution

EXAMPLE 1:

X is a normally distributed random variable with mean 30 ($\mu = 30$) and standard deviation 7 ($\sigma = 7$).

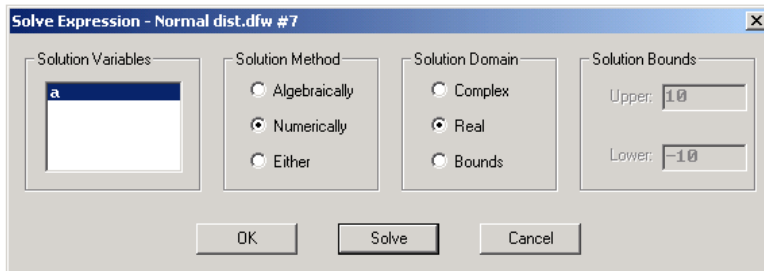
Find the value of a such that $\Pr(X < a) = 0.95$.

X is normally distributed with mean 30 ($\mu = 30$) and standard deviation 7 ($\sigma = 7$).
Find the value of a such that $\Pr(X < a) = 0.95$.

Need to solve $\text{NORMAL}(a, 30, 7) = 0.95$, numerically

Step 1: Type $\text{NORMAL}(a, 30, 7) = 0.95$, then [Enter]

Step 2: SOLVE >Expression. Select "Numerically" and "Real". Click "Solve" button



#7: $\text{NORMAL}(a, 30, 7) = 0.95$

#8: $\text{NSOLVE}(\text{NORMAL}(a, 30, 7) = 0.95, a, \text{Real})$

#9: $a = 41.51397526$

This means that 95% of students have a study score below 41.5

EXAMPLE 2: Assume $X =$ "weight of eggs" is normally distributed
25% of eggs produced on a particular farm weigh less than 40 gram.
That is, $\Pr(X < 40) = 0.25$.

The mean weight is known to be 55 grams ($\mu = 55$).

Find the standard deviation ($\sigma = ?$).

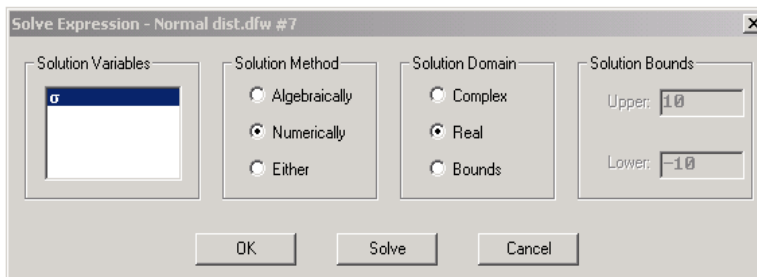
EXAMPLE 2: Assume $X =$ "weight of eggs" is normally distributed
25% of eggs produced on a particular farm weigh less than 40 gram. That is, $\Pr(X < 40) = 0.25$.
The mean weight is known to be 55 grams ($\mu = 55$).
Find the standard deviation ($\sigma = ?$).

Need to solve $\text{NORMAL}(40, 55, \sigma) = 0.25$, numerically

STEP 1: Type $\text{NORMAL}(40, 55, \sigma) = 0.25$, then [ENTER]

#7: $\text{NORMAL}(40, 55, \sigma) = 0.25$

STEP 1: SOLVE >Expression. In dialog box select: "Numerically" and "Real". Click "Solve" button



#8: $\text{NSOLVE}(\text{NORMAL}(40, 55, \sigma) = 0.25, \sigma, \text{Real})$

#9: $\sigma = 22.23903179$

The standard deviation is 22 grams