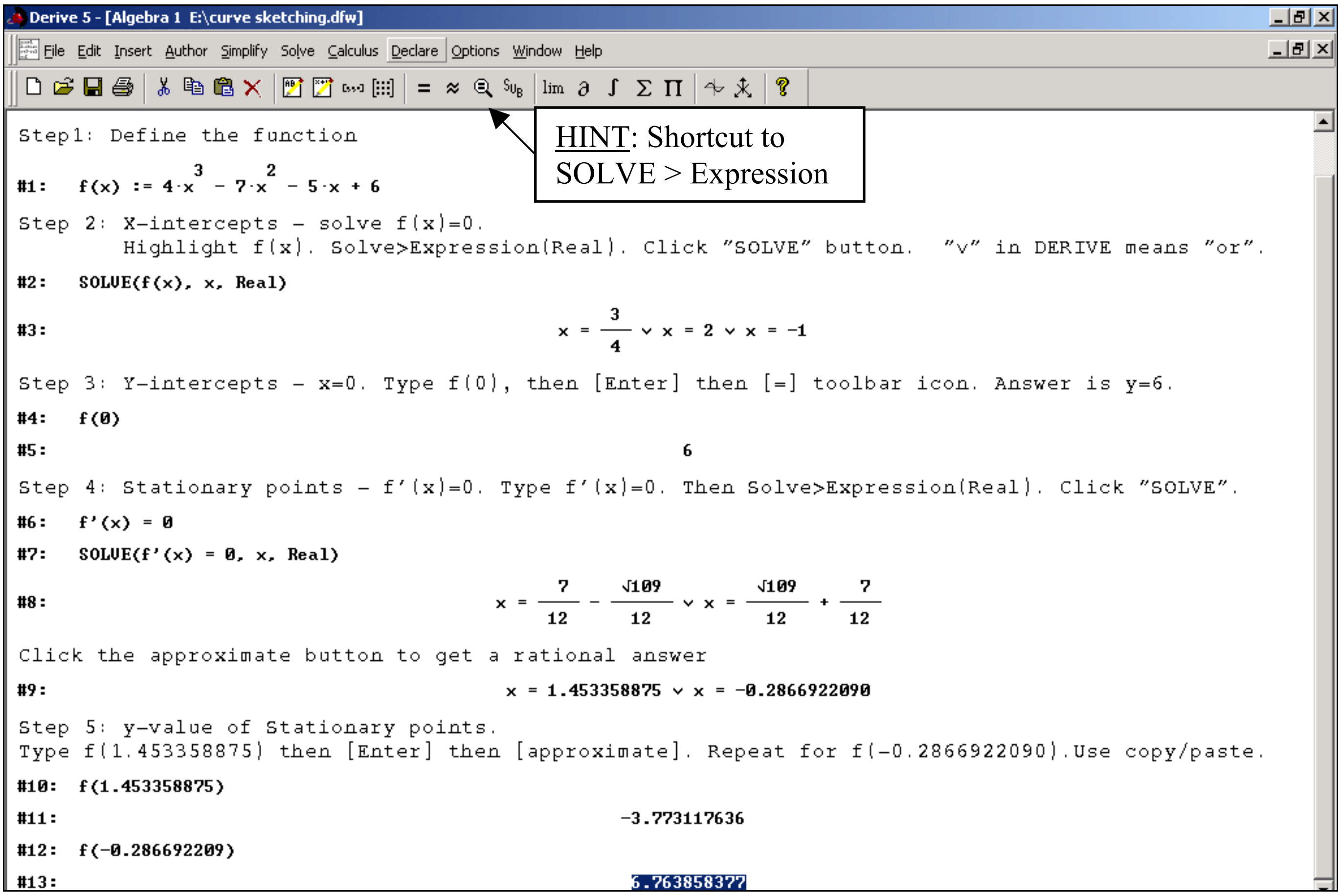
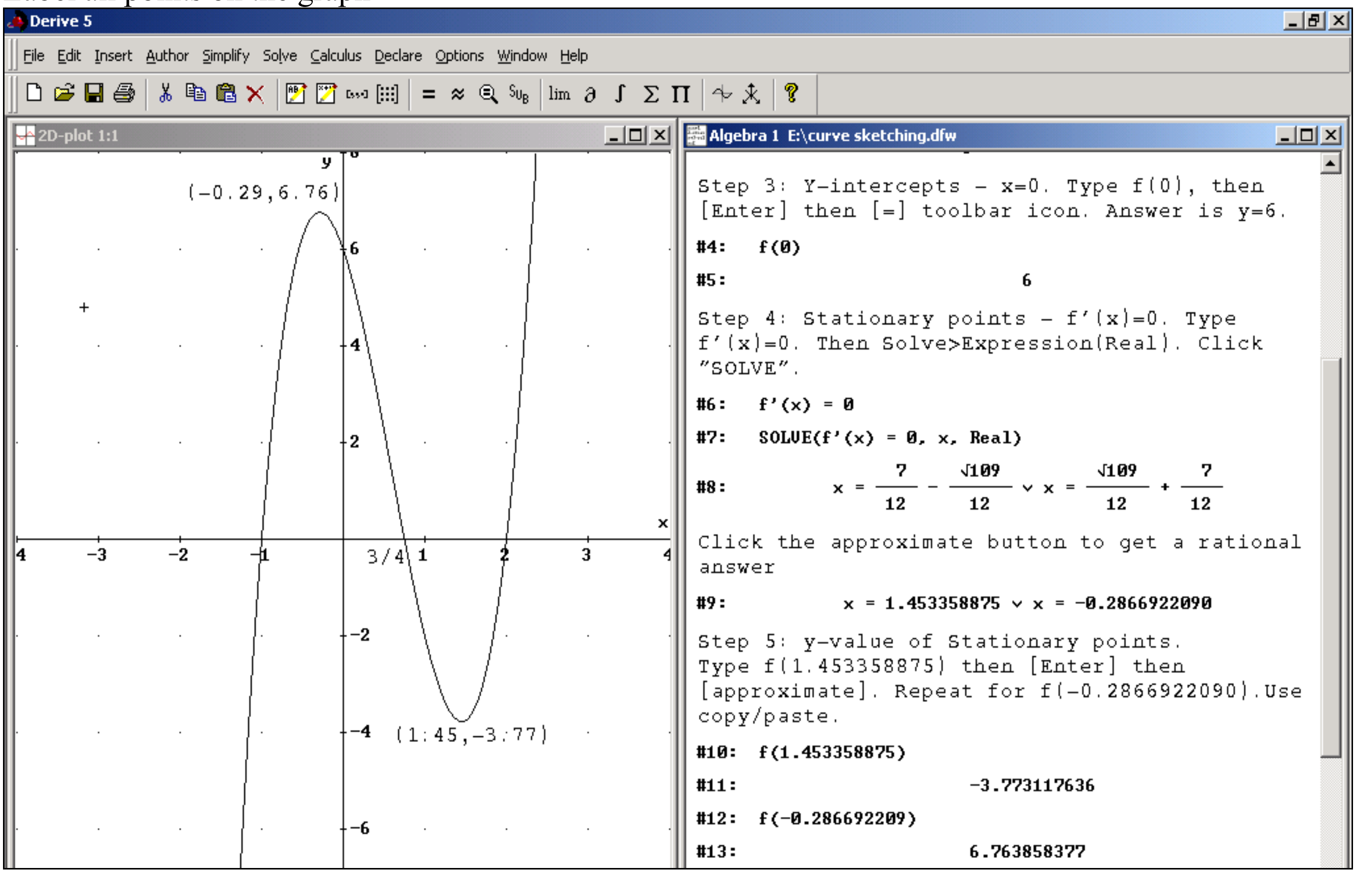
USING DERIVE

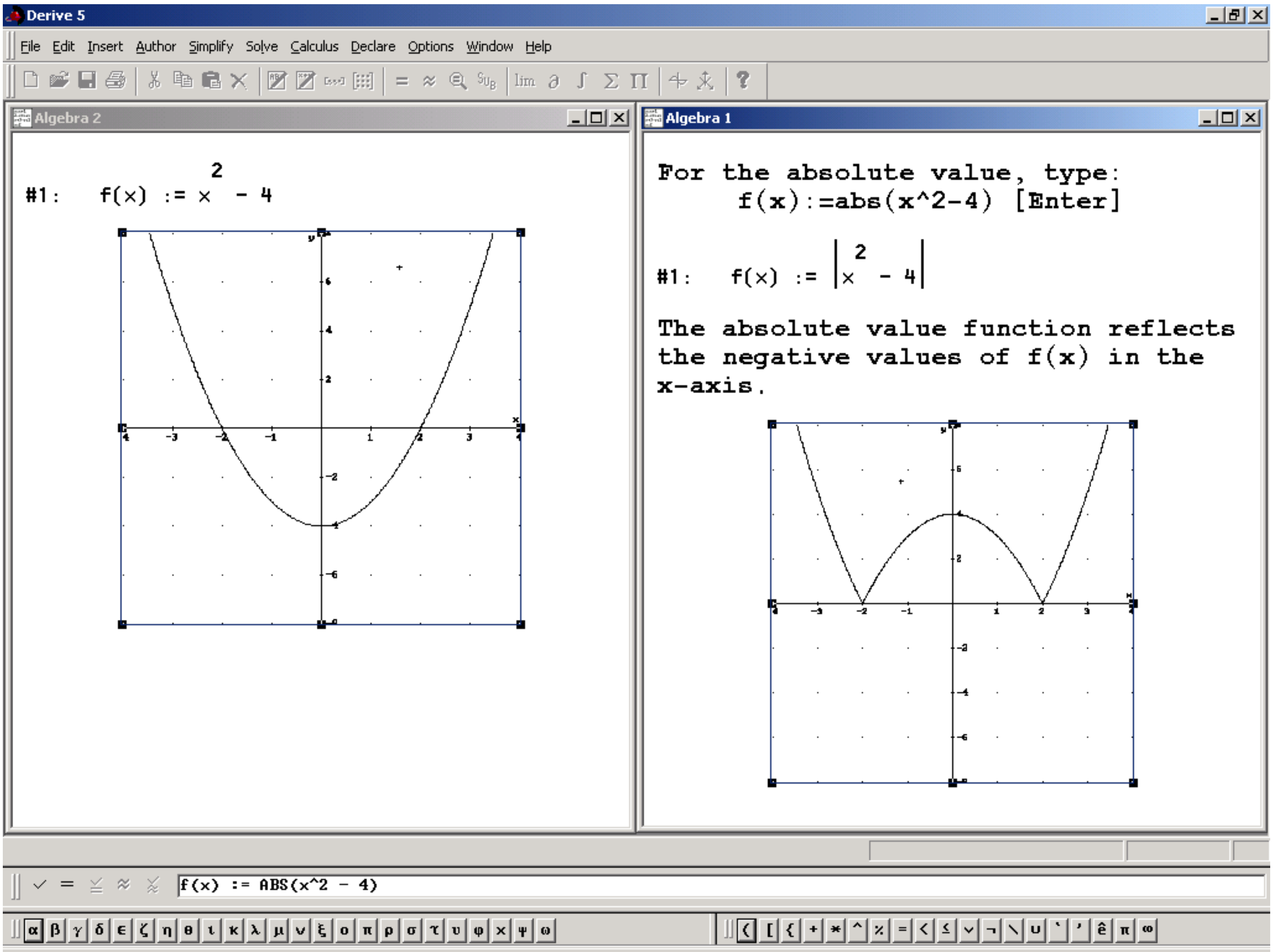
1. CURVE SKETCHING



Label all points on the graph

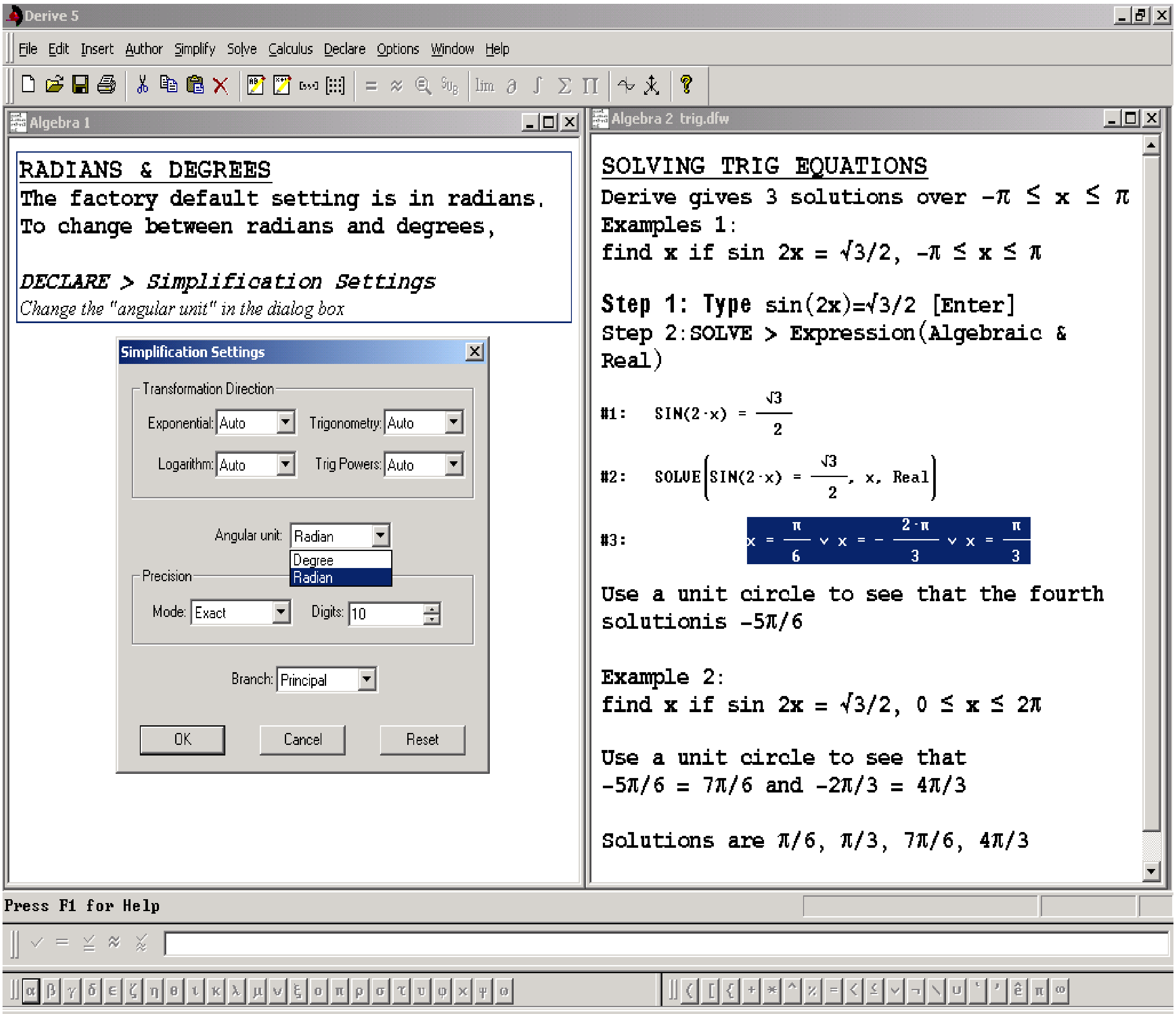


2. ABSOLUTE VALUE FUNCTION

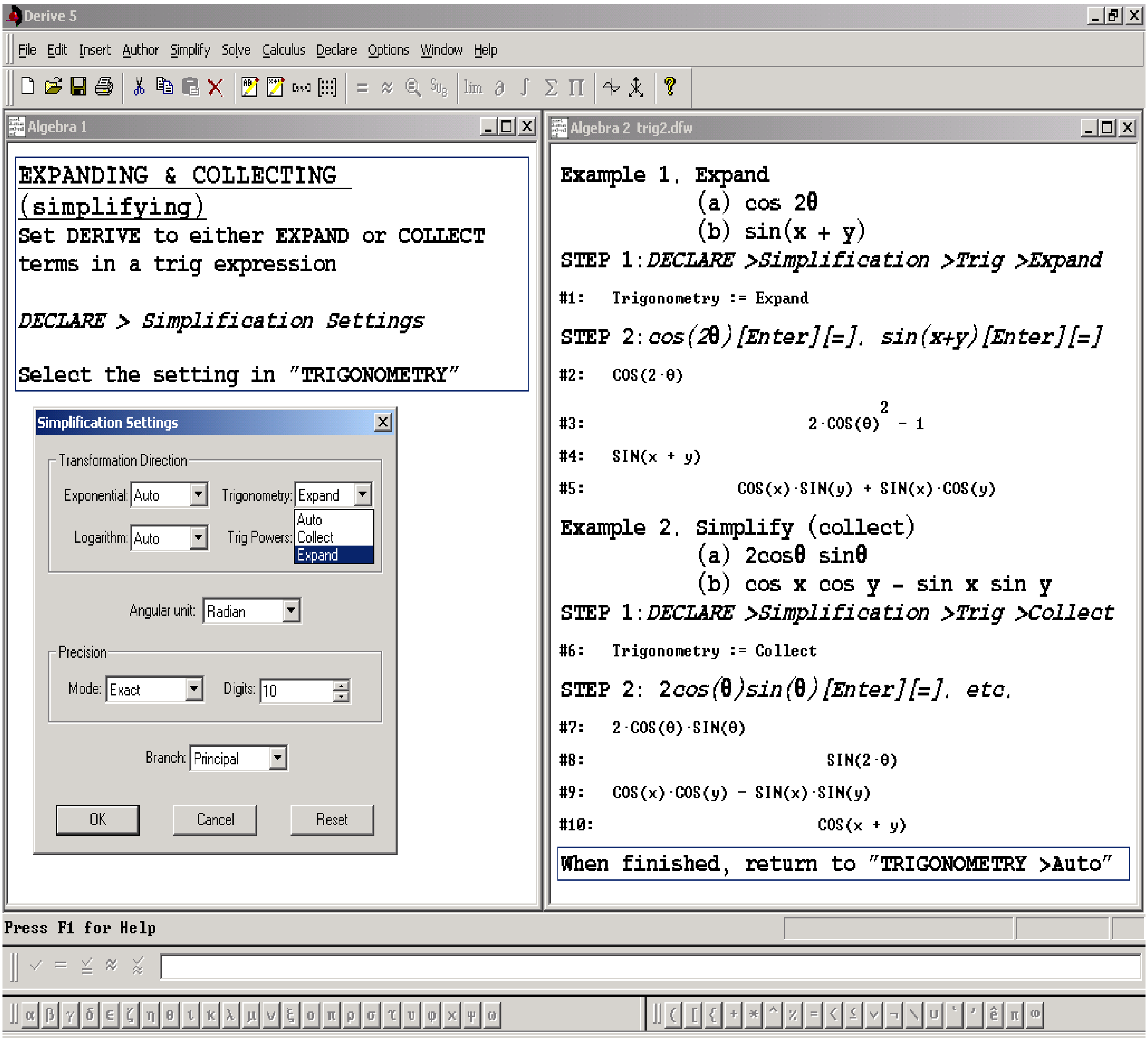


3. TRIGONOMETRY

(a) Solving Trig equations in either radians or degrees



(b) Simplifying & Expanding Trig expressions (Double angle formulas)



4. Exponentials

NOTE: ***e^{-2x}*** **is entered as** $\hat{e}$^ **(-2x) in DERIVE**

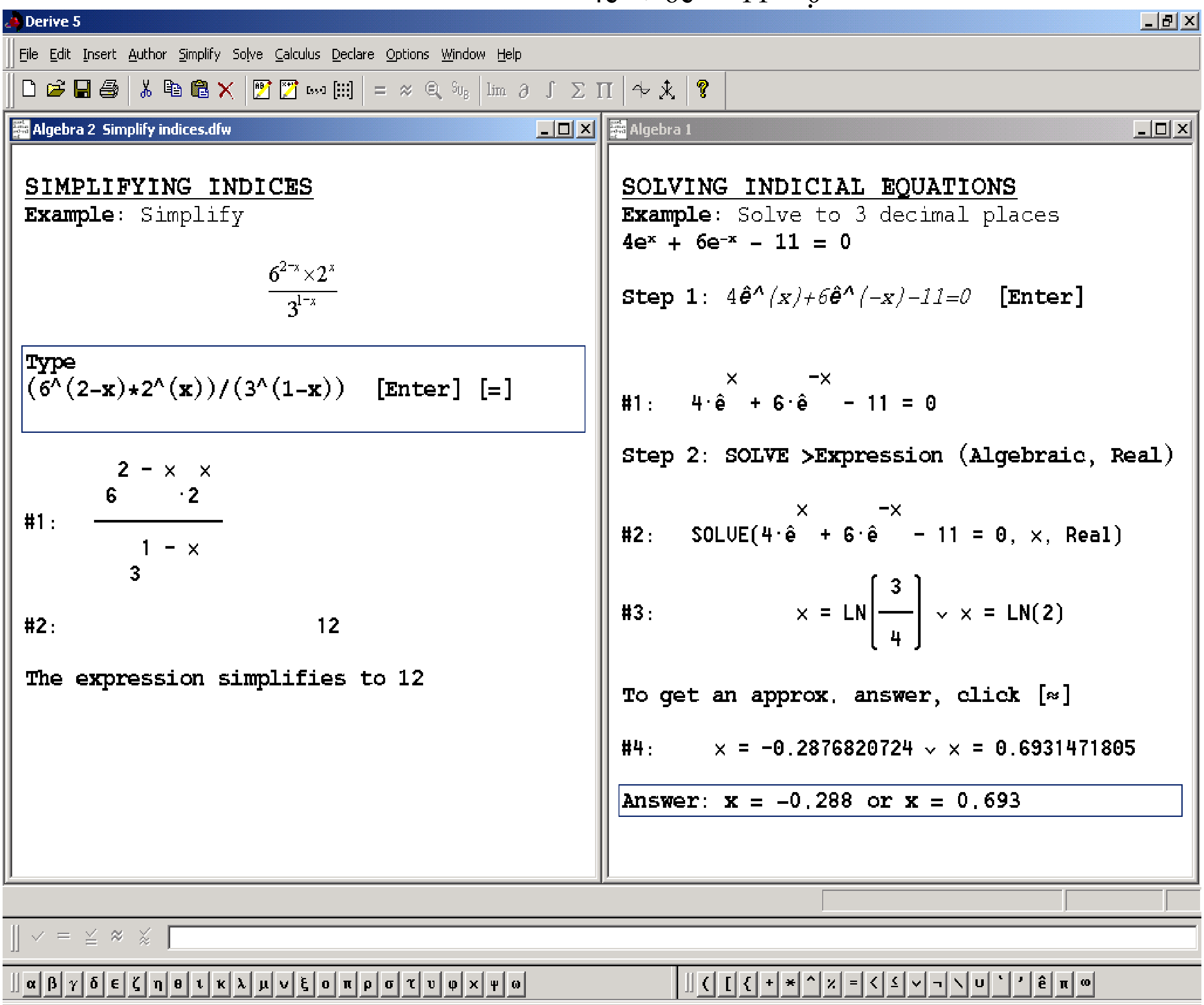
(a) Simplifying

Example: Simplify $\dfrac{6^{2-x} \times 2^{x}}{3^{1-x}}$

(b) Solving indicial equations

Example: Solve to 3 decimal places

$4e^{x} + 6e^{-x} - 11 = 0$

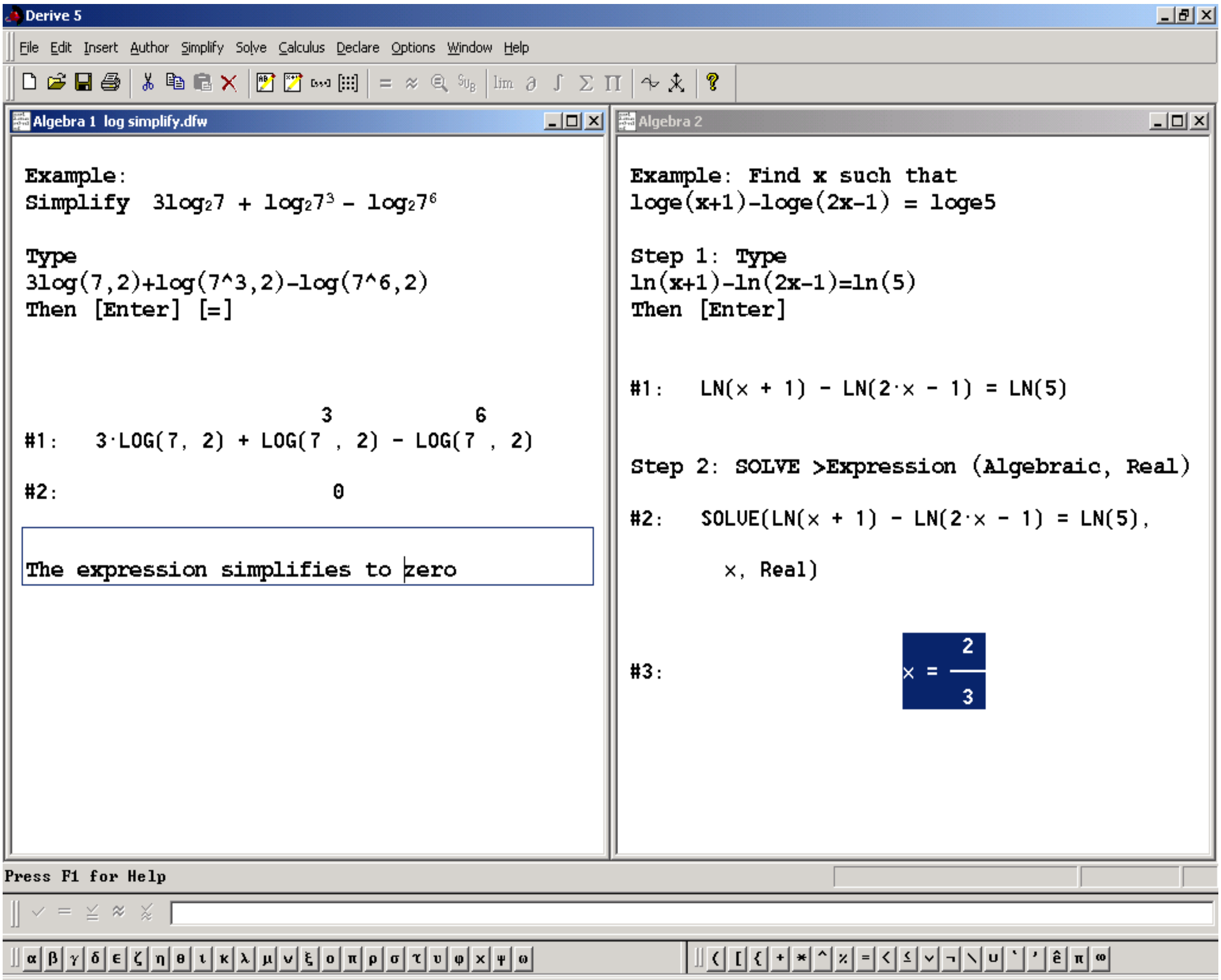


5. Logarithms

NOTE: ***$log_3 x$* is entered as *log(x,3)* in DERIVE**
$log_e x$* is entered as *ln(x)* in DERIVE, or as *log(x,ê)

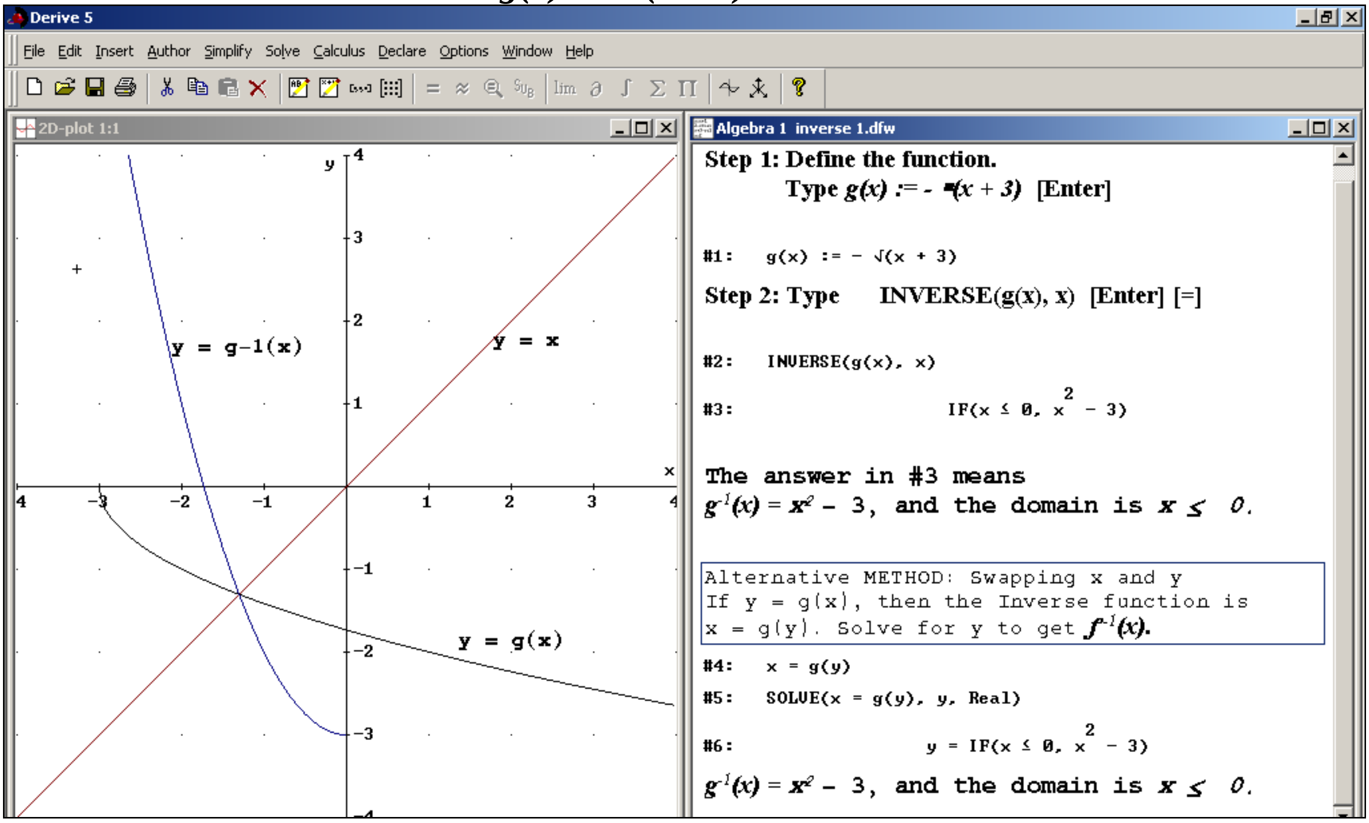
(a) Simplifying
Example: Simplify
$\log_4(x-2)^3 - \log_4(x-2)$

(b) Solving log equations
Example: Solve
$\log_e(x+1) - \log_e(2x-1) = \log_e 5$



6. Inverse functions

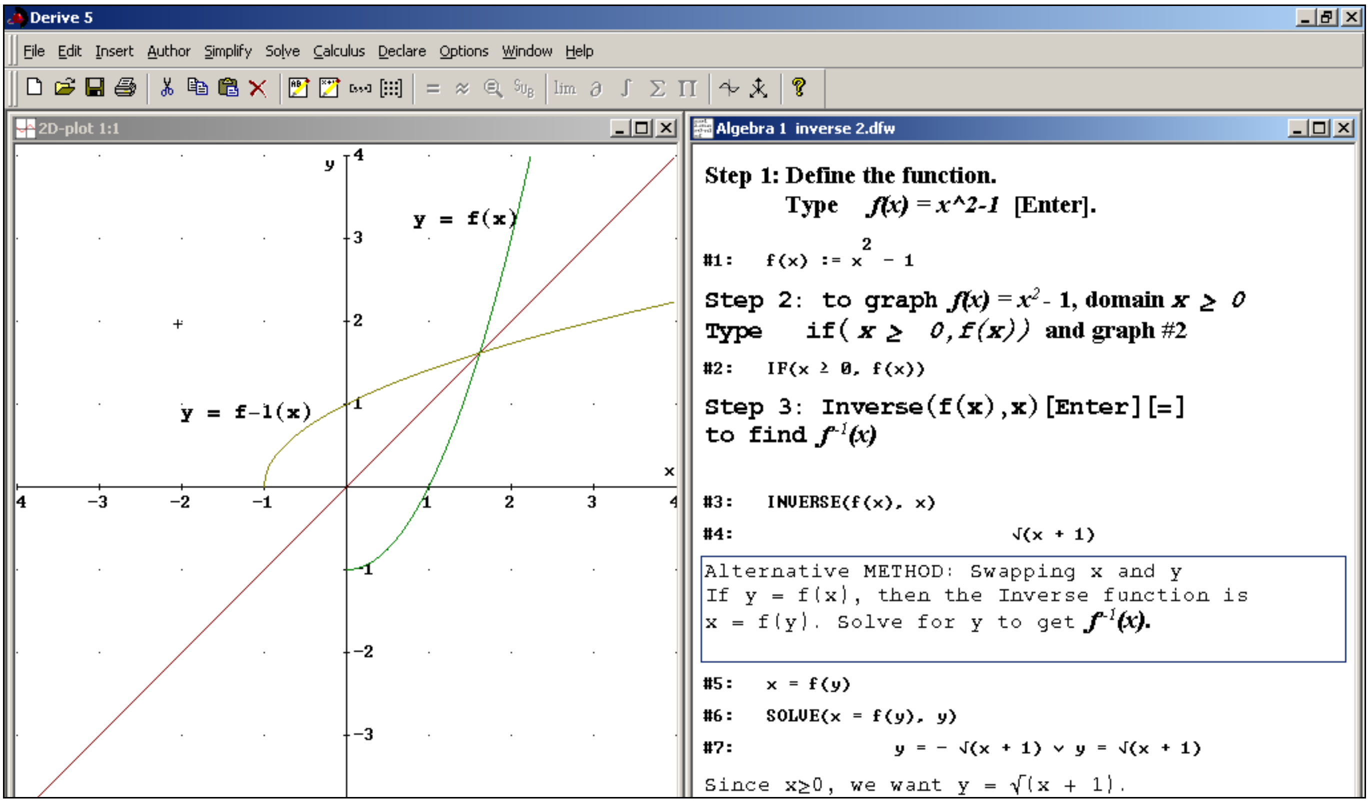
- **Find the inverse function of $g(x) = -\sqrt{(x + 3)}$.**



Restricted Domain

Example:

- **Find the inverse function of $f(x) = x^2 - 1$, domain $x \geq 0$**
- **Draw graphs of $f(x)$, $f^{-1}(x)$ and $y = x$, on the same set of axes**

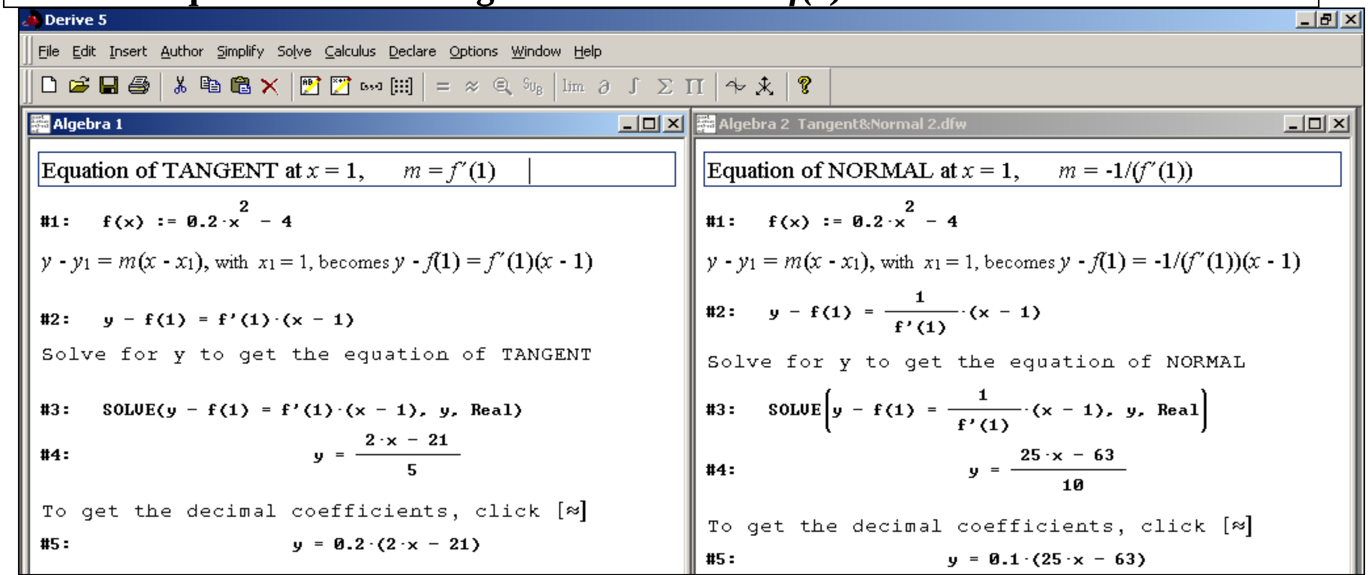


7. Tangents & Normals

Method 1: Using $y - y_1 = m(x - x_1)$

EXAMPLE

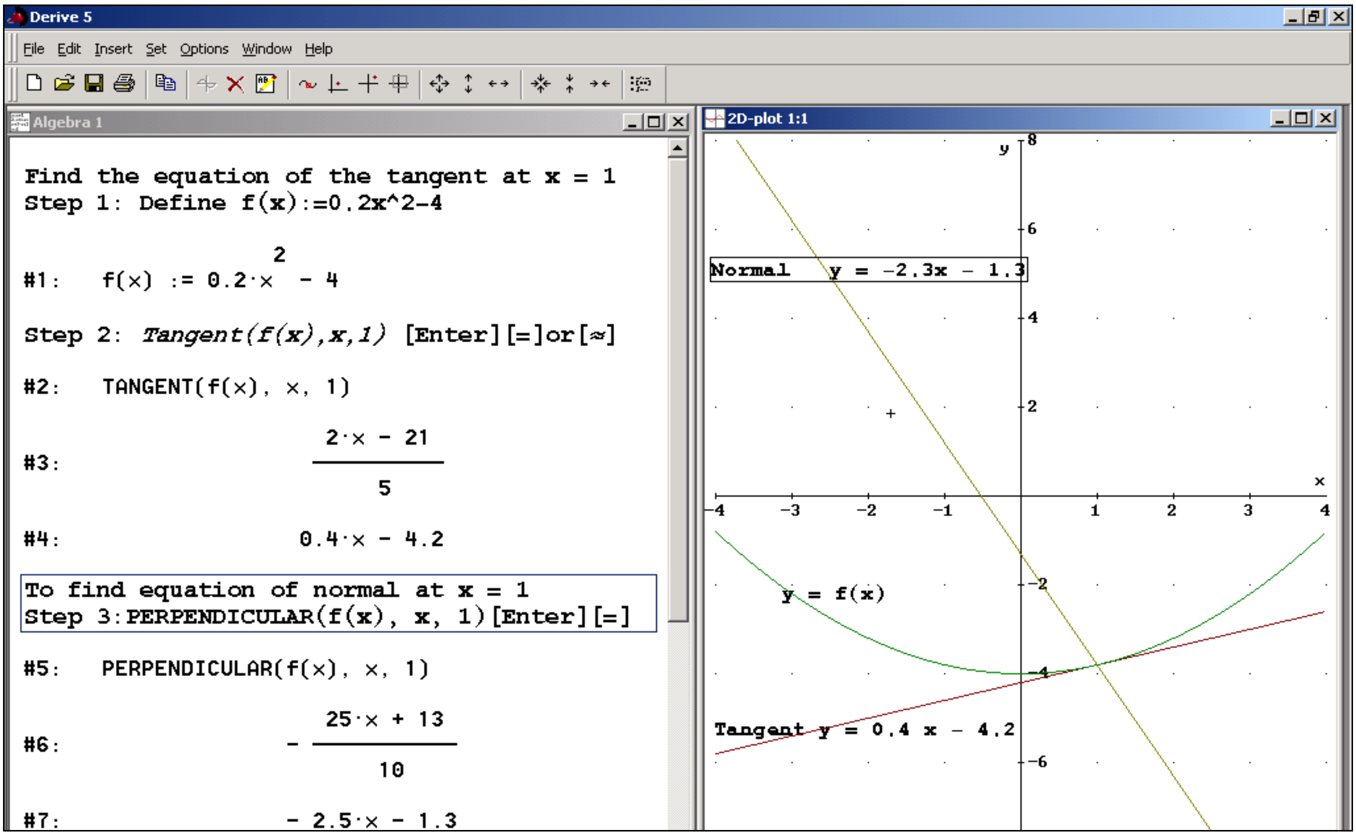
Find the equations of the tangent and normal to $f(x) = 0.2x^2 - 4$ **at** $x = 1$



Method 2: Using ***TANGENT(f(x),x,1) and PERPENDICULAR(f(x),x,1)***

Find the equations of the tangent and normal to $f(x) = 0.2x^2 - 4$ **at** $x = 1$

USING DERIVE



NOTE: The equation of the tangent must NOT be written as: $0.4\,x - 4.2$.
The equation MUST be written as: $y = 0.4\,x - 4.2$

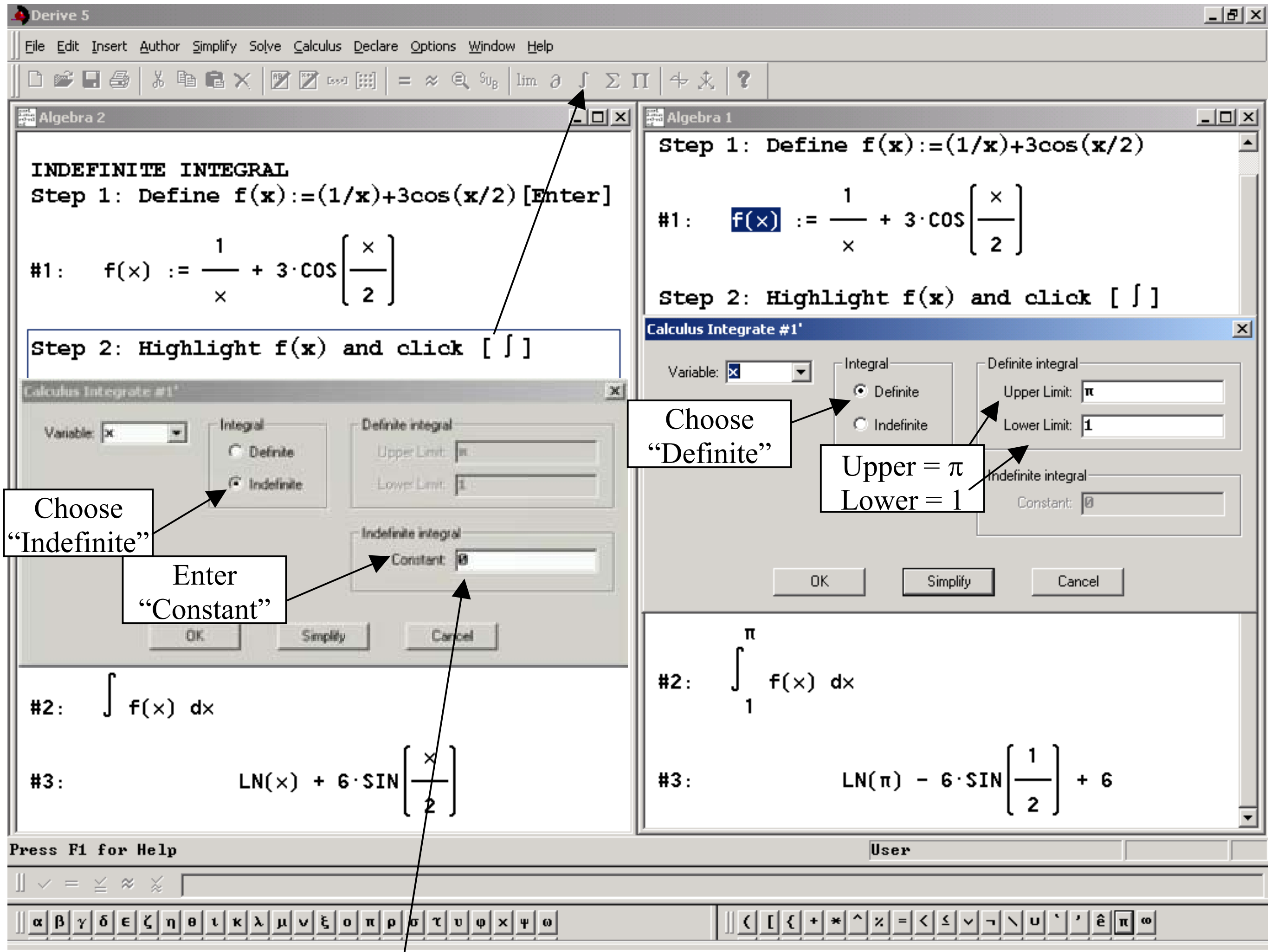
8. Integration

(a) Indefinite integral

Find $\int\left(\frac{1}{x}+3\cos\frac{x}{2}\right)dx$

(b) Definite integral

Evaluate $\int_1^{\pi}\left(\frac{1}{x}+3\cos\frac{x}{2}\right)dx$



Further Notes on Indefinite Integral

(1) #3 (left window) gives the *exact* answer to $\int\left(\frac{1}{x}+3\cos\frac{x}{2}\right)dx$ = ***$\log_e x$ + 6sin(x/2) + c***

(2) You can set the "constant of integration" as *c*, *0*, or any other value.

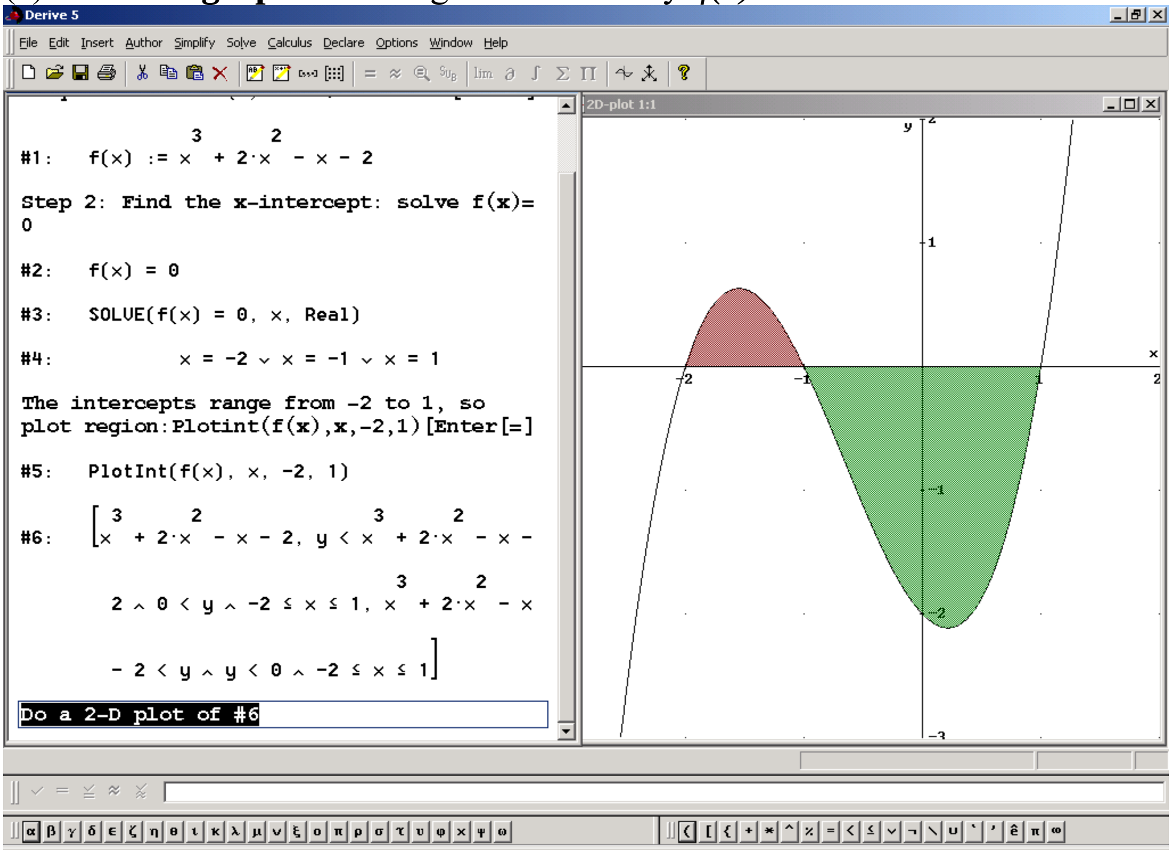
Further Notes on Definite Integral

(3) #3 (right window) gives the *exact* answer to $\int_1^{\pi}\left(\frac{1}{x}+3\cos\frac{x}{2}\right)dx$ = ***$\log_e\pi$ + 6sin(1/2) + 6.***

(4) To obtain a rational (decimal) approximation, highlight #3 and click [≈].

USING DERIVE : 9. Signed region

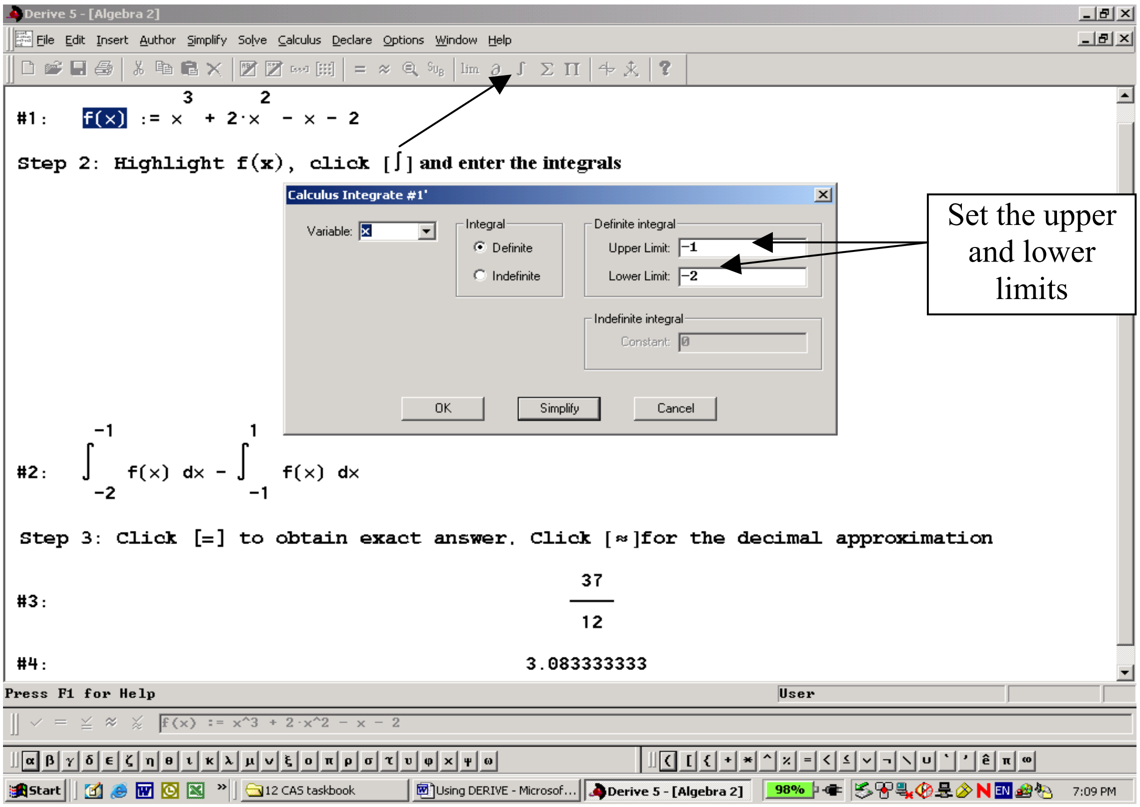
(a) Draw the **graph** of the region bounded by $f(x) = x^3 + 2x^2 - x - 2$ and the x-axis.



(b) Area of signed region

Find the **area** of the region bounded by $f(x) = x^3 + 2x^2 - x - 2$ and the x-axis.

NOTE: From the graph, Area = $\int_{-2}^{-1} f(x)dx - \int_{-1}^{1} f(x)dx$



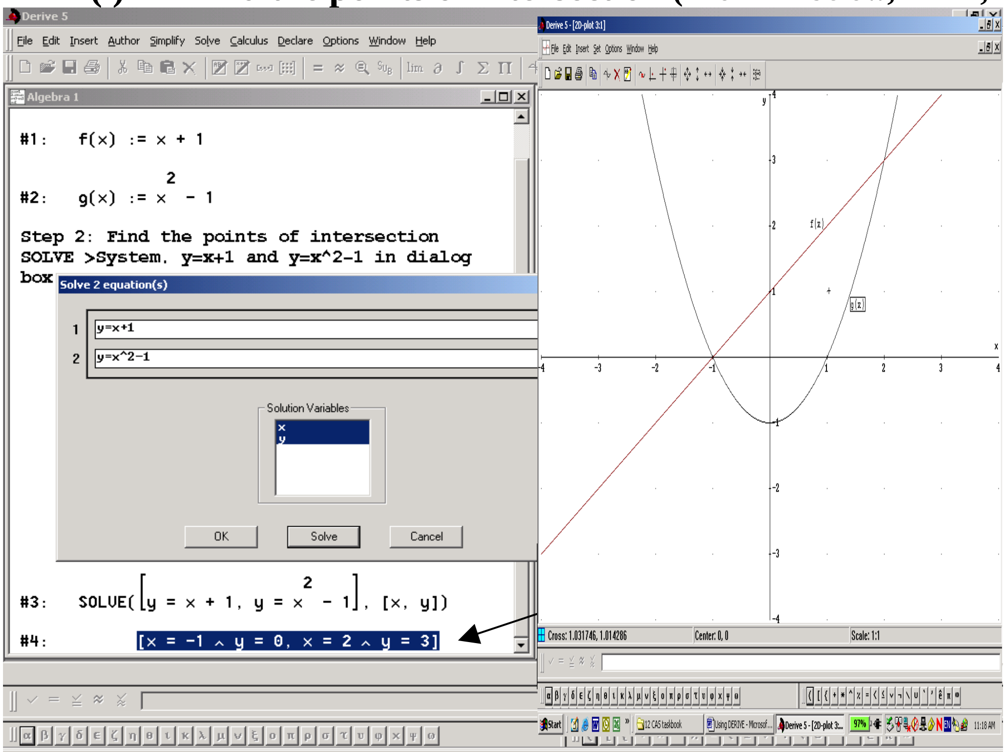
(c) Area between two curves

NOTE: Area = $\int_a^b [f(x) - g(x)]dx$, if *f(x) > g(x)* in the interval [a,b]

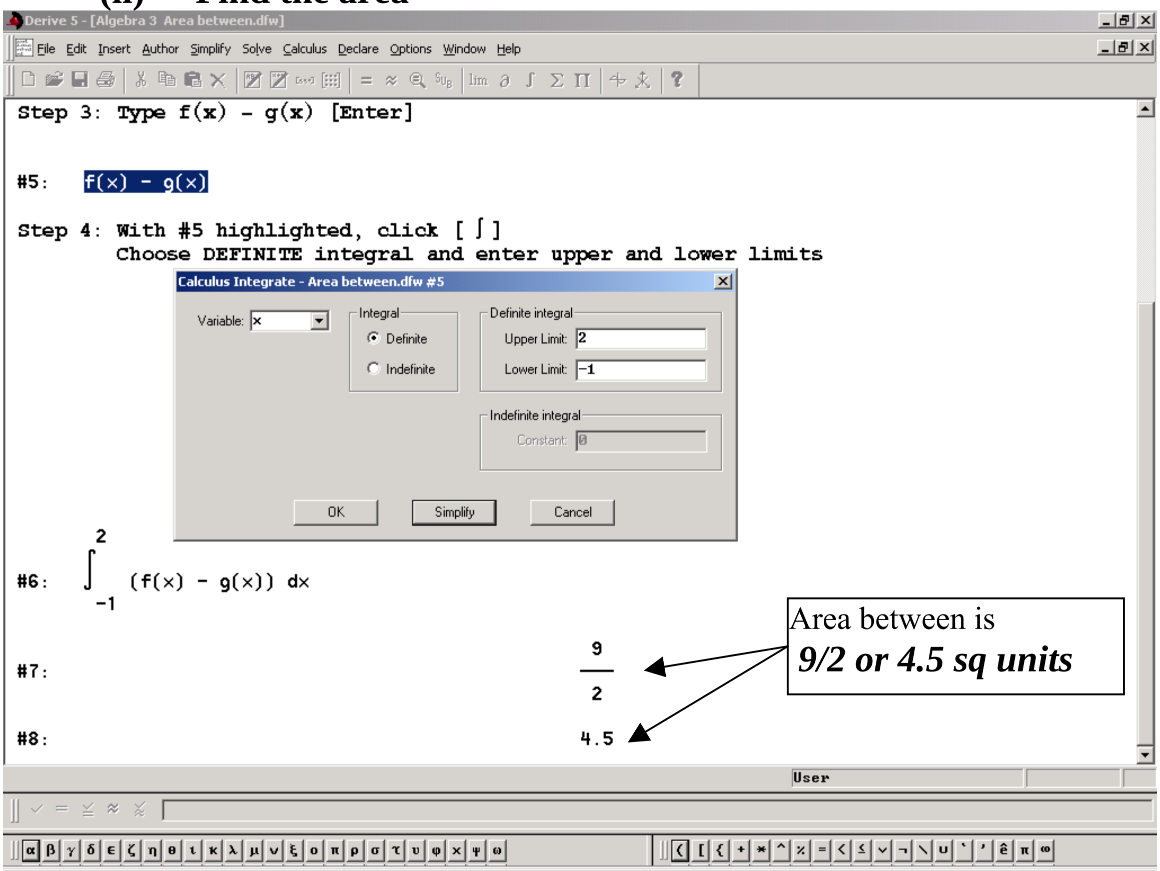
Example

Find the **area** of the region bounded by *f(x) = x + 1 and g(x) =* x^2 *- 1.*

(i) Find the points of intersection (From #4 below, x = -1, x = 2)



(ii) Find the area



USING DERIVE
10. Binomial Distribution

- Binomial coefficients, ${}^nC_x$, evaluated in DERIVE with " **comb(*n*,*x*)**
- $\Pr(X = x) = {}^nC_x\, p^x (1-p)^{n-x}$, evaluated in DERIVE with " **binomial_density(*x*,*n*,*p*)** "
- $\Pr(X \le a)$, evaluated in DERIVE with $\sum_{x=0}^{a} binomial\_density(x,n,p)$
- $\Pr(X \ge a)$ evaluated in DERIVE with $\sum_{x=a}^{n} binomial\_density(x,n,p)$
- $\Pr(a \le X \le b)$ evaluated in DERIVE with $\sum_{x=a}^{b} binomial\_density(x,n,p)$

Example:

Jo rolls a die 4 times (**n = 4**). She defines "success" as rolling an even number (**p = 0.5**).
The random variable, X, denotes "rolling an even number".

(a) What is the probability of rolling exactly 2 even numbers ($\Pr(X = 2)$)?
(b) What is the probability of rolling no more than 2 even numbers ($\Pr(X \le 2)$)?
(c) What is the probability of rolling at least 3 even numbers ($\Pr(X \ge 3)$)
(d) What is the probability of rolling between 1 and 3 even numbers ($\Pr(1 \le X \le 3)$)

Algebra 1 Binomial.dfw

Jo rolls a die 4 times (**n = 4**). "Success" is rolling an even number (**p = 0.5**).
(a)What is the probability of rolling exactly 2 even numbers (Pr(**X = 2**))?

#1: BINOMIAL_DENSITY(2, 4, 0.5)

#2: 0.375

(b)What is the probability of rolling no more than 2 even numbers Pr(X ≤2)

#3: BINOMIAL_DENSITY(x, 4, 0.5)

With #3 highlighted, click 'Σ'button.
Enter 'Lower limit'=0, 'Upper limit'=2

#4: $\sum_{x=0}^{2}$ BINOMIAL_DENSITY(x, 4, 0.5)

#5: 0.6875

(c)What is the probability of rolling at least 3 even numbers (Pr(X ≥3))
Copy and paste #4, but change the limits to " x,3,4 "

#6: $\sum_{x=3}^{4}$ BINOMIAL_DENSITY(x, 4, 0.5)

#7: 0.3125

What is the probability of rolling between 1 and 3 even numbers (Pr(1≤ X ≤3)
Copy and paste #4, but change the limits to " x,1,3 "

#8: $\sum_{x=1}^{3}$ BINOMIAL_DENSITY(x, 4, 0.5)

#9: 0.875

USING DERIVE
11. Hypergeometric Distribution

- Binomial coefficients, ${}^nC_x = \binom{n}{x}$, evaluated in DERIVE with " **comb(*n*,*x*)**
- $\Pr(X = x) = \dfrac{\binom{D}{x}\cdot\binom{N-D}{n-x}}{\binom{N}{n}}$, evaluated in DERIVE with ***hypergeometric_density(x,n,D,N)***
- $\Pr(X \le a)$, evaluated in DERIVE with $\sum_{x=0}^{a} hypergeometric\_density(x,n,D,N)$
- $\Pr(X \ge a)$ evaluated in DERIVE with $\sum_{x=a}^{n} hypergeometric\_density(x,n,D,N)$
- $\Pr(a \le X \le b)$ evaluated in DERIVE with $\sum_{x=a}^{b} hypergeometric\_density(x,n,D,N)$

Example:
Eggs are sold in cartons of 12 (**N = 12**). A carton contains 5 brown eggs (**D = 5**). Alex selects 3 eggs at random (without replacement) and breaks them to make an omelet (**n = 3**).

(a) What is the probability of selecting exactly 2 brown eggs ($\Pr(X = 2)$)?
(b) What is the probability of selecting no more than 2 brown eggs, $\Pr(x \le 2)$?
(c) What is the probability of selecting at least 2 brown eggs, $\Pr(x \ge 2)$?
(d) What is the probability of selecting between 1 and 3 brown eggs, $\Pr(1 \le x \le 3)$?

Eggs are sold in cartons of 12 (**N = 12**). A carton contains 5 brown eggs (**D = 5**). Alex selects 3 eggs at random and breaks them to make an omelet (**n = 3**).

(a) What is the probability of selecting exactly 2 brown eggs (**Pr(X = 2))?**

#1: HYPERGEOMETRIC_DENSITY(2, 3, 5, 12)

#2: $\dfrac{7}{22}$

(b) What is the probability of selecting no more than 2 brown eggs, $\Pr(X \le 2)$

#3: HYPERGEOMETRIC_DENSITY(x, 3, 5, 12)

With #3 highlighted, click 'Σ'button.
Enter 'Lower limit'=0, 'Upper limit'=2

#4: $\sum_{x=0}^{2}$ HYPERGEOMETRIC_DENSITY(x, 3, 5, 12)

#5: $\dfrac{21}{22}$

(c) What is the probability of selecting at least 2 brown eggs, **$\Pr(X \ge 2)$?**
Copy and paste #4, but change the limits to " x,2,3 "

#6: $\sum_{x=2}^{3}$ HYPERGEOMETRIC_DENSITY(x, 3, 5, 12)

#7: $\dfrac{4}{11}$

(d) What is the probability of selecting between 1 and 3 brown eggs , $\Pr(1 \le X \le 3)$?
Copy and paste #4, but change the limits to " x,1,3 "

#8: $\sum_{x=1}^{3}$ HYPERGEOMETRIC_DENSITY(x, 3, 5, 12)

#9: $\dfrac{37}{44}$

USING DERIVE

12. Normal Distribution

- $\Pr(X < a)$, evaluated in DERIVE with "**normal(*a*,*μ*,*σ*)**"
- $\Pr(X > a) = 1 - \Pr(X < a)$ evaluated in DERIVE with "1 – normal(*a*,*μ*,*σ*)"
- $\Pr(a < X < b) = \Pr(X < b) - \Pr(X < a)$

  evaluated in DERIVE with "normal(*b*,*μ*,*σ*) – normal(*a*,*μ*,*σ*)".

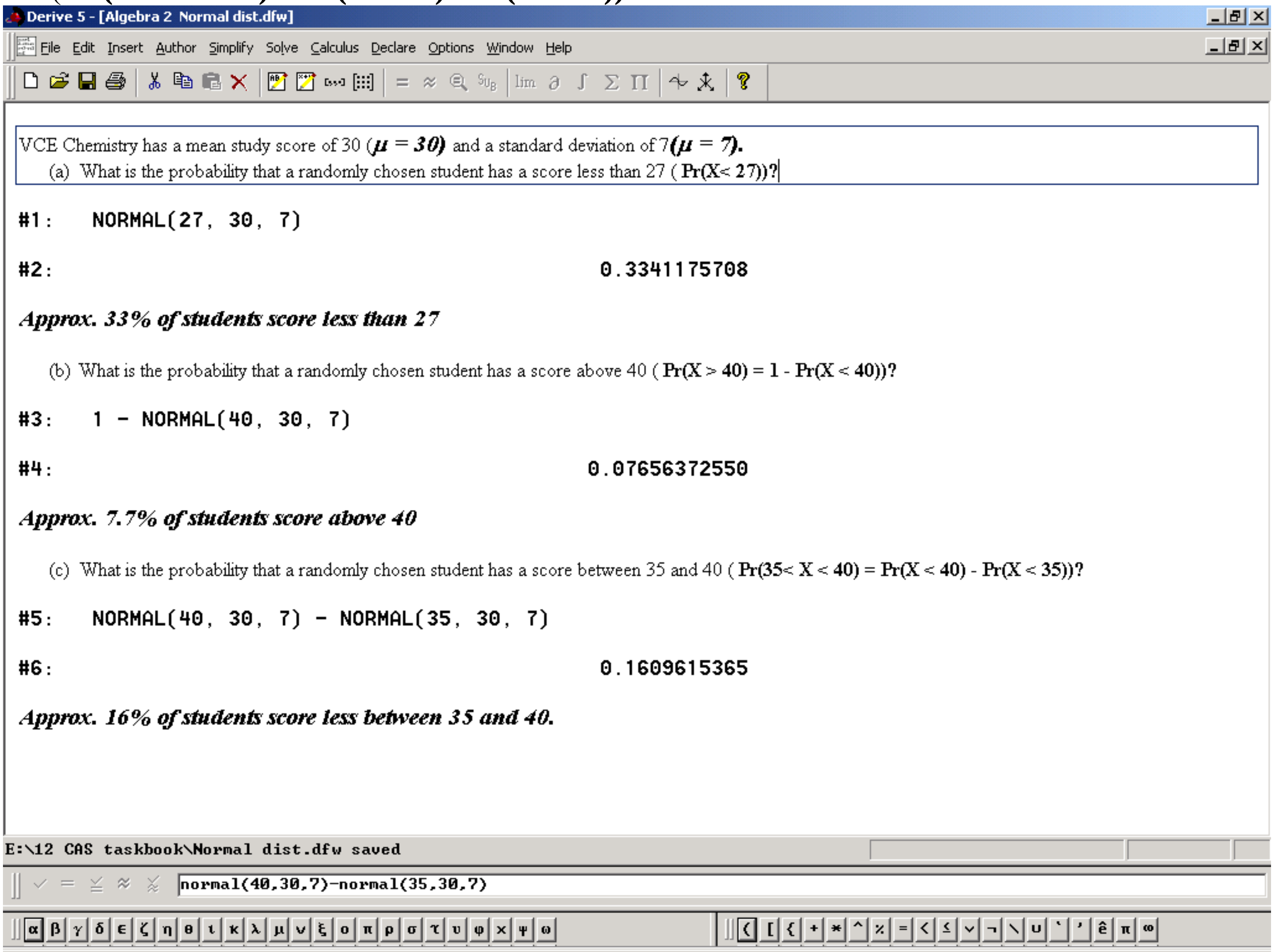
Example: Assume *X* = "VCE study score" is normally distributed.

VCE Chemistry has a mean study score of 30 (***μ* = 30)** and a standard deviation of 7**(*μ* = 7).**

(a) What is the probability that a randomly chosen student has a score less than 27 (**Pr(X< 27))?**

(b) What is the probability that a randomly chosen student has a score above 40
(**Pr(X > 40) = 1 - Pr(X < 40))?**

(c) What is the probability that a randomly chosen student has a score between 35 and 40
(**Pr(35< X < 40) = Pr(X < 40) - Pr(X < 35))?**



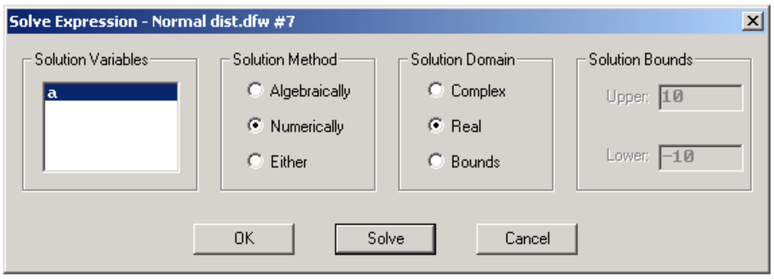
Inverse Normal Distribution

EXAMPLE 1:

X is a normally distributed random variable with mean 30 ($\mu = 30$) and standard deviation 7 ($\sigma = 7$).
Find the value of *a* such that $\Pr(X < a) = 0.95$.

X is normally distributed with mean 30 (**μ = 30**) and standard deviation 7 (**σ = 7**).
Find the value of **a** such that **Pr(X < a) = 0.95**.

Need to solve NORMAL(a,30,7)=0.95, numerically
Step 1: Type NORMAL(a,30,7)=0.95, then [Enter]
Step 2: SOLVE >Expression. Select "Numerically" and "Real". Click "Solve" botton



#7: NORMAL(a, 30, 7) = 0.95

#8: NSOLVE(NORMAL(a, 30, 7) = 0.95, a, Real)

#9: a = 41.51397526

This means that 95% of students have a study score below 41.5

EXAMPLE 2: Assume X = "weight of eggs" is normally distributed
25% of eggs produced on a particular farm weigh less than 40 gram.
That is, $\Pr(X < 40) = 0.25$.
The mean weight is known to be 55 grams ($\mu = 55$).
Find the standard deviation ($\sigma = ?$).

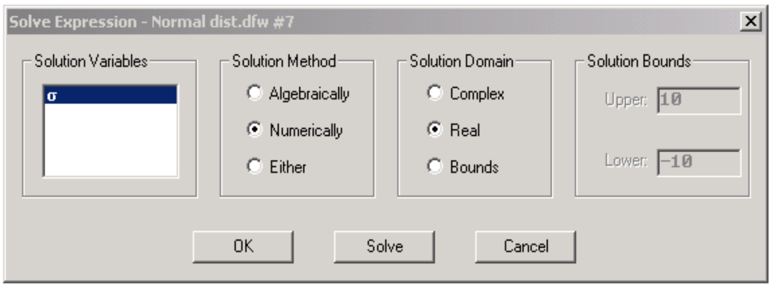
EXAMPLE 2: Assume X = "weight of eggs" is normally distributed
25% of eggs produced on a particular farm weigh less than 40 gram. That is, **Pr(X < 40) = 0.25**.
The mean weight is known to be 55 grams (**μ = 55**).
Find the standard deviation (**σ = ?**).

Need to solve NORMAL(40,55,σ) = 0.25, numerically

STEP 1: Type NORMAL(40,55,σ) = 0.25, then [ENTER]

#7: NORMAL(40, 55, σ) = 0.25

STEP 1: SOLVE >Expression. In dialog box select: "Numerically" and "Real". Click "Solve" button



#8: NSOLVE(NORMAL(40, 55, σ) = 0.25, σ, Real)

#9: σ = 22.23903179

The standard deviation is 22 grams