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Slope/Triangle Area Exploration

ID: 9863

Time required
60 – 90 minutes

Topics: Linear Functions, Triangle Area, Rational Functions

- *Graph lines in slope-intercept form*
- *Find the coordinate of the x- and y-intercepts of a line.*
- *Calculate the area of a triangle.*
- *Identify vertical, horizontal, and oblique asymptotes for a rational function.*
- *Find values of functions on a graph.*

Activity Overview

In this activity, students investigate the following problem (Adapted from Philips Exeter Academy, 2007):

> The equation $y - 5 = m\,(x - 2)$ represents a line, no matter what value m has.
> (a) What are the x- and y-intercepts of this line?
> (b) For what value of m does this line form a triangle of area 36 square units with the positive x- and y-axes?
> (c) Describe the areas of the first-quadrant triangles.

Using multiple representations afforded by the TI-Nspire CAS – in particular, graphs, spreadsheets and scatter plots – students will explore how the area of a triangle formed by the x- and y-axes and a line through the point (2,5) is related to the slope of the line.

This activity is appropriate for students in Algebra 1, Geometry, and Algebra 2. It is assumed that students understand how to graph lines in point-slope form and that students know how to find the area of a triangle. For students in Algebra 2, knowledge of rational functions and asymptotes is helpful. However, knowledge of rational functions is not a prerequisite for this activity. In fact, this activity can be used as an introduction to rational functions by providing concrete examples of vertical asymptotes, oblique asymptotes, and zeros. This activity will allow students to begin to explore the concept of a limit and rates of change.

Questions for further investigation will allow students to explore similar triangles, triangle centers, and to formulate generalizations relating the coordinates of the pivot point to the minimum triangle area.

Teacher Preparation

- *The following pages provide a preview of the TI-Nspire CAS document and function as the Teacher Answer Key.*
- *If you are planning on having your students investigate this activity individually or in pairs, you will need to download the .tns files to the student handhelds before hand. .*
- *Distribute the student worksheet to your class.*

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Classroom Management

- *Pages in this .tns activity can be used by students individually or in pairs. The entire activity is appropriate for a whole-class format using the TI-Nspire CAS handheld or TI-Nspire CAS computer software and the questions found in this document. Depending on your students, the only part of this activity that may require teacher guidance is when students are deriving the triangle area formula.*
- *All the guided inquiry questions for your students are on notes pages in the .tns file and are included in the Teacher Answer Key. The guided inquiry questions can be used to engage in a whole-class discussion. In addition to possible content-specific things that may be addressed in this activity, there are informal introductions to pre-calculus and Calculus concepts that should be pointed out.*
- *Optional investigations are provided at the end of this activity in Problems 4 and 5 of the student .tns file. You may delete these problems from the .tns file if you prefer.*

TI-Nspire™ Applications

Graphs & Geometry, Lists & Spreadsheet, Notes

Reference

Philip Exeter Academy (July 2007). *Mathematics 2 (Problem 1, page 25).* Available online at http://www.exeter.edu/documents/math2all.pdf

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TEACHER ANSWER KEY

Problem 1 – Problem Introduction

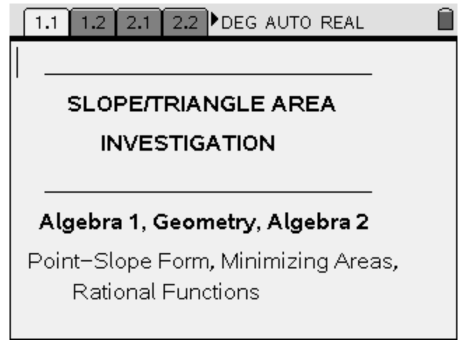
Pages 1.1 and 1.2 of this document introduce the problem. Students should be encouraged to explore this problem *by hand* prior to working through the document. One suggestion is a whole-class activity in which pairs of students calculate the intercepts and areas for random values of the slope of the line. This data is then organized in a table for the entire class to analyze. Guided inquiry questions are:

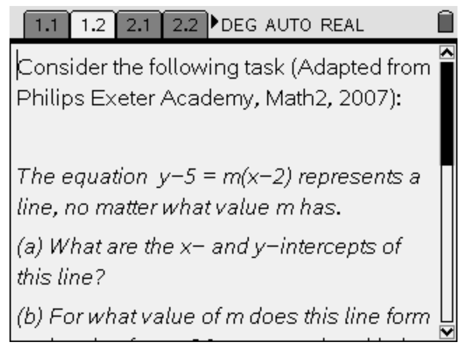
1. As the slope changes, how does the *y*-intercept change?

   Answer:In order to maintain a triangle in the first quadrant, the slope of the line must always be negative. As the line gets "steeper" (or as the slope of the line decreases or as the absolute value of m increases – both of these characterizations of the slope may be difficult for students to verbalize), the value of the y-intercept increases (without bound). As the slope of the line approaches zero, the y-intercept approaches (0,5).
2. As the slope changes, how does the *x*-intercept change?

   Answer: As the line gets "steeper," (or as the slope of the line decreases) the x-intercept approaches (2,0), but never reaching (2,0). As the slope of the line approaches zero, the value of the x-intercept increases without bound.
3. As the slope changes, how does the triangle area change?

   Answer: The answer to this question is the goal of this investigation. The purpose of this "by hand" part of the investigation is for students to get a "feel" for how the area of the triangle depends on the slope of the line. As the slope of the line changes from values just less than zero to values much less than zero, the area of the triangle decreases from a very large area to an area close to 20 square units, then increases to a very large area.





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Problem 2 – Initial Investigation

The next two pages (2.1 and 2.2) allow students to explore a dynamic first-quadrant triangle. The questions on page 2.1 are intended to get the students to focus on manipulating the slope (ultimately, the independent variable in this problem) and its effect on the area of the first-quadrant triangle.

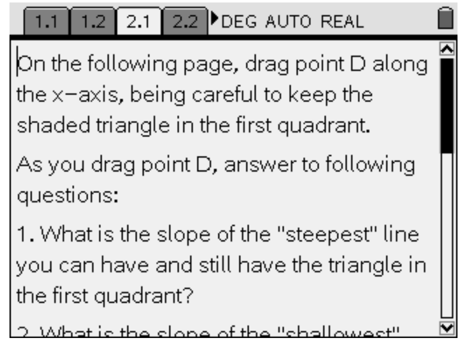
4. What is the slope of the "steepest" line you can have and still have the triangle in the first quadrant?
   Answer: The characterizations of the slope may be difficult for the students to verbalize. The idea is that the slope can be "a very large" negative number (the absolute value of the slope is very large). There is no single correct numerical answer. The slope may be -100, or -1000, or -10000, as long as the line is not a vertical line.

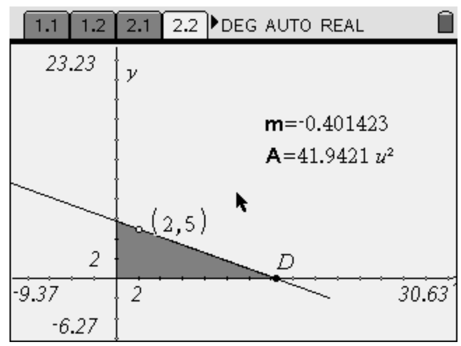
5. What is the slope of the "shallowest" line you can have and still have the triangle in the first quadrant?
   Answer: As with the answer to #5, the slope must be negative, just less than zero. Again, there is no correct numerical answer, as long as the line is not a horizontal line.

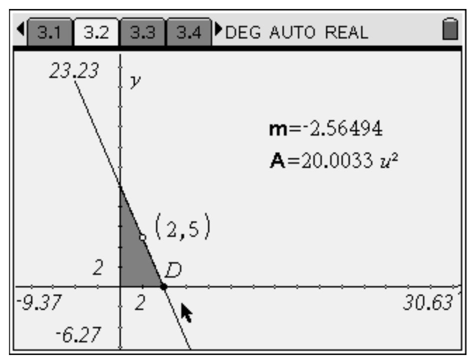
6. Where is the triangle when the slope is positive?
   Answer: When the slope is greater the 2.5, the triangle is located in the 4th quadrant. When the slope is between 0 and 2.5, the triangle is in the 2nd quadrant. When the slope is 2.5, there is not triangle.

7. What happens to the triangle when the slope is zero?
   Answer: When the slope is zero, there is not x-intercept, so there is no triangle (just an unbounded region).

8. What happens to the triangle when the slope is undefined?
   Answer: When the slope is undefined, the line is vertical, and there is no y-intercept. There is no triangle, just an unbounded region.







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9. What values of the slope always produce a triangle in the first quadrant?

   Answer: Any slope that is negative.

10. What value(s) of the slope produces a triangle with the smallest possible area?

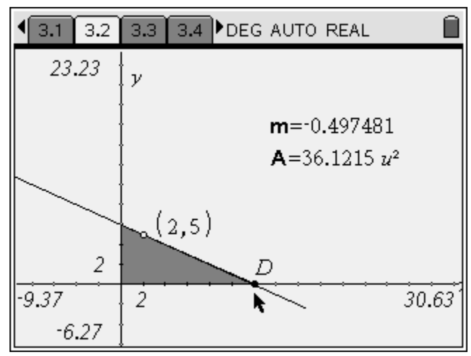
    Answer: -2.5

11. What value(s) of the slope produces a triangle with an area of 36 square units?

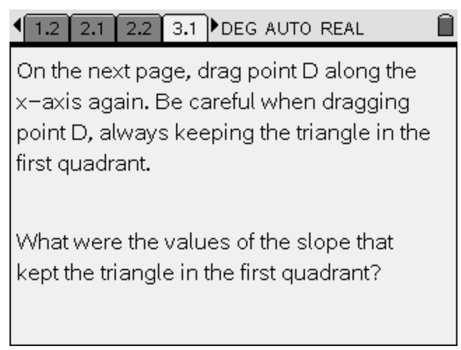
    Answer: -0.5 and -12.5

Page 3.2 at right illustrates an anticipated student response to question #7 displaying an area close to 20 square units when the slope is close to −2.5.

Page 3.2 at right illustrates an anticipated student response to question #8 displaying an area close to 36 square units when the slope is close to −0.5.



Problem 3 – Multiple Representations
These pages – there are 17 of them – are intended to lead to students through an exploration of this problem from multiple points of view.



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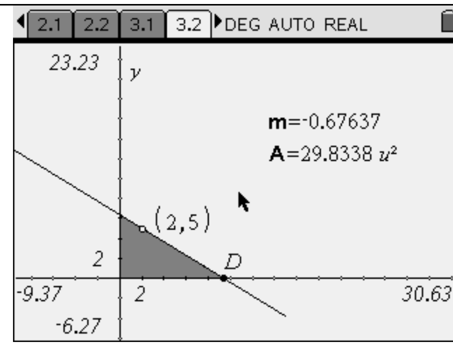
Page 3.2 is the page that generates the slope and area data for the entire problem. As students drag point D to generate the data, they should drag point D *slowly*, being sure to generating data for just the first-quadrant triangle.



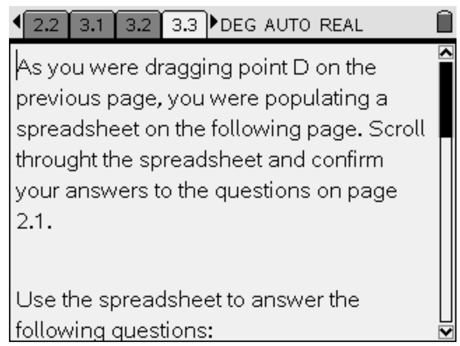
Pages 3.3 and 3.4 – Exploring data in a spreadsheet. The data generated on page 3.2 is displayed here. Guided questions for exploring this data are:

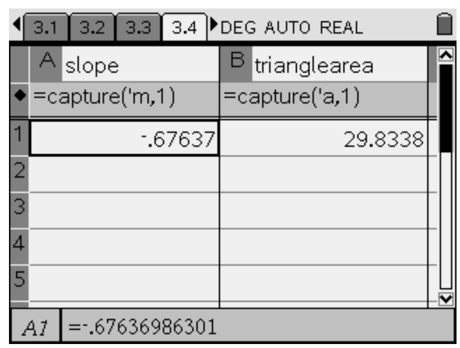
12. Find the slope of a line that produces a triangle with an area close to 40 square units.
    Answer: Approximately -14.57 and -0.43.

13. Find the slope of a line that produces an area close to 30 square units.
    Answer: Approximately -9.33 and -0.67

14. Find a slope that is close to -2 and another that is close to -3. What is the difference in the areas of the triangles?
    Answer: When the slope is -2, the area is approximately 20.25. When the slope is -3, the area is approximately 20.1667. The difference in the areas is approximately 0.083.

15. Find a slope that is close to -4 and another that is close to -5. What is the difference in the areas of the triangles?
    Answer: When the slope is -4, the area is approximately 20.125. When the slope is -5, the area is approximately 22.5. The difference in the area is approximately 1.375.





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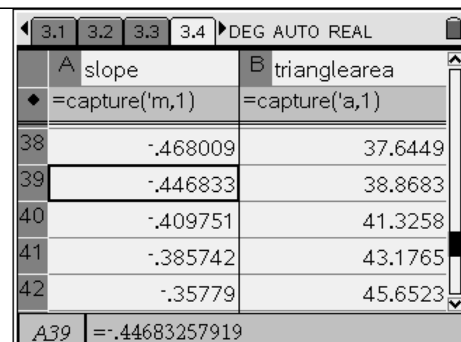
16. Find a slope that is close to -0.2 and another that is close to -0.3. What is the difference in the areas of the triangles?
    Answer: When the slope is -0.2, the area is approximately 72.9. When the slope is -0.3, the area is approximately 52.27. The difference in the areas is approximately 20.63.

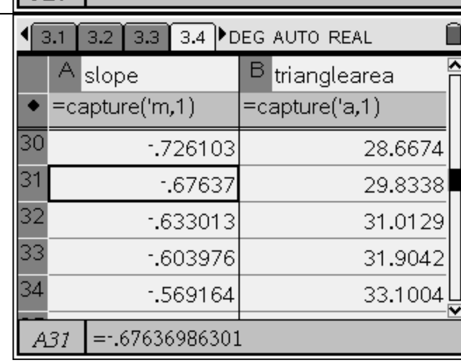
17. Find a slope that is close to -0.4 and another that is close to -0.5. What is the difference in the areas of the triangles?
    Answer: When the slope is -0.4, the area is approximately 42.05. When the slope is -0.5, the area is approximately 36. The difference in the areas is approximately 6.05.

18. For what values of the slope does the area of the triangle appear to be changing the fastest?
    Answer: The area of the triangle is changing fastest for values of the slope close to zero, for very small changes in the value of the slope produce very large changes in the value of the area.

Page 3.4 at right shows an anticipated student response to question #1, displaying an area of 38.8683 square units when the slope is −0.446833.



Page 3.4 at right shows an anticipated student response to question #2, displaying an area of 31.0129 square units when the slope is −0.67637.



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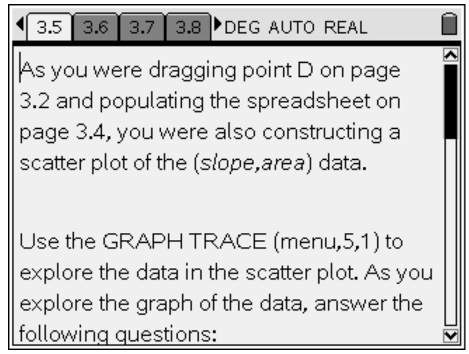
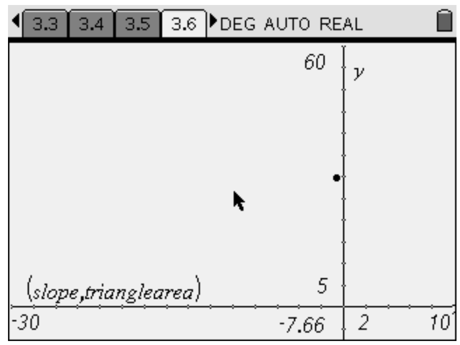
Paged 3.5 and 3.6 – Exploring data in a scatter plot

On these pages, the data generated on page 3.2 and collected in the spreadsheet on page 3.4 is displayed in a scatter plot on page 3.6. Questions for guided inquiry are:





19. The data looks like it gets very close to the y-axis. Will the graph of the data ever cross the y-axis? Use the slope of the line and the area of the triangle to explain your answer.
    Answer: The graph of the data will never cross the y-axis. Any data values on the y-axis represent a slope of zero, which is a horizontal line. There is no triangle when the line is horizontal.

20. Will the graph of the data ever cross the negative x-axis? Use the slope of the line and the area of the triangle to explain your answer.
    Answer: The graph of the data will never cross the negative x-axis. Values of data on the x-axis represent triangles with an area of zero. With the slope of the line always being negative, the area of the first-quadrant triangle will always be positive.

21. Why does the graph of the data show that there are two triangles with an area of 22 square units?
    Answer: If you draw a horizontal line a y=22, it will cross the graph of the data at two distinct points, showing that there are two distinct slopes that produce triangles with an area of 22 square units.

22. The data appears to have a "U" shape. Explain why the slope of the line and the area of the triangle lead to data that has a "U" shape.
    Answer: As the slope is very close to zero (but still negative), the area is very large. As the slope of the line becomes smaller, the area of the triangle decreases, reaching a minimum at x=-2.5, then increasing for values of x less than -2.5.

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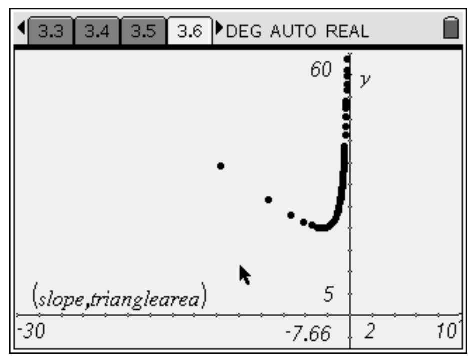
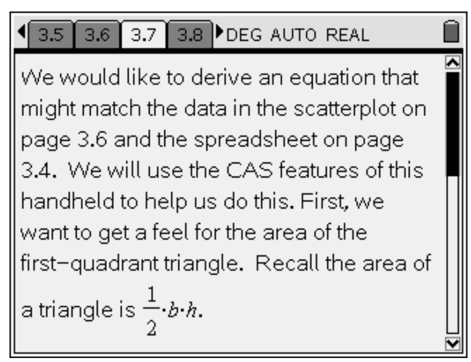
23. THOUGHT EXPERIMENT: What do you think would happen to the graph of the data if you used the point (5,2) as the pivot point instead of the point (2,5)? Explain your reasoning using the slope of the line and the area of the triangle.
Answer: The graph of the data would have the same basic shape. The slope of the line that produces the minimum area is -0.4(though the students do not know this yet), so the minimum value of the graph will move closer to the y-axis. The minimum area is 20 square units (though the students do not know this yet, either).

Page 3.6 at right shows an anticipated student scatter plot.



Pages 3.7 through 3.10 – Deriving an area formula using CAS

On the next four pages, students are guided through a derivation of a formula for the area of the triangle in terms of the slope of the line. This formula is stored at *f1*(*x*) and is used to answer questions in the problem. Guided questions prior to the derivation are intended to have students consider what goes into calculating the area of a triangle.



24. Using the axis intercepts of the line, what is the formula for the area of the first quadrant triangle?
Answer: Area = 0.5(x-intercept)(y-intercept)

25. In terms of the axis intercepts of the line, when would two triangles have the equal areas?
Answer: When the product of the respective axis intercepts are equal

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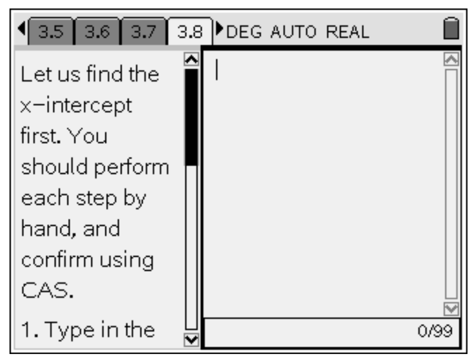
26. If a triangle had an area of 36 square units, what are some possible values of the x- and y-intercepts?

    Answer: Possible intercept values are 1 and 72, 2 and 36, 4 and 18, 6 and 12, 8 and 9 (any pair of numbers whose product is 72). Be aware that these values DO NOT satisfy our particular problem. It is just that these values would produce triangles with an area of 36 square units.

27. Which of these intercept values are possible for THIS problem?

    Answer: Since the x-intercept is greater than 2, and the y-intercept is greater than 5, it is <u>possible</u> that 4 and 18, or 6 and 12, or 8 and 9 could be possible intercept values in this problems (but they are not!).

When students solve for the intercepts on pages 3.8 and 3.9, they should be reminded that they are solving for *x* (or *y*) in terms of the slope of the line. Students should do as much solving as possible *by hand* and confirm or check their results with CAS.



Page 3.8 – Solving for the *x*-intercept.

1. Type in the equation of the line. Use *m1* for the slope.

   Answer: y-5=m1(x-2)

2. Substitute *y*=0 and solve for *x* in terms of *m1*.

   Answer: $x=(-5+2m1)/m1$

3. There was a small error message that appeared. Why?

   Answer: You are dividing by m1, so the domain of the result no longer contains m1=0.

4. Store what x is equal to at *xint*.

   Answer:

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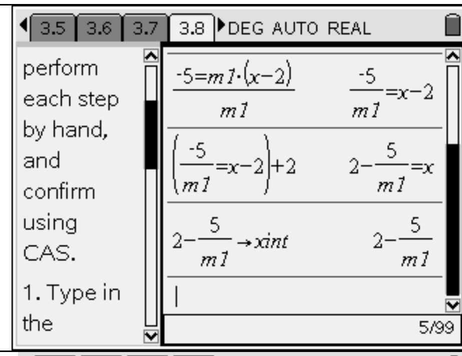
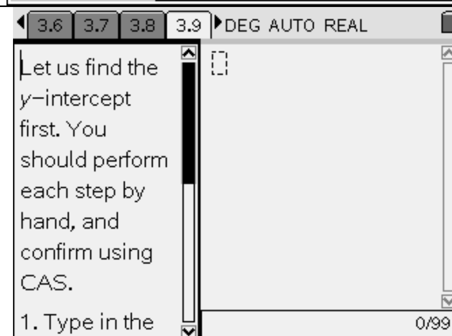
Page 3.8 at right shows an anticipated derivation of the x-intercept in terms of m1.

When you divide by m1, you will see an error message regarding the size of the domain. This is something that should be discussed with your students.

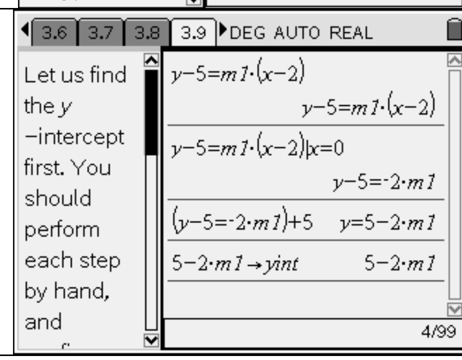


Page 3.9 – Solving for the *y*-intercept.

1. Type in the equation of the line. Use *m1* for the slope.
2. Substitute *x*=0 and solve for *y* in terms of *m1*.
3. Store what *y* is equal to at *yint*.

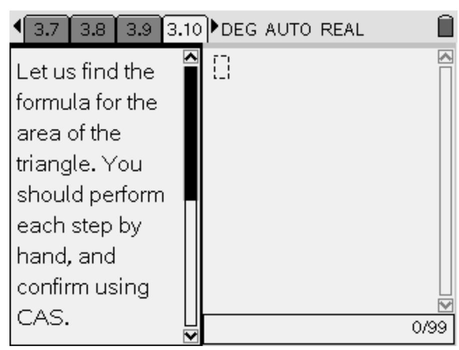


Page 3.9 at right shows an anticipate derivation of the y-intercept.



Page 3.10 – The area formula.

1. Enter *.5(yint)(xint)*
2. Make the Substitution *m1=x*.
3. Store this expression at *f1(x).*



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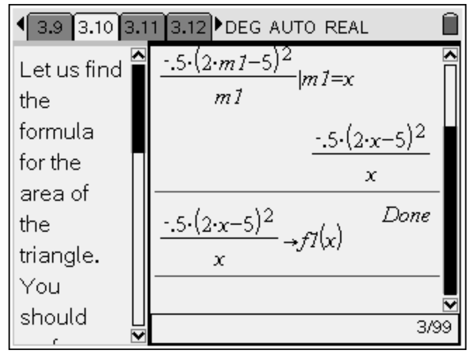
Page 3.10 at right shows an anticipated derivation of the area formula, the substitution m1=x, and how it is stored at f1(x).

You should discuss with your students the need to substitute x for m1.

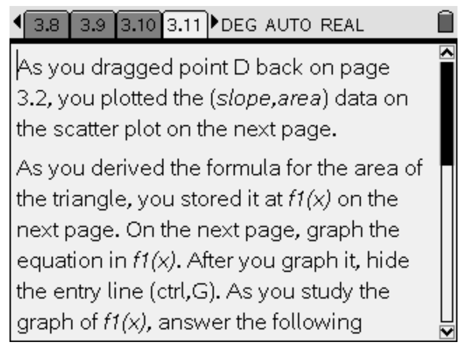


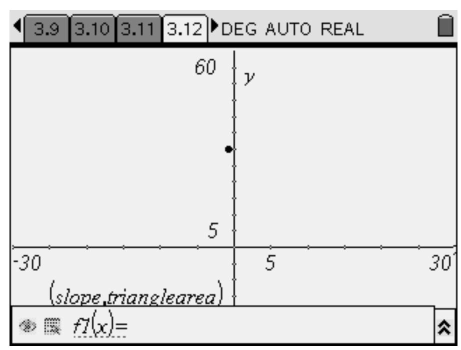
Pages 3.11 and 3.12 – Is our formula good?
On these two pages, the formula derived and stored on page 3.10 is now graphed on top of the data. Students are asked to judge how well our formula fits the data through these guided questions:

1. How well does your area formula fit the data? How can you tell?
   Answer: An acceptable answer for now is based on how the graph looks. The graph of f(x) "looks" as if it matches the data points well, so it it fits the data well.

2. Pretend you are a bug crawling along this graph from left to right. What is happening to the values of the slope and the values of the triangle area as the bug crawls?
   Answer: As the bug crawls along the graph from left to right, the values of the slope is increasing, the area of the triangle slowly decreases, reaching a minimum at x=-2.5, then the area quickly increases.

3. A part of this graph lies in the fourth quadrant. Does this part of the graph have any meaning in light of this triangle problem? Explain.
   Answer: The part of the graph BELOW the x-axis would represent a negative area, which can not happen. However, if you were to graph the absolute value of f(x), then THIS graph would match the triangle area data for ANY line slope regardless of the quadrant the triangle lies in. This is certainly a worthwhile extension.





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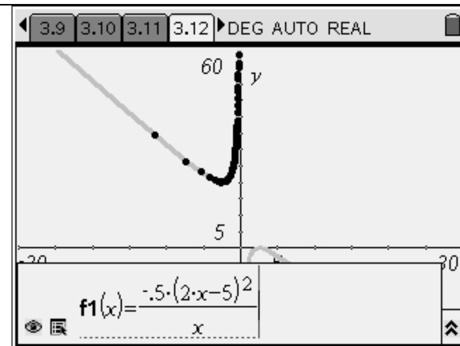
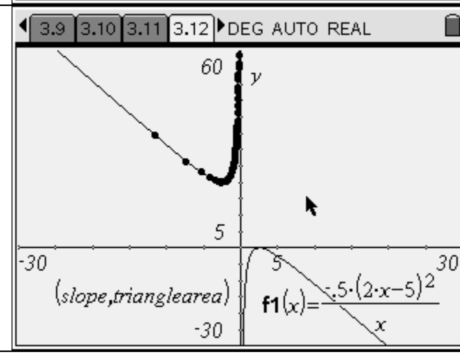
Page 3.12 at right shows the graph of the f1(x). The area formula will be stored at f1(x) from your work on page 3.10. You will need to press enter to graph the function.



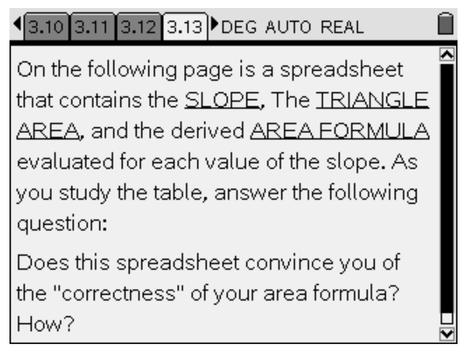
Page 3.12 at right shows the graph after hiding the entry line (ctrl,G).

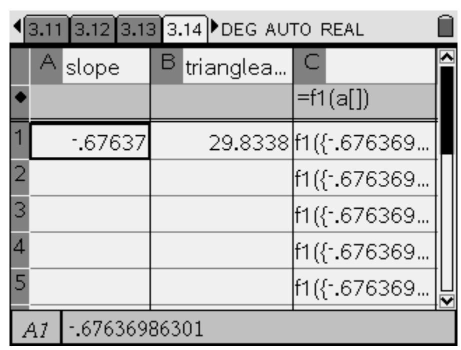


Pages 3.13 and 3.14 – Is our formula good (part 2)?
On these two pages, students once again are asked to explore a spreadsheet that contains data on the slope, triangle area, and the derived area formula evaluated at each value of the slope.

There is only one guided inquiry question for this spreadsheet:

1. Does this spreadsheet convince you of the "correctness" of your area formula? How?
   Answer: One possible answer is that the column for the triangle area matches the function value column.





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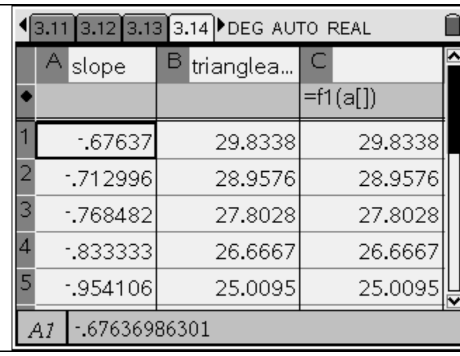
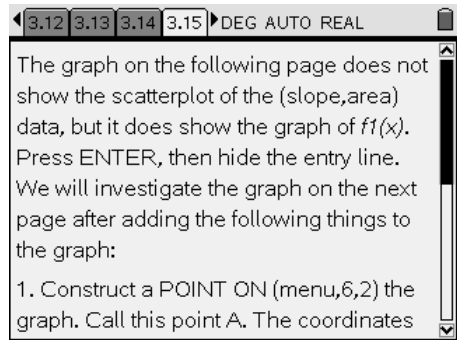
Page 3.14 at right shows the revised spreadsheet from page 3.4 with the function values of f1(x) displayed in column C.

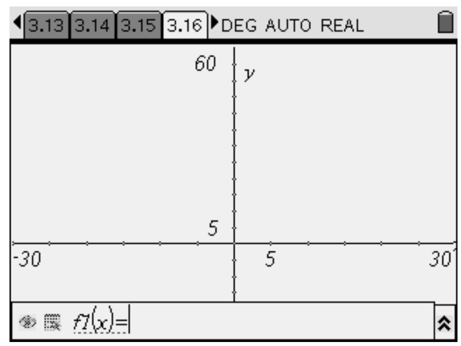


Pages 3.15, 3.16, and 3.17 – Use the formula to answer questions.

In these pages, students are guided to explore the graph of the area formula by tracing points along the graph and exploring changes in the values of the slope and area. Students first are asked on page 3.15 to construct some things on the graph to aide in the exploration of the graph:

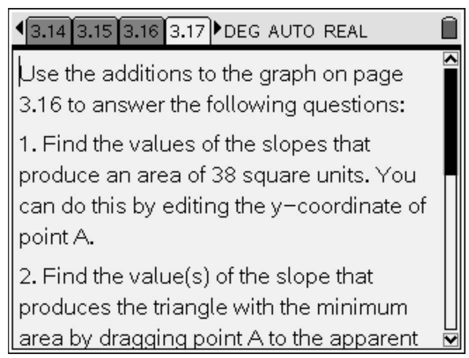
2. Construct a POINT ON (menu,6,2) the graph. Call this point A. The coordinates of this point should be displayed.
3. Construct a line PARALLEL to the x-axis through the point on the graph (menu,9,2).
4. Construct the INTERSECTION POINT of this line and the graph (menu,6,3).
5. Find the COORDINATES of this intersection point (menu,1,6).





On page 3.17, students are asked the following guided inquiry questions:

1. Find the values of the slopes that produce an area of 38 square units. You can do this by editing the y-coordinate of point A.
   Answer: -13.54 or -0.46
2. Find the value(s) of the slope that produces the triangle with the minimum area by dragging point A to the apparent minimum on the graph. What are the values of the slope and area?
   Answer: Slope = -2.5. Area = 20.
3. When point A is at the minimum point on the graph, is the parallel line you constructed TANGENT to the graph?
   Answer: Yes



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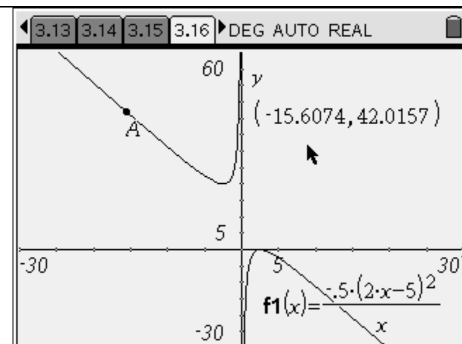
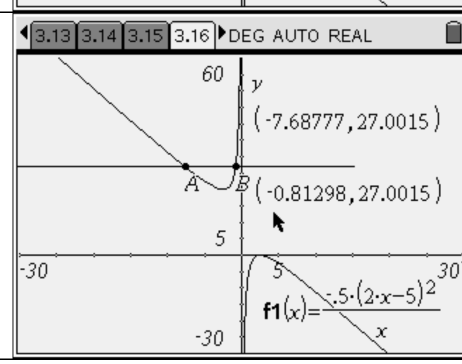
4. For all but one area value, there are two different slope values. Why? *Answer: There is only one minimum value for the area. There are slopes less than -2.5 and greater that -2.5 that produce different triangles with equal areas.* 5. When the triangle area increases by 1 square unit, by how much does the slope increase? *Answer: This depends where you are on the graph. Different points on the graph will have different rates of change, so there are many possible answers.* 6. How do you explain these different increases? *Answer: The graph is "steeper" to the right of the minimum value with a positive slope, and is less steep to the left of the minimum value with negative slope*	
Page 3.16 at right shows the graph of f1(x) with point A on the graph. When you first arrive at this page, you will need to press enter to graph f1(x), and then hide the entry line (ctrl,G). The coordinates of A have been moved to the upper right corner of the view screen.	
Page 3.16 at right shows the parallel line constructed through A, and the other intersection of this line with the graph of f1(x).	

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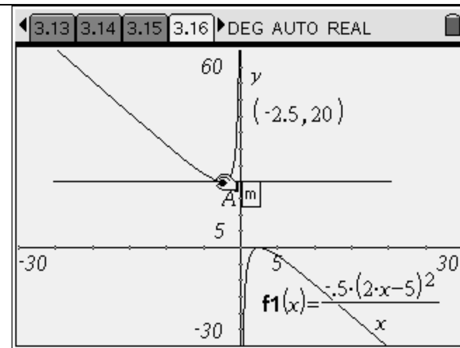
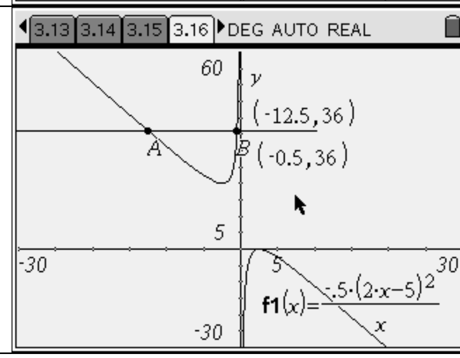
Page 3.16 at right shows point A after dragging it to the apparent minimum on the graph. Notice the lower case m that appears, signifying the minimum on the graph. Notice that the coordinates for point B no longer appear, suggesting the parallel line is tangent to f1(x) at this point. The coordinates of the minimum point confirm what the students may already know; the minimum area is 20 square units, and the slope of the line is −2.5.



Page 3.16 at right shows the graph after editing the y-coordinate of point A to be 36. The coordinates of A and B may confirm what the students already know; slopes of −12.5 and −0.5 produce an area of 36 square units.

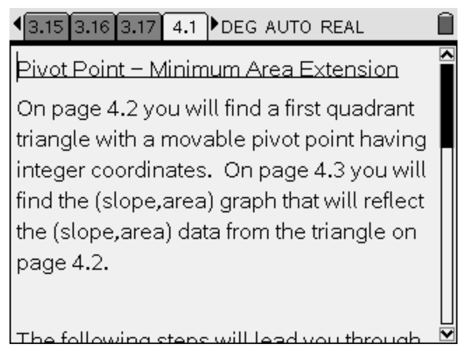


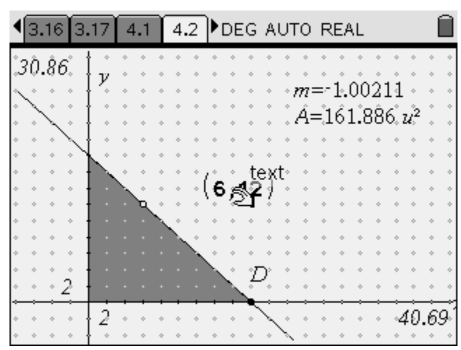
Problem 4 – Pivot Point and Minimum Area Extension.
On these optional pages, students are engaged in finding patterns among the coordinates of the pivot point and the area of the triangle, formulating conjectures about any relationships they discover, and ultimately proving these conjectures.

On page 4.2, students are asked to move the pivot point to another location and to drag point D to find the triangle with the minimum area. On page 4.3, the graph of the area function is automatically updated to reflect the coordinates of the pivot point. Students drag point A on the graph to the find the coordinates of the minimum (slope,area) value.

The steps for the investigation are as follows:

1. Move the pivot point on page 4.2, drag point D along the x-axis, and estimate the minimum area and the slope that produces this area.
2. On page 4.3, drag point A to the minimum point on the graph. What are the coordinates of the minimum point on the graph? You MAY need to adjust the window settings (menu,4,1) to see a good graph.
3. Repeat steps 1 and 2 until you can conjecture how the minimum area and corresponding slope is related to the coordinates of the pivot point.
4. Prove your conjecture.





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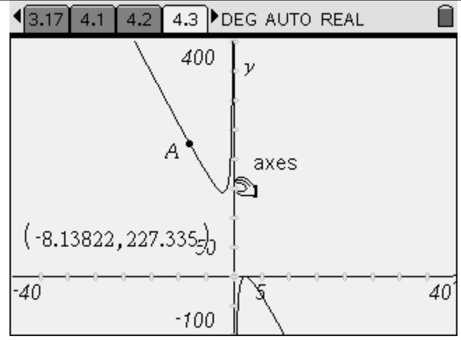
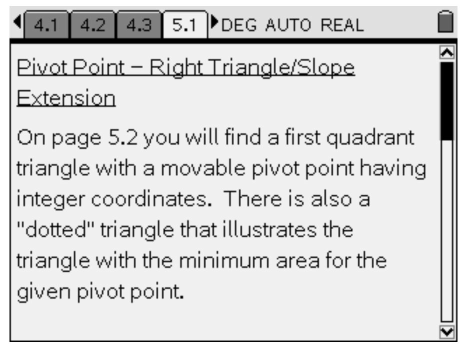
Answer: The minimum area is twice the product of the coordinates of the pivot point. The slope that produces the minimum area is $\dfrac{y-coordinate}{x-coordinate}$.

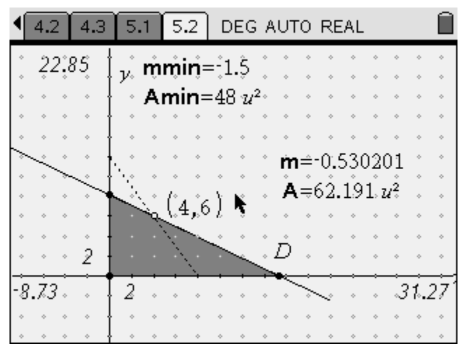


Problem 5 – Pivot Point, Right Triangle, and Slope Investigation.

On these optional pages, students are engaged in finding patterns among the coordinates of the pivot point, the slope of the line and the area of the triangle. They are also looking at how the pivot point is related to the right triangle with minimum area. Students are formulating conjectures about any relationships they discover, and ultimately proving these conjectures.



On page 5.2 is the now familiar first-quadrant triangle. There is also a dashed segment showing the position of the first-quadrant triangle with minimum area. As students drag the pivot point to different locations, this segment will always show the triangle with minimum area.



The steps for the investigation are as follows:

1. Move the pivot point, and drag point D to the minimum triangle.
2. Observe the value of the slope, the minimum area, and the relationship of the pivot point to the minimum area triangle.
3. Repeat steps 1 and 2 until you can formulate a conjecture as to how the coordinates of the pivot point are related to the minimum area and to the corresponding slope, and how the pivot point is related in general to the minimum area triangle.
4. Prove your conjectures.

   Answer: The pivot point is the Circumcenter of the right triangle with the minimum area. This means the pivot point is equidistant from the origin, the x-intercept, and the y-intercept.

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STUDENT WORKSHEET

Problem 1

Go to page 1.2 of the TI-Nspire document. Using the graph, answer these questions.

1. As the slope changes, how does the *y*-intercept change?

2. As the slope changes, how does the *x*-intercept change?

3. As the slope changes, how does the triangle area change?

Problem 2

Go to page 2.2 of the TI-Nspire document. Using the graph, answer these questions.

4. What is the slope of the "steepest" line you can have and still have the triangle in the first quadrant?

5. What is the slope of the "shallowest" line you can have and still have the triangle in the first quadrant?

6. Where is the triangle when the slope is positive?

7. What happens to the triangle when the slope is zero?

8. What happens to the triangle when the slope is undefined?

9. What values of the slope always produce a triangle in the first quadrant?

10. What value(s) of the slope produces a triangle with the smallest possible area?

11. What value(s) of the slope produces a triangle with an area of 36 square units?

Problem 3

For problems 12-18 below, go to pages 3.3 and 3.4 in the Nspire document

12. Find the slope of a line that produces a triangle with an area close to 40 square units.

13. Find the slope of a line that produces an area close to 30 square units.

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14. Find a slope that is close to -2 and another that is close to -3. What is the difference in the areas of the triangles?

15. Find a slope that is close to -4 and another that is close to -5. What is the difference in the areas of the triangles?

16. Find a slope that is close to -0.2and another that is close to -0.3. What is the difference in the areas of the triangles?

17. Find a slope that is close to -0.4 and another that is close to -0.5. What is the difference in the areas of the triangles?

18. For what values of the slope does the area of the triangle appear to be changing the fastest?

For questions 19-23 below, please refer to pages 3.5 and 3.6 in the TNS file

19. The data looks like it gets very close to the y-axis. Will the graph of the data ever cross the y-axis? Use the slope of the line and the area of the triangle to explain your answer.

20. Will the graph of the data ever cross the negative x-axis? Use the slope of the line and the area of the triangle to explain your answer.

21. Why does the graph of the data show that there are two triangles with an area of 22 square units?

22. The data appears to have a "U" shape. Explain why the slope of the line and the area of the triangle lead to data that has a "U" shape.

23. <u>THOUGHT EXPERIMENT</u>: What do you think would happen to the graph of the data if you used the point (5,2) as the pivot point instead of the point (2,5)? Explain your reasoning using the slope of the line and the area of the triangle.

For questions 24 – 27 below, please refer to pages 3. 7 in the TNS file

24. Using the axis intercepts of the line, what is the formula for the area of the first quadrant triangle?

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25. In terms of the axis intercepts of the line, when would two triangles have the equal areas?

26. If a triangle had an area of 36 square units, what are some possible values of the x- and y-intercepts?

27. Which of these intercept values are possible for THIS problem?

For question 28-31, please refer to pages 3.11 through 3.14 of the .tns file.

28. How well does your area formula fit the data? How can you tell?

29. Pretend you are a bug crawling along this graph from left to right. What is happening to the values of the slope and the values of the triangle area as the bug crawls?

30. A part of this graph lies in the fourth quadrant. Does this part of the graph have any meaning in light of this triangle problem? Explain.

31. Does this spreadsheet convince you of the "correctness" of your area formula? How?

For questions 32 through 37, please refer to page 3.17 of the .tns file

32. Find the values of the slopes that produce an area of 38 square units. You can do this by editing the y-coordinate of point A.

33. Find the value(s) of the slope that produces the triangle with the minimum area by dragging point A to the apparent minimum on the graph. What are the values of the slope and area?

34. When point A is at the minimum point on the graph, is the parallel line you constructed TANGENT to the graph?

35. For all but one area value, there are two different slope values. Why?

36. When the triangle area increases by 1 square unit, by how much does the slope increase?

37. How do you explain these different increases?