Alternating Series

ID: 10283

Time required
45 minutes

Activity Overview

Students will explore alternating series and the method to approximate its sum. They will use the capabilities of their handhelds to first be introduced to the idea of an alternating series, and then testing this type of series for convergence.

Topic: Series & Taylor Polynomials

- *Prove and apply the alternating series test for convergence.*

Teacher Preparation and Notes

- *This investigation offers opportunities for students to use alternating series. This investigation offers opportunities for review and consolidation of their skills. As such, care should be taken to provide ample time for ALL students to engage actively with the requirements of the task, allowing some who may have missed aspects of earlier work the opportunity to build new and deeper understanding.*
- *This activity is designed to be* ***teacher-led.*** *You may use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds. Although the majority of the ideas and concepts are only presented in this document, be sure to cover all the material necessary for students' comprehension.*
- *Students should review p-series, the sum of a geometric series as well as the definition of a harmonic series prior to the beginning of this activity.*
- *Students should be able to use the calculator and set up tables.*
- *Before starting this activity, students should go to the home screen and select* ***F6:Clean Up > 2:NewProb****, then press* ENTER*. This will clear any stored variables, turn off any functions and plots, and clear the drawing and home screens.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "10283" in the keyword search box.***

Associated Materials

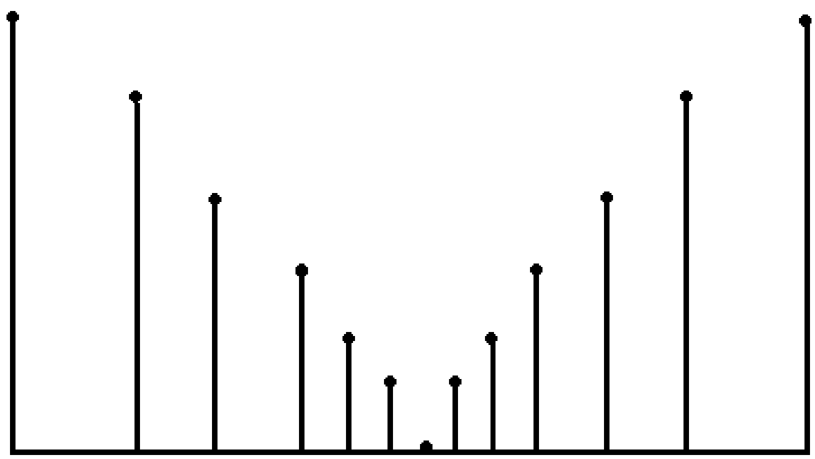
- *AlternatingSeries_Student.doc*

Problem 1 – Introduction to an alternating series

Instruct students to use the tip of their pencil and touch the top of each vertical line. They must go from left to right. Students should see that they go back and forth until the tip of their pencil goes to the middle of the figure; the process repeats.

Discuss this with students and have them answer the questions on their worksheet.

When the terms of an infinite series alternate in sign for every term (+, –, +, –, +…) or (–, +, –, +, –…), then the series is called an alternating series.



1. What do you notice about the motion of your pencil point?

The tip of the pencil goes back and forth. (Answers may vary)

2. Relate your illustration to a number line with both positive and negative values. What can you now say about your pencil point?

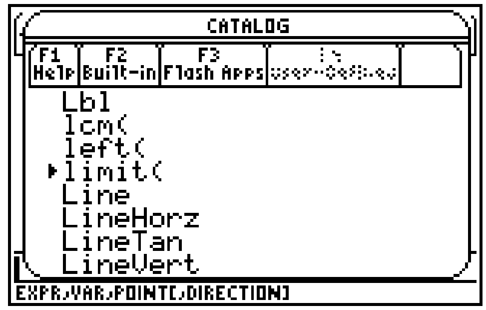
The pencil point goes from negative to positive and continues this pattern OR signs alternate. (Answers may vary)

3. If the center is 0 and each line is a term belonging to a series, what can you say about the series and its terms?

The series is alternating and its values approach 0. (Answers may vary)

Problem 2 – Alternating Series Test

The Alternate Series Test emphasizes that certain conditions must be met for an alternating series to be convergent.

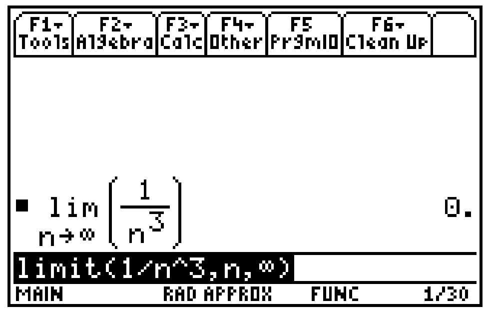


If an alternating series

$\sum_{n=1}^{\infty}(-1)^n a_n$ or $\sum_{n=1}^{\infty}(-1)^{n+1} a_n$ converges, then these conditions must hold.

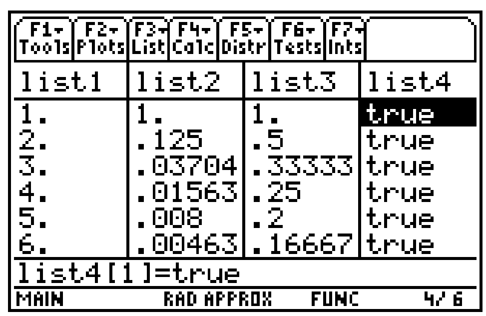
- $\lim_{x\to\infty} a_n = 0$.
- $a_{n+1} \le a_n$ for all *n.*

Students first go to the home screen to find the limit. Select **limit(** by visiting the catalog. For **Question 4**, students can enter **limit(1/n^3,n,∞)**.



Students then use the **Stats & List Editor** to compare each value. They enter in **list1**, **seq(n,n,1,50)** for the first 50 values for *n*. In **list 2**, students can enter **1/list1^3**, and in **list3** enter **1/list1**.

In **ListEditor**, students should compare list 1 and list 2 using **= l2 ≤ l3**. For the less than or equal to sign, students can press [2nd] [MATH] and go to the **test** menu.



Instruct students to investigate convergence of the alternating series below.

4. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^3}$

Converges

5. $\sum_{n=1}^{\infty}\frac{(-1)^{n}}{\sqrt{n}}$

Converges

6. $\sum_{n=1}^{\infty}\frac{n}{(-3)^{n-1}}$

Converges

7. $\sum_{n=1}^{\infty}\frac{(-1)^{n}2n}{3n-1}$

Diverges

Problem 3 – Alternating Series Estimation

A convergent alternating series, has a partial sum S_n. S_n can be used to find the approximation for the total sum S of the series.

If $\lim\limits_{x\to\infty} a_n = 0$ and $a_{n+1} \le a_n$ hold true then the absolute value of the remainder (R_n) is less than or equal to the first neglected term.

That is

$$\left|S - S_n\right| = \left|R_n\right| \le a_{n+1}$$

R_n can also be referred to as the error.

Students will again use the table to the right to observe the partial sums of an alternating series.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n!}$$

In the first column contains the number of the term, the second column shows the terms of the series, and the third column shows the partial sum to that point.

n	Terms	Sum of Terms
1	0.5	0.5
2	–0.25	0.25
3	0.08333	0.333333
4	–0.02083	0.312500
5	0.00417	0.316667
6	–0.00069	0.315972
7	0.0001	0.316071
8	–0.00001	0.316059
9	0.00000137	0.3160604
10	–0.000000137	0.3160603

8. Approximate the sum of an alternating series

i) by its first three terms

$S_3 \approx 0.33333$ and $a_4 = \frac{1}{48} \approx 0.02083$

Therefore

$0.33333 - 0.02083 \le S \le 0.33333 + 0.02083$

$0.31250 \le S \le 0.35416$

ii) by its first six terms

$S_6 \approx 0.31597$ and $a_7 = \frac{1}{10080} \approx 0.0001$

Therefore

$0.31597 - 0.0001 \le S \le 0.31597 + 0.0001$

$0.31587 \le S \le 0.31607$

9. What do you notice about the intervals?

The interval is becoming smaller.

10. What do you think will occur as the series approaches infinity?

The interval where the actual sum lies will become more precise and a better approximation can be made.