Integration by Substitution

TIMATH.COM: CALCULUS

TEACHER NOTES

ID: 9889

Activity Overview

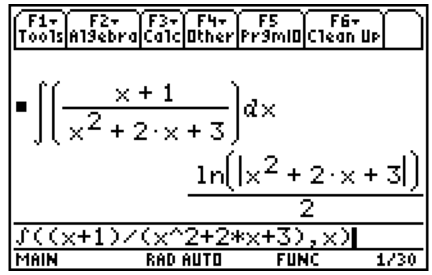
In this activity, students explore methods for computing integrals of functions that are not in one of the standard forms. The focus here is on the use of substitution to transform the given integral into a standard form.

Topic: Integration

- Use substitutions such as $u = \sqrt{ax + b}$ to compute an integral.
- Use **Integral** (in the Calculus menu) to verify manual calculation of integrals.

Teacher Preparation and Notes

- This investigation offers opportunities for review and consolidation of key concepts related to methods of substitution and integration of composite functions. Opportunities are provided for skill development and practice of the method of taking integrals of suitable functions. As such, care should be taken to provide ample time for ALL students to engage actively with the requirements of the task, allowing some who may have missed aspects of earlier work the opportunity to build new and deeper understanding.
- This activity can serve to consolidate earlier work on the product rule and methods of integration. It offers a suitable introduction to integration by substitution.
- This activity requires the use of CAS technology for students to check their answers.
- Before starting this activity, students should go to the Home screen, select **F6:Clean Up > 2:NewProb**, and then press ENTER. This will clear any stored variables, turn off any functions and plots, and clear the drawing and Home screens.
- **To download the student worksheet, go to education.ti.com/exchange and enter "9889" in the keyword search box.**



This activity includes screen captures taken from the TI-89 Titanium.

Compatible Devices:

- TI-89 Titanium

Associated Materials:

- IntegrationBySubstitution_Student.pdf
- IntegrationBySubstitution_Student.doc

Click [HERE]() for Graphing Calculator Tutorials.

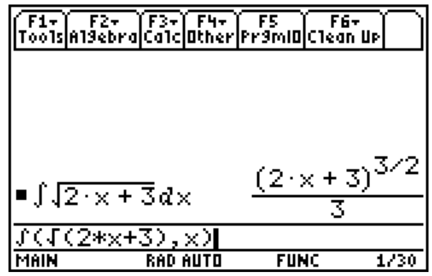
Problem1 – Introduction

Step 1: Begin with discussion and review of both the chain rule for differentiation of composite functions and of the integrals of standard function forms.

Ensure that students are comfortable with these and then challenge them to consider more difficult forms—in this case, composite functions of the form $y = f(g(x))$ which may be suitable for integration by substitution methods.

Step 2: Students are to use substitution to integrate $\int\sqrt{2x+3}dx$. They can use the table on the worksheet to help guide them through the steps.

Students should use the selection of u to compute du and rewrite the function in terms of u. Then they integrate it with respect to u and rewrite the evaluation of the integral in the original variable.



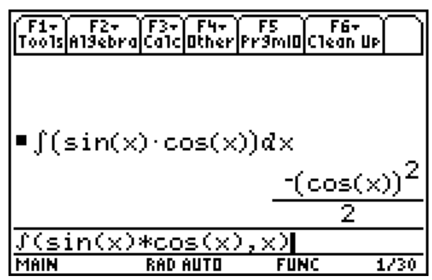
Note: Some students may not realize that if they need a 2, they have to multiply the integral by $\frac{1}{2}$ to keep the value the same.

In addition, they should use the **integral** command (**Home > F3:Calc > 2: integrate**) or (**Home >** [2nd] [7]) to check their answer. Students should be aware that integrals evaluated by CAS generally do not include the constant term.

Step 3: The integral for $\sin(x)\cos(x)dx$ is developed three ways. First, students let $u = \sin(x)$ and then let $u = \cos(x)$. Students should see that the results are different, but only a constant apart.

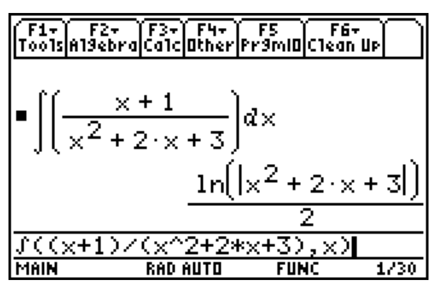
The third way students find the integral is by transforming $\sin(x)\cos(x)dx$ into $\frac{1}{2}\sin(2x)$ using the Double Angle formula, letting $u = \sin(2x)$.

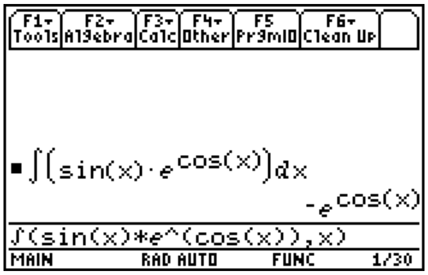
Students can check the answer with their calculator.

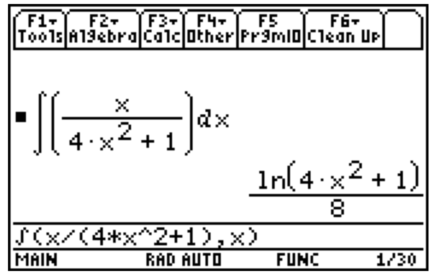


Problem 2 – Common Feature

Step 4: Students are to work through three problems using substitution to integrate. First, have them discuss what they should use for u in each of the problems. If students are having trouble with the steps, have them create a table like the one on the worksheet.







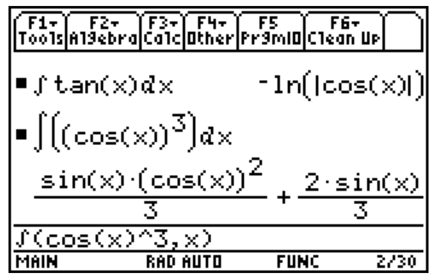
Step 5: After working through the problems, students are challenged to identify the common feature: that each of the given functions in some way includes the derivative of the function substituted. It is critical for students to understand that this method will not work for all functions, but only for certain well chosen forms.

Extension

The challenge to the students in these problems is to use trig identities to rewrite the integral so that they have a substitution format.

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\cos(x)^3 = \cos(x)\cos(x)^2 = \cos(x)(1 - \sin(x)^2)$$



Solutions

1. u = 2x+3; du = 2dx; $\int\sqrt{2x+3}\,dx = \frac{1}{2}\int u^{\frac{1}{2}}du = \frac{1}{3}u^{\frac{3}{2}} + C = \frac{1}{3}(2x+3)^{\frac{3}{2}} + C$

2. u = sin(x); du = cos(x)dx; $\int\sin(x)\cos(x)dx = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}\sin(x)^2 + C$

3. u = cos(x); du = –sin(x)dx; $\int\sin(x)\cos(x)dx = -\int u\,du = -\frac{1}{2}u^2 + C = -\frac{1}{2}\cos(x)^2 + C$

4. u = 2x; du = 2dx; $\int\frac{1}{2}\sin(2x)dx = \frac{1}{4}\int\sin(u)du = -\frac{1}{4}\cos(u) + C = -\frac{1}{4}\cos(2x) + C$

5. $u = x^2$ + 2x + 3; du = (2x + 2)dx; $\int\frac{x+1}{x^2+2x+3}dx = \frac{1}{2}\int\frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+2x+3| + C$

6. u = cos(x); du = –sin(x)dx; $\int\sin(x)\,e^{\cos(x)}dx = -\int e^u\,du = -e^u + C = -e^{\cos(x)} + C$

7. $u = 4x^2$ + 1; du = 8x dx; $\int\frac{x}{4x^2+1}dx = \frac{1}{8}\int\frac{1}{u}\,du = \frac{1}{8}\ln|u| + C = \frac{1}{8}\ln|4x^2+1| + C$

8. Each contains the derivative of the substitution element: if u is the substitution, then du/dx exists as part of the expression, or at least a constant multiple of it.

9. u = cos(x); du = –sin(x); $\int\frac{\sin(x)}{\cos(x)}dx = -\int\frac{1}{u}\,du = -\ln|u| + C = -\ln|\cos(x)| + C$

10. u = sin(x); du = cos(x)dx; $\int\cos(x)(1-\sin(x)^2)dx = \int(1-u^2)\,du = u - \frac{u^3}{3} + C = \sin(x) - \frac{\sin(x)^3}{3} + C$