Count the Differences

ID: 12092

Time required
15–20 minutes

Activity Overview

In this activity, students are given data and asked to find the finite differences. They will use this to find a polynomial that models the data.

Topic: Finite Differences

- *Polynomial Functions*
- *Matrices (Extension)*
- *Systems of equations (Extension)*

Teacher Preparation and Notes

- *Problem 1 is meant to be a teacher demonstration to review finding slope and connect the process to finite differences.*
- *Teachers can lead Problems 2 and 3 or let students work on their own. Students are asked to find an equation for a quadratic and cubic equation. If students have gone over a process for how to find an equation, then they can solve for them algebraically. If not, then students can use guess and check to select the equation from the multiple-choice question following the spreadsheet.*
- *Students will need to know how to plot functions.*
- *The extension uses inverse matrices to solve a system of equations.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "12092" in the quick search box.***

Associated Materials

- *FiniteDifferences_Student.doc*
- *FiniteDifferences.tns*

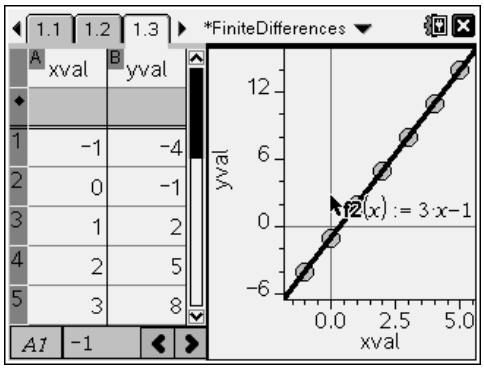
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Finite Differences (TI-84 Plus) — 8212*

Problem 1 – Review Linear Functions

In this problem, students will find the slope of data given in a table and shown to be a linear function in a graph. If students are struggling, make the connection for students that the first finite differences are the same and the linear function is a first degree polynomial.

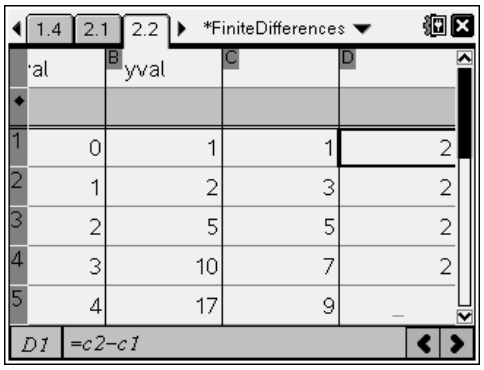


Discussion Questions:

- What is the change (difference) between consecutive *y*-values?
- What degree is the function that models the data?

Problem 2 – Finding Finite Differences, Part 1

In this problem, students will use a spreadsheet to find the differences between the *y*-values given in spreadsheet. Students must find the second-order difference to find the finite difference, or where each difference of consecutive term is the same. Teachers should make sure students are subtracting the correct terms. Students should try to find the function any way they can. The multiple-choice question after the graph can help students narrow the choices.



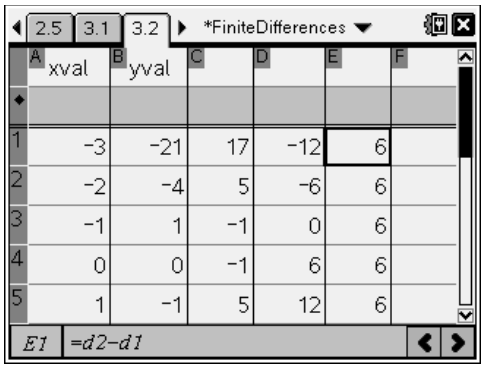
On page 2.2, students could enter formulas into the cells or type the numbers in directly. For example, in cell C1 type **=b2–b1**, then use the **Fill Down** command (**MENU > Data > Fill Down**) to fill through cell C5. Remind students that they should only fill down through cell C5, not C6 because there will be one less difference than number of values.

Discussion Questions:

- Did anyone use formulas in the spreadsheet to find the differences faster?
- What is the relationship between the degree of the polynomial that models the data and the number of times you need to find the difference between the term values?

Problem 3 – Finding Finite Differences, Part 2

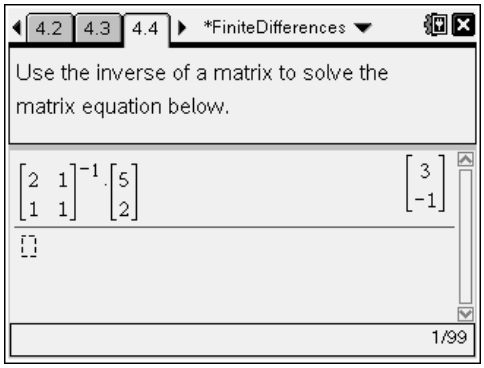
In this problem, students will have to find the third-order difference and determine a cubic polynomial. They will repeat the same process as in Problem 2.



Extension – Using Matrices to find Equations

In the extension, students will write a system of equations and solve the system using matrices to find the polynomial that fits the data.

If students are unsure of the general form of a polynomial, remind them that it is in the following form: $y = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots$



Student Solutions

1. A linear function is first degree and the first differences were the same.

2. 2

3. $f(x) = x^2 + 1$

4. 3

5. $f(x) = x^3 - 2x$

6. $5 = 2a + b$
$2 = a + b$

7. $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

8. $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}^{-1}\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}^{-1}\begin{pmatrix} 5 \\ 2 \end{pmatrix}$

$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

9. Sample answers:

Problem 2: $1 = c$
$2 = a + b + c$
$5 = 4a + 2b + c$

$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$

Problem 3: $0 = d$
$-1 = a + b + c + d$
$4 = 8a + 4b + 2c + d$
$21 = 27a + 9b + 3c + d$

$\begin{pmatrix} 27 & 9 & 3 & 1 \\ 8 & 4 & 2 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 21 \\ 4 \\ -1 \\ 0 \end{pmatrix}$