NUMB3RS Activity: Angling for Distance
Episode: “End of Watch”

Topic: Maximizing the distance traveled by a projectile **Grade Level:** 10 - 12
Objective: To calculate the angle for maximum distance of a projectile using trigonometry.
Materials: Paper, pencil, TI-83 Plus/TI-84 Plus graphing calculator
Time: 15 - 20 minutes

Introduction

In “End of Watch,” Charlie and Amita are shooting flaming projectiles at a model they built. Charlie explains to Alan that he is “deriving a set of coupled differential equations to check against a fourth-order Runge-Kutta method.” Amita simplifies matters by explaining, “He’s seeing how far he can fling a ball of fire.” The question of how far a projectile can travel is a very old one. Throughout history, people have tried to launch weapons over great distances. Two primary factors play a role in solving this problem—the force behind the projectile and the angle at which it is launched. In this activity, students will understand how the angle at which a projectile is launched affects the distance it travels.

As time permits, the “Extensions” activity using the shot put ball (which takes an entire class period) can be done with the class. It works effectively with students who are concrete or visual learners. The reason for using the shot put ball is that it does not travel too far and it moves more slowly than most other objects. This makes it easier for students to capture the image with a fixed camera and to trace the path when they play back the video. In addition, the angle and the force with which it is launched does not vary as much as when throwing a baseball. The markings on a football field allow a quick tabulation of the distance.

This activity deliberately uses traditional customary units. It acts as a continuation of the activity “Where’s the Bullet?” from the *NUMB3RS* Season 2 Episode “Convergence” and is consistent with the yard markings on a football field in the extension. However, conversion to the metric system is quite simple, should that be necessary.

Discuss with Students

If students have not studied vectors in either a math or physics, give a little background in the basics of breaking down a vector into horizontal and vertical components. The *NUMB3RS* activity “Where’s the Bullet?” could serve as an excellent source for teaching this concept from the beginning because it also addresses the parabolic path.

Review basic definitions of sines and cosines as well as the double angle formulas, particularly $\sin(2x) = 2\sin x\cos x$.

Note that while some students may find it beneficial to actually key in the CATAPULT program on their calculators, the program file can also be downloaded for free by going to **http://education.ti.com/exchange** and searching for “7859.”

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Student Page Answers:

1. *$0 = v_0 t \sin\theta - 16t^2 = t(v_0 \sin\theta - 16t)$, which means $t = 0$ (beginning) or $v_0 \sin\theta - 16t = 0$ (hits the ground); the solution is $t = \frac{v_0 \sin\theta}{16}$.* **2.** *$x = v_0 \frac{v_0 \sin\theta}{16} \cos\theta = \frac{(v_0)^2 \sin\theta \cos\theta}{16}$.*

3. *$2 \sin\theta \cos\theta = \sin(2\theta) \rightarrow \sin\theta \cos\theta = \frac{\sin(2\theta)}{2}$, so $x = \frac{(v_0)^2 \sin(2\theta)}{32}$* **4.** *Between angles of 0° and 90°, sine has maximum value at 90°. $2\theta = 90° \rightarrow \theta = 45°$.* **5.** *Answers may vary. Sample answer: Because $\sin(90 - \theta) = \cos(\theta)$ and $\cos(90 - \theta) = \sin(\theta)$,
$\sin(90 - \theta)\cos(90 - \theta) = \cos\theta \sin\theta$, so the distances are equal.*

6. *$\frac{(100)^2 \sin(90)}{32} = \frac{1,000}{32} = 312.5$ feet* **7.** *Answers will vary. A player may choose to not maximize the distance of the ball because of a desire to have the ball arrive as quickly as possible. This requires more of the velocity to be used in the horizontal component.*

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Name: ________________________________ Date: _______________

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Suppose a projectile is launched from the ground at an angle θ. Let x be the horizontal distance, in feet, that the projectile traveled after t seconds and let y be its height, in feet. The projectile travels in a parabolic path, determined by the parametric equations $x = v_0 t \cos \theta$ and $y = v_0 t \sin \theta - 16t^2$, where v_0 is the initial velocity in feet per second, θ is the angle at which it is launched in degrees, and t is time in seconds.

1. The height y is 0 feet at the start ($t = 0$) and again when the projectile hits the ground. Calculate the time t in terms of θ when the object hits the ground.

2. Substitute the value of t from Question 1 into the formula for x to find the horizontal distance traveled by the projectile.

3. Simplify the formula in Question 2 using trigonometric identities.

4. Using the formula for x found in the previous questions, which value of θ will give the maximum distance?

5. What is the relationship between the distances of two objects launched at complementary angles? Why?

6. What is the maximum distance of an object launched with an initial velocity of 100 feet per second?

7. When a baseball outfielder throws a ball to home plate, the player usually throws the ball at an angle other than the one calculated in Question 4. Why might the player choose *not* to maximize the distance the ball is thrown?

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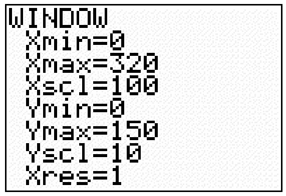
You can also explore this concept by using the following calculator program. It shows a comparison of the distance traveled for various angles. It first asks how many angles you would like to compare and then prompts you to enter the number of degrees in each of these angles. As each new trajectory appears, the previous graphs remain so that the differences are apparent.

Use this window setting: and these modes:




```
:ClrDraw
:FnOff
:0→C
:Input "HOW MANY?",N
:Lbl 1
:Input "DEGREES?",A
:A*π/180→A
:0→T
:Repeat Y<0
:T+.1→T
:100*cos(A)*T→X
:100*sin(A)*T-16T^2→Y
:Pt-On(X,Y)
:End
:C+1→C
:If C<N
:Goto 1
:Stop
```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The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

This activity considers the simplest case: that the ball starts on the ground (as in golf), and there is no friction. It also assumes that the launching velocity is constant for any angle. This is true of a mechanical device, but is not the case when a human throws a ball. Eliminating assumptions adds more interesting mathematics.

As a class, assemble on the school's football field (the yard markings make measuring distance as easy as reading a number line). Recruit a member of the track team to give a brief lesson on throwing the shot put ball. Set a camcorder on a tripod at a distance that allows the entire flight of the shot to be captured without any camera movement or zooming. Have students throw the shot three times at angles approximating 30°, 45°, and 60°. The student running the camcorder should record the distance of each throw when the shot hits.

Back in the classroom, show the video on a whiteboard. Mark the paths of the throws on the whiteboard with three different color markers. Then measure the angles that each makes with the horizontal (they need not be exactly 30°, 45°, or 60°). Make a chart recording the angle and distance for each throw. While the throws will differ from each other (and the track team member will most likely have the longest), the chart will generally show that for each student (who makes each throw with approximately consistent velocity), the maximum distance occurs at the angle closest to 45 degrees.

For the Student

Work with a physics teacher to find out what other forces act on an object in motion. Develop experiments that take into account friction due to air, wind, the shape of the projectile, etc. The modern military launches weapons at targets well beyond viewing distance. This activity assumed a flat surface. Investigate how the curvature of the Earth comes into play over long distances and what effect, if any, it has on the angle involved.

Research the history of the math of artillery fire for a social studies class. For example, Napoleon enlisted mathematicians as part of his army. Investigate the contributions of 16th-century mathematician Nicolo Fontana Tartaglia (who first discovered the result of this activity) and scientist Galileo Galilei to the study of trajectory.

Additional Resources

A page from Galileo's notebook that shows a sketch and calculations for projectile motion, as well as Tartaglia's illustration of how to aim a cannon based on the 45° result can be found at: **http://www.yorku.ca/bwall/glimpses/12.pdf**.

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