Derivative of Composite Functions

ID: 13418

Time required
15 minutes

Activity Overview

Students will practice using the Chain Rule to differentiate compositions of functions. The approach taken here is largely symbolic and makes use of the computer algebra facilities of TI-Nspire CAS technology. Prepared algebraic spreadsheets are utilized for skill development and consolidation.

Topic: Chain Rule

- *Differentiation of standard forms, fundamentals of derivatives, Chain Rule*

Teacher Preparation and Notes

- *This activity can serve to consolidate earlier work on differentiation. It offers a suitable introduction to derivatives of more difficult functions.*
- *Begin by reviewing the method of differentiation of first principles, both graphically and symbolically, and methods of differentiation of the standard function forms.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "13418" in the quick search box.***

Associated Materials

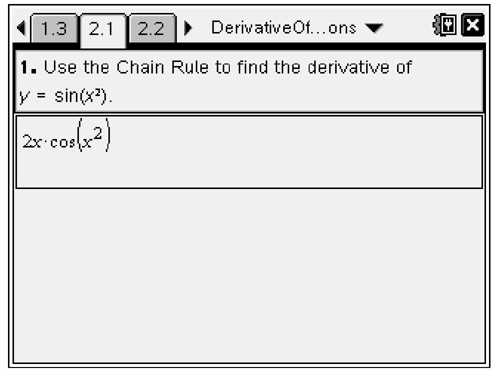
- *DerivativeOfCompositeFunctions_Student.doc*
- *DerivativeOfCompositeFunctions.tns*
- *DerivativeOfCompositeFunctions_Soln.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

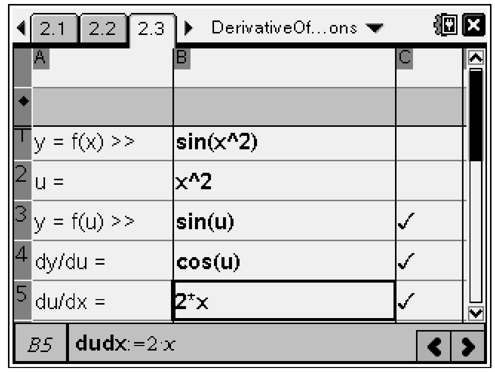
- *Chain Rule (TI-Nspire technology) — 11363*
- *Derivative of Composite Functions (TI-Nspire CAS technology) — 10223*
- *Discovery of the Chain Rule (TI-Nspire CAS technology) — 11331*

The Chain Rule is given for students to recall. They will practice using the Chain Rule to solve several problems.

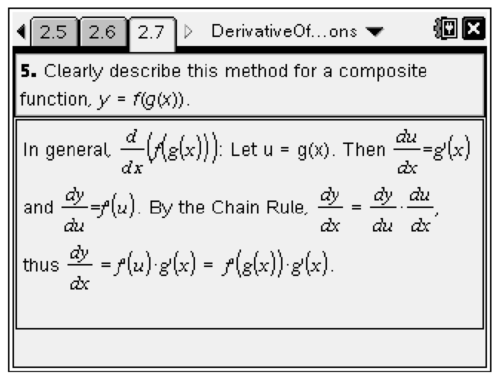


An algebraic spreadsheet is available to support students in working through the process: students supply each step, which is checked for algebraic equivalence.

This scaffolding tool may be used as much or as little as desired, noting that it does offer a model for a well-structured worked-out solution for such questions.



Finally, after working through several examples, students should be encouraged to apply these ideas to the general case.



Student Solutions

1. For $\boldsymbol{y = \sin(x^2)}$: Let $\boldsymbol{u = x^2}$, then $\boldsymbol{du/dx = 2x}$ and $\boldsymbol{dy/du = \cos(u)}$.

By the Chain Rule, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$, thus $\frac{dy}{dx} = \cos(u) * 2x = 2x\cos(x^2)$

2. For $\boldsymbol{y = \sin(x)^2}$: Let $\boldsymbol{u = \sin(x)}$, then $\boldsymbol{du/dx = \cos(x)}$ and $\boldsymbol{dy/du = 2u}$.

By the Chain Rule, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$, thus $\frac{dy}{dx} = 2u * \cos(x) = 2\sin(x)\cos(x) = \sin(2x)$

3. $\frac{d}{dx}(\ln(x^2))$: Let $\boldsymbol{u = x^2}$ then $\boldsymbol{du/dx = 2x}$ and $\boldsymbol{dy/du = 1/u}$

By the Chain Rule, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$, thus $\frac{dy}{dx} = \frac{1}{u} \cdot 2x = \frac{2}{x}$

4. $\frac{d}{dx}(\sin(\tan(x)))$: Let $\boldsymbol{u = \tan(x)}$ then $\boldsymbol{du/dx = \sec^2(x)}$ and $\boldsymbol{dy/du = \cos(u)}$

By the Chain Rule, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$, thus $\frac{dy}{dx} = \cos(u) \cdot \sec^2(x) = \frac{\cos(\tan(x))}{\cos(x)^2}$

5. In general, $\frac{d}{dx}(f(g(x)))$: Let $\boldsymbol{u = g(x)}$ then $\boldsymbol{du/dx = g'(x)}$ and $\boldsymbol{dy/du = f'(u)}$

By the Chain Rule, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$, thus $\frac{dy}{dx} = f\,'(u) \cdot g\,'(x) = f\,'(g(x) \cdot g\,'(x)$