

Journey to the Stratosphere



Student Activity

7 8 9 10 11 12



TI-30XPlus
MathPrint™



Activity



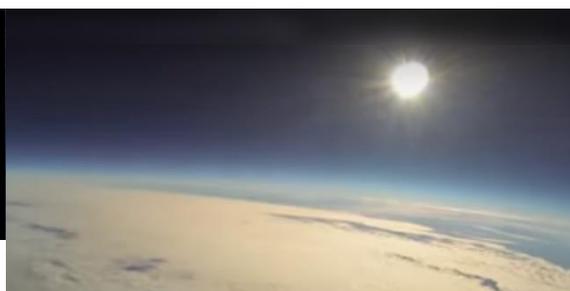
Student



30 min

Introduction

BHEEM-KC 001 - Telemetry
Time: 11:31:07
Flight Time: 01:01:41
Altitude: 16,934 m
Pressure: 90.74 hPa
Outside Temperature: -20.3 c
Internal Temperature: 0.83 c



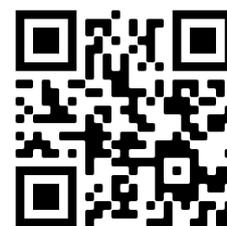
<http://bit.ly/HighAltitudeBalloon>



Stratosphere Video

<https://bit.ly/LinearRegression30XPlus>

Release a helium filled balloon and watch it race skyward. How high will it go? Where will it end up? Does it continue hurtling upward at the same rate that it left your hand? Maxime Dehaye Web Developer and avid photographer released a weather balloon with a payload consisting of a bundle of telemetric devices and some cameras. The result has been uploaded to YouTube for all to see and share. In this investigation you will use the first two minutes of data from Dehaye's video to predict the altitude of the balloon some 30 minutes later, well beyond extrapolation norms.



A series of videos to assist with data entry, manipulation and regression analysis have been created and available on YouTube. Scan the QR codes opposite to view the videos.

Calculator Instructions

Collecting the Data

Watch Maxime's video on YouTube and record the time (10 second intervals) and corresponding altitude for the first two minutes of the balloon flight.

Store the time data in List 1 (L₁) and the altitude data in List 2 (L₂).

Data entry can be accessed by pressing the [DATA] key.

A screenshot of a TI-30XPlus calculator screen showing data entry. The screen displays a table with two columns: time (L1) and altitude (L2). The data points are: (0, 127), (10, 169), (20, 223), and (30, 275). The bottom of the screen shows 'L1(1)=0'.

L1	L2	DEG	L3
0	127		-----
10	169		
20	223		
30	275		
L1(1)=0			

Question: 1.

At what altitude does the balloon flight commence?

Question: 2.

Study the altitude values, in particular, the difference between consecutive altitudes.

- What is the 'average' change in the altitude in each 10 second interval?
- What is the 'average' change in the altitude per second?

Question: 3.

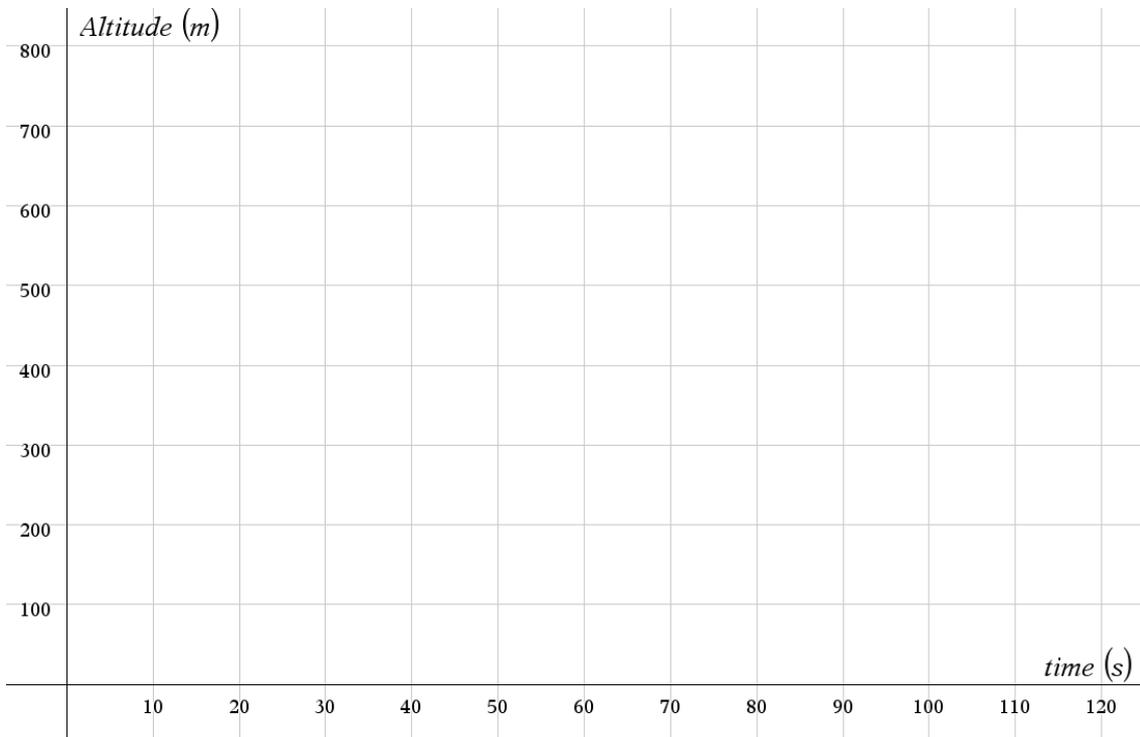
Based on your answers from Questions 1 and 2; suggest a possible straight line equation to model the data.

Equation Quality

In this section we explore how well your equation models or predicts the altitude of the balloon.

Question: 4.

Graph the altitude of the balloon against time and include your straight line graph from Question 3.



Question: 5.

Calculate the average (mean) time and altitude for each data set and plot this as a point on your graph.

Question: 6.

Does your straight line pass through the point plotted in Question 5?

The straight line graph passes quite close to each point. The distance from each point can be *measured* or *quantified* by comparing predicted values with the data gleaned from the video. On the calculator this can be done very efficiently.

The first step is to define your rule.

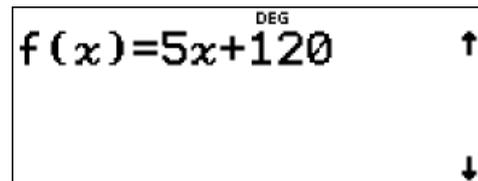
Press the [table] key then select option 1: Add / Edit Function.



Type in the equation that you derived in Question 3.

Use the variable key [x,y,z,t ...] to access 'x'.

Once the rule or function has been defined press [2nd] [mode]



[Sample equation only shown above]

Now it is time to enter this formula into a list.

Press the [data] key and navigate across to list 3: L₃.

L1	L2	DEG	L3
0	127		-----
10	169		
20	223		
30	275		

L3(1)=

Press the [data] key again and navigate across to the FORMULA menu, select option 1: Add / Edit Formula.

DEG		
CLR	FORMULA	OPS
1: Add/Edit	Frmla	
2: Clear L1	Frmla	
3: Clear L2	Frmla	

To use the rule or function to calculate the predicted values, press [table] and select option 2: f(

Press the [data] key to access the list names and select List 1: L₁.

DEG	
NAMES	
1: L1	
2: L2	
3: L3	

Close the parenthesis to complete the formula: f(L₁) and press [enter] to generate the set of predictions in List 3.

L1	L2	DEG	L3
0	127		-----
10	169		
20	223		
30	275		

L3=f(L1)

[Predicted values will populate List 3 after pressing enter]

Question: 7.

Determine the differences between the predicted values (using your rule) and the actual data.



The formula in List 3 can be modified to calculate the numerical differences between the predicted value and the actual data. Formula: $f(L_1) - L_2$

Question: 8.

Determine the average (mean) of the numerical differences and suggest reasons why this calculation is not a true representation of the 'average' distance.

Question: 9.

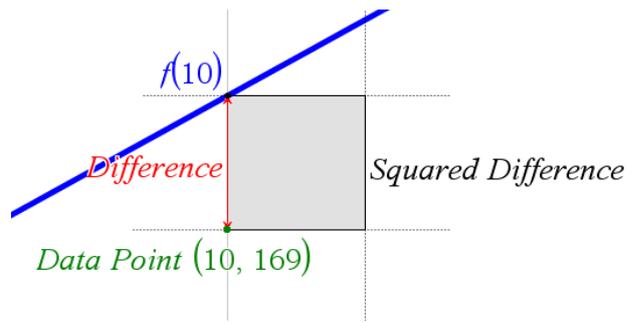
Determine sum and mean of the squared differences.



The formula in List 3 can be edited to calculate the squared differences between the predicted value and the actual data. Formula: $(f(L_1) - L_2)^2$. To 'insert' characters press [2nd] followed by [delete]. The data > statistics menu can be used to calculate the sum and the mean.

Least Squares Regression

One numerical approach for obtaining a 'line of best' fit is called Least Squares Regression. As the name suggests the equation is determined by generating the minimum sum of the square areas drawn such that one vertex of the square is on the line and another through the data point.



The Least Squares Regression line can be computed directly by the calculator.

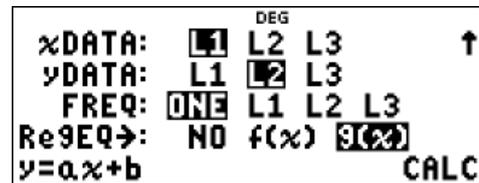
Press [2nd] [data] and navigate to option 4: Linear Regression.



Match the settings shown opposite, remembering that:

L_1 = Time

L_2 = Altitude



The regression equation will be stored in $g(x)$. Select Calculate!

Question: 10.

Write down the Least Squares Regression equation generated by the calculator and include the r^2 value. The calculator stores the equation in the form: $y = ax + b$.

Question: 11.

Determine the sum of the squared differences using this new equation. You can use the steps provided previously and changing $f(x)$ to $g(x)$ in the formula. Comment on the result and compare with the value obtained in Question 9.

Question: 12.

Does the least squares regression line pass through the point: (\bar{x}, \bar{y}) ?

Question: 13.

Which line is a better fit? The original line determined in Questions 1 – 3 or the Least Squares Regression line?

Question: 14.

Use your original equation and the Least Squares Regression equation to calculate the altitude of the weather balloon at the following times:

- i) 5 minutes (300 seconds)
- ii) 10 minutes
- iii) 30 minutes
- iv) 2 hours

In each case above, comment on the accuracy of the prediction.

Least Squares Regression - Extension

Question: 15.

Use the statistical features of the calculator to determine the following:

- i) Sample standard deviation of the time data. (S_T)
- ii) Sample standard deviation of the altitude data. (S_A)
- iii) Pearson's correlation coefficient for the time – altitude data. (r)

Question: 16.

Use your answers from Question 15 to calculate: $r \frac{S_A}{S_T}$ and compare the result with the gradient of the Least Squares Regression equation.

Question: 17.

Assuming the Least Squares Regression equation passes through the point: (\bar{x}, \bar{y}) , use your answer to Question 16 to determine the Least Squares Regression equation.