

EXPLORATIONS

Chapter 7

Features Used

[J], when(), NewProb,
[ANS]

Setup

1
NewFold conv

Convolution This chapter shows the convolution of two functions. To simplify the details, the functions are finite, piecewise, and continuous.

Topic 32: The Convolution Integral

Given a linear, time-invariant system with an impulse response of

$$h(t) = 2e^{-\frac{t}{2}} \text{ for } t > 0$$

$$h(t) = 0 \text{ for } t \leq 0$$

find the output $y(t)$ for the input function $x(t) = 1$ for $1 < t < 3$ and zero elsewhere. $y(t)$ is found by solving the convolution integral

$$y(t) = \int_{-\infty}^{\infty} h(\alpha)x(t-\alpha)d\alpha$$

Topic 33: Piecewise Convolution

Both $x(t)$ and $h(t)$ are piecewise, continuous functions. That is, they are continuous everywhere within sub-ranges and discontinuous only at the boundaries between subranges. As such, they can be entered using the **when** function.

1. Clear the TI-89 by pressing [2nd] [F6] 2:NewProb [ENTER].
2. Enter the piecewise function for $x_1(t)$ as shown in screen 1.

[CATALOG] when(1 [2nd] [<] t [CATALOG] and t [2nd] [<] 3 , 1 , 0
[] [STO] x1 [] t []

The **when** function says $x_1(t)$ has the value 1 for $1 < t < 3$ and the value 0 for all other values of t . $x_1(t)$ is used instead of x since the TI-89 uses x when graphing.



- Enter the piecewise function for $h(t)$ as shown in screen 2.

CATALOG when(t **2nd** [>] 0 [,] 2 [♦] [e^x] [(-)] 1 [÷] 2 t [)] , 0 []
STO h [(] t [)]

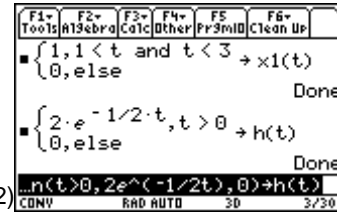
To be sure the functions are entered correctly, graph them. This book follows the standard electrical engineering convention of writing these as functions of time, t . The TI-89, however, displays graphs as functions of x .

- Enter $h(t)$ and $x1(t)$ in the Y= Editor as functions of x as shown in screen 3.
- Set the plot ranges in the Window Editor as shown in screen 4.
- Press **♦** [I] and set **Grid** to **ON**.
- Press **♦** [GRAPH] to graph the functions (screen 5).

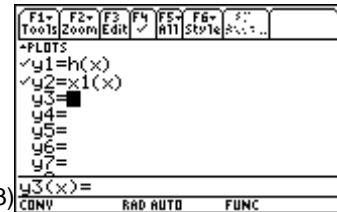
If you enter the convolution integral from Topic 32, an error message is displayed as in screen 6. Therefore, the piecewise integral must be divided into sub-ranges “by hand.”

- First, graph $x(t-\alpha)$ versus α .

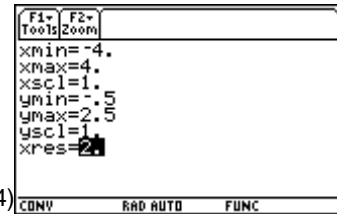
To do this, pick a value for t , such as $t=0$, and enter it on the Home screen (screen 7).



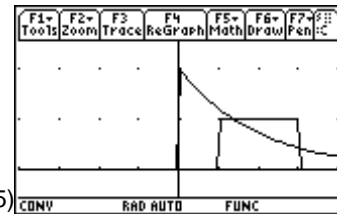
(2)



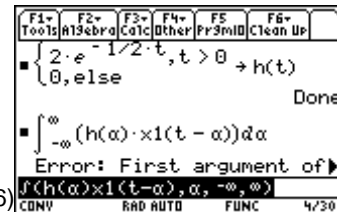
(3)



(4)

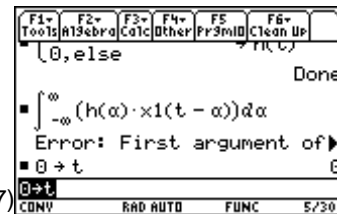


(5)



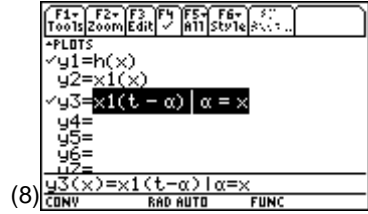
(6)

Note: To enter the integral press **2nd** [∫]. To enter α press **♦** [alpha] α . (Save keystrokes by entering “a” instead of “ α .”)



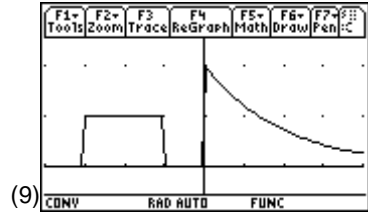
(7)

2. Then define the functions in the Y= Editor as shown in screen 8. Notice that $y_2(x)$ is deselected (use [F4]) and $y_1(x)$ and $y_3(x)$ are selected.

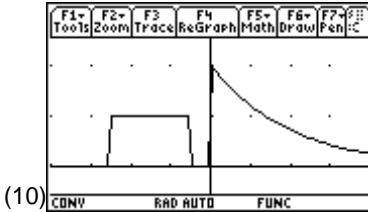


3. Press [GRAPH].

$y_3(x)$ graphs a version of $x_1(t)$ that is “flipped” about the y-axis. The graph for $t=0$ is shown in screen 9. $x_1(t)$ is flipped so that its edges are at -3 and -1. For each value of $t > 0$ $x_1(t)$ is positioned further to the right.



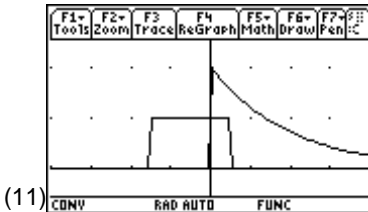
4. Return to the Home screen and set $t=0.5$. Press [GRAPH] to see the result (screen 10). Notice that for $t=0.5$, $x_1(t)$ is closer to $h(t)$.



5. Continue to consider the convolution integral for various ranges of t . The following ranges are chosen so that the integrals are easy to define.

Try a value of $t \leq 1$. From screen 10, the product of $x_1(t-\alpha)h(\alpha)$ is 0 since there are no values of t where both functions are non-zero. Therefore, $y(t) = 0$ for $t \leq 1$.

6. Use a value of t such that $1 < t < 3$. Set t to 1.5, and graph the functions (screen 11).



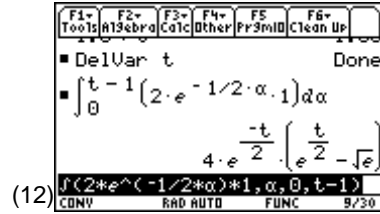
For these values of t , the product of $x_1(t-\alpha)h(\alpha)$ is non-zero over a range where there is some overlap between the two functions. The convolution integral is now

$$y(t) = \int_{-\infty}^{\infty} h(\alpha)x(t-\alpha)d\alpha$$

$$= \int_0^{t-1} 2e^{-\frac{\alpha}{2}}(1)d\alpha$$

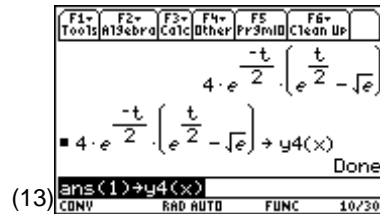
7. On the Home screen, use **DelVar** to delete the variable t before doing the integral since it was previously set to 1.5. Then enter the integral as shown in screen 12.

$\boxed{2nd} \boxed{[j]} \boxed{2} \boxed{\blacktriangleright} \boxed{[e^x]} \boxed{(-)} \boxed{1} \boxed{\div} \boxed{2} \boxed{\times} \boxed{\blacktriangleright} \boxed{[]} \boxed{\alpha} \boxed{a} \boxed{)} \boxed{\times} \boxed{1} \boxed{,} \boxed{\blacktriangleright} \boxed{[]}$
 $\boxed{\alpha} \boxed{a} \boxed{,} \boxed{0} \boxed{,} \boxed{t} \boxed{-} \boxed{1} \boxed{)} \boxed{[]}$



8. Once the integral is calculated, store the result in $y_4(x)$ as shown in screen 13 so that it can be graphed in Topic 34.

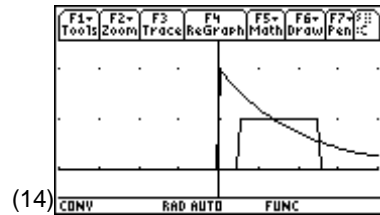
$\boxed{2nd} \boxed{[ANS]} \boxed{[STO\blacktriangleright]} \boxed{y_4} \boxed{[]} \boxed{x} \boxed{[]}$



Therefore, $y(t)$ has the value shown in screen 12 for a range of values of t . The graphs show that the overlap starts when the $t-1$ edge of $x_1(t)$ passed $t=0$. Therefore, when $t-1>0$, or when $t>1$, this form of $y(t)$ is valid. However, when the $t-3$ edge of $x_1(t)$ passes $t=0$, the integral takes on a different form. This form of $y(t)$ is valid when $t-3<0$, or $t<3$. Therefore, this graph is valid for $1<t<3$.

9. Now use the range $t>3$. In this range, $x_1(t)$ lies completely within $h(t)$.

Set $t = 3.5$ on the Home screen. Then graph $y_1(x)$ and $y_3(x)$ as shown in screen 14.



For this range, the integral is

$$y(t) = \int_{-\infty}^{\infty} h(\alpha)x(t-\alpha)d\alpha$$

$$= \int_{t-3}^{t-1} 2e^{-\frac{\alpha}{2}}(1)d\alpha$$

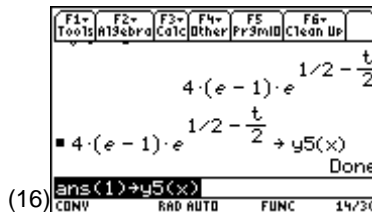
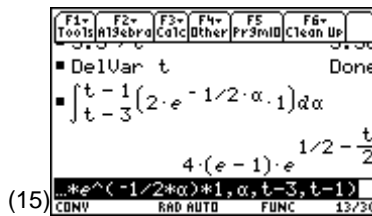
which is like the previous integral except the lower limit is changed.

10. On the Home screen, use **DelVar** to delete **t** and then enter the integral as shown in screen 15.

$\int_0^t 2e^{-\alpha} (-1)^{2 \times \alpha} \alpha \times 1 \alpha \times 1 \alpha$
 $\int_0^t 2e^{-\alpha} \alpha \times 1 \alpha$

11. Store the results in the variable **y5(x)** as shown in screen 16 for plotting in Topic 34.

This is valid for $t-3 > 0$, or $t > 3$.



Topic 34: Graphing Piecewise Convolution Results

The output $y(t)$ is given in three different pieces.

$$y(t) = 0 \quad \text{for } t \leq 1$$

$$y(t) = 4e^{-t/2}(e^{t/2} - e^{1/2}) \quad \text{for } 1 < t < 3$$

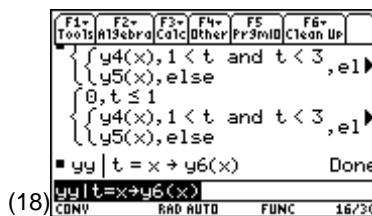
$$y(t) = 4(e-1)e^{1/2-t/2} \quad \text{for } t > 3$$

1. Combine these using the **when()** function as shown in screen 17.

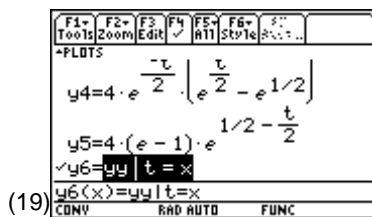
$\text{when}(t \leq 1, 0, \text{when}(1 < t < 3, y4(x), y5(x)))$
 $\text{and } t < 3, y4(x), y5(x) \rightarrow yy$



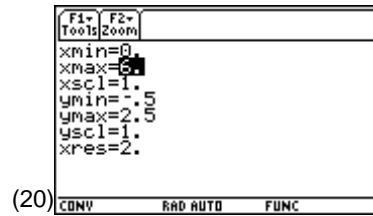
2. In preparation for graphing **yy**, change all the **t**'s in **yy** to **x**'s and save **yy** in **y6(x)** as in screen 18.



3. Press \square [Y=]. Use \square [F4] to deselect **y1(x)**, **y3(x)**, **y4(x)**, and **y5(x)** and select **y2(x)** and **y6(x)** (screen 19).

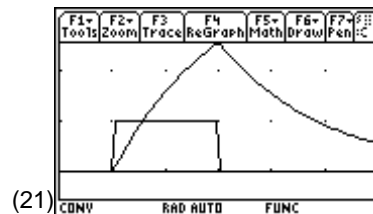


4. In the Window Editor, change the plot range on x so that x_{min} is 0 and x_{max} is 6 as shown in screen 20.



5. Press \square [GRAPH] to see a graph of the convolution integral as shown in screen 21.

The effects of the system on the input pulse $x_1(t)$ are clearly seen in screen 21. The input pulse is amplified and “smeared” or broadened as it passes through the system.



Tips and Generalizations

The **when()** function is a powerful feature of the TI-89 that allows piecewise functions to be manipulated easily. Here, the three pieces of the solution to a piecewise convolution were combined into a single function (**yy**), allowing it to be graphed as if it were a single continuous function. The **when()** function can be used anytime a piecewise function is needed.

Sometimes a new function is built by defining pieces over different time intervals. Other times it is better to define a function by adding sinusoids of different frequencies. In Chapter 8, the TI-89 will be used to find the Fourier series coefficients of a signal and reconstruct that signal from some of the coefficients.