EXPLORATIONS

Activity 3

Take the Train

Have you ever ridden on a train? In 1869, the first railroad that crossed the continent was built. Now, railroad tracks cover the United States – you probably know of some near where you live. Railway companies such as Amtrak® can take you to almost any city or town in the country. In this activity, you will travel on one of Amtrak's most famous routes and follow its path through towns like Kankakee, Illinois and Yazoo City, Mississippi.

Introduction

Using the TI InterActive!™ web browser, you will visit Amtrak's web site and access the City of New Orleans timetable. After collecting a list of stops, times and distances, you will determine which mathematical model can best track the train.

Equipment Required

- TI InterActive! software
- A working Internet connection
- Adobe® Acrobat® Reader software (http://www.adobe.com)

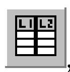
Collecting the Data

1. Start TI InterActive! The software opens to a new, blank document.
2. Title your new document *Take the Train* and add your name and the date. Click the Save button  to save and name your document.
3. Click the Web Browser button  to open the TI InterActive! browser. Click on the Data Sites button . Under the **Activity Book Links** category, click on *TI InterActive! Data Collection and Analysis*. Choose **Activity 3: Take the Train.**
4. Once the page has been loaded in the browser, scroll down to the "City of New Orleans" and click this link. Adobe Acrobat Reader automatically launches, and the timetable for this route appears. Scroll through the timetable and note that the train starts in Chicago, and ends in New Orleans. The arrival times for the train are shown in the leftmost column, and the distances in the column just beside it. Note that the distance traveled is cumulative.
5. Record the data for the train's first ten stops in the table provided.

   Note*: You can print the web page to use in the* Working With the Data *section, rather than writing it in the table and transferring it to the Data Editor.*

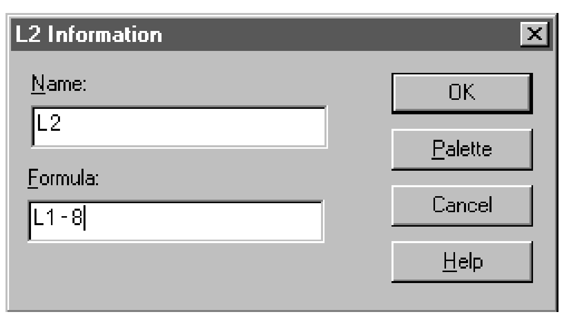
City	Time	Total Distance
Chicago, IL	8:00	0
Homewood, IL	8:51	25
Kankakee, IL	9:24	
Champaign-Urbana, IL		
Matoon, IL		
Effingham, IL		
Centralia, IL		
Carbondale, IL		
Fulton, KY		
Newbern-Dyersburg, TN		

Working With the Data

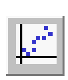
1. Click the List button , and then click the empty cell at the top of list **L1**. The times you wrote on paper will be changed slightly, and be written as decimals. Type the starting time value, 8:00, as **8**, and then press the down arrow key to move to the next cell. Enter the next time, 8:51, as a fraction, for instance **8+51/60**. Notice that TI InterActive!™ automatically enters the time in decimal form, 8.85.

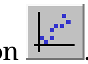
   Also, since you are interested in calculating total elapsed time, write the stops that occur after midnight as 12, plus the hour. For example, write 1:30 a.m. as **13+30/60**, and 4:25 a.m. as **16+25/60**. Continue entering the elapsed times until you have entered all of the time values into **L1**.

2. You will now use **L2** to calculate total elapsed time. Double-click on the gray box that reads **L2**. In the dialog box that appears, click in the Formula box and type **L1-8**. Click **OK**.

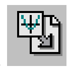


3. Click the empty cell at the top of list **L3**. Type the starting distance, **0**, and press the down arrow key to move to the next cell. Continue entering the distances until you are finished.

4. Click the Scatter Plot button  and then click the Stat Plots tab. In the uppermost text box, type **L2** to specify it as the list containing the *x*-coordinates. Press the Tab key and move to the second text box, and type **L3** to specify the list containing the *y*-coordinates.

5. Press Enter, then click the Zoom Statistics button . The viewing boundaries will be adjusted automatically to show all the plotted data.

6. The plot of elapsed time versus total distance traveled should go up and to the right. You can make a sketch on one of the blank grids in the Appendix of the data that you collected for elapsed time versus the total distance traveled. Label the horizontal and vertical axes on your sketch.

7. Click the Save to Document button  to save the graph in your TI InterActive!™ document.

Analysis and Questions

1. When two quantities are related so that when one changes, the other one changes by a constant multiple of the first, we say that the quantities are *directly proportional*. Mathematically, this type of relationship can be expressed as:

$$y = kx$$

where, for this activity, y represents the total distance traveled, x represents the total elapsed time, and k is called the *constant of variation*.

In order to find a direct variation model for elapsed time and distance, you will need to find the value of k, the constant of variation. We will use the guess-and-check method. Click the f(x) tab in the upper left corner of the Functions dialog box. Start with an initial guess of $k = 10$. Type **f(x):= 10*x** in the uppermost text box of the f(x) tab. Press Enter to superimpose the graph on the plotted data.

It is unlikely that your first guess for the value of k produced a model that matches the data closely. Click in the text box of the f(x) tab again and edit the direct variation equation, replacing the old value, $k = 10$, with your new guess for k. Press Enter to update the graph. Repeat the guess-and-check procedure until you find a k-value that models the data well and record it in the space below.

$k =$

2. In science class, you have probably used the equation: $distance = rate \times time$. This is really the same as the equation $y = kx$, because the y-values represent total *distance* traveled and the x-values represent total elapsed *time*.

For this activity, what is the real-world meaning of the k value you computed earlier using the guess-and-check method? What are the units of measure for the k value?

3. You probably noticed that increasing and decreasing the constant of variation, k, changes the steepness of the line you are graphing. For this reason, the constant k in the linear equation $y = kx$ is called the *slope* of the line. Using guess-and-check, you determined an overall slope for the train data. You also determined that this number approximates the average speed of the train over *all* of its ten stops.

We know, however, that the train was not traveling that *exact* speed between every city. You may be able to get a pretty good idea of when the train was moving fastest just by looking at the graph and finding the steepest sections. You can use another feature of TI InterActive! to figure out where the train was going fastest and slowest. Mathematically, slope is defined as the change in *y*-values divided by the change in *x*-values, $\Delta y/\Delta x$.

Click the Graph close box. Choose **Yes** when asked if you want save changes. Click the Save to Document button to save the graph in your TI InterActive! document.

To find the slope of the segment between each pair of points (the speed of the train between each of its stops), double-click on the gray box that reads **L4**. In the dialog box that appears, click in the Formula box and type **deltaList(L3)/deltaList(L2)**, and click **OK**.

The numbers that appear in list **L4** represent the average speeds for the train between stops. Between which two cities was the train moving slowest? What was the average speed between these cities?

4. TI InterActive!™ lets you check the value of *k* you found by calculating the line of best fit.

a. Click the Graph close box to return to the Data Editor.

b. Click Statistical Regressions.

c. Click the down arrow next to Calculation Type, scroll down the list and click on **Linear Regression (ax + b).**

d. In the text box labeled **X List**, type **L2**; the the box labeled **Y List**, type **L3**.

e. Click **Calculate** to find the regression equation, $y = ax + b$, and its variables. Record the regression equation values of *a* and *b* below:

$y = ax + b$
$a =$
$b =$

f. Click **Save Results**. TI InterActive! stores the results in variables, closes the Statistical Regressions tool, and displays the selected results in your document. How does the value of *a* in the linear regression equation compare with the *k*-value you found by guess-and-check?

In theory, what should the *b*-value from the regression equation be? Explain.

5. Do you think that the linear modeling equation you developed in this activity could be used to accurately predict when the train will arrive in New Orleans? (Hint: What happens to the train in Memphis?) What factors would you have to take into account if you wanted to use your equation to predict arrival times?

6. Save and print your TI InterActive!™ document.

Extensions

- Edmund and Ted are going skiing for the weekend. After work, they pack the car and set off for their trip. Leaving town, they drive on a small road for 30 minutes at a constant speed of 40 mph. When they reach the highway, they speed up to 65 mph and drive for two hours. What is the average speed (total distance divided by total time)? Why isn't the answer the same as the average of the speeds: 52 ½ mph? (Hint: Think about how long they were traveling at each speed).

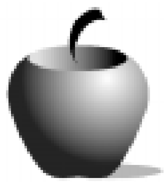
- Suppose that, in the previous question, Edmund and Ted had to drive an additional 30 minutes at 30 mph on a windy road from the highway to the ski mountain. What is the average speed for the entire trip now?

- Tonya is going to Barbara's house for a slumber party, but has to stop and pick up Mary first. Tonya walks ¼ mile to Mary's house in 5 minutes, but has to wait there while she packs her sleeping bag. Mary's dad offers to drive them, but only if they sit down and try some of his experimental grapefruit and marshmallow cookies. By the time they explain to Mary's dad that they would prefer a good walk to a stomach ache, 15 minutes have passed. They leave the house, and walk ¾ mile to Barbara's house in 10 minutes. What is the total distance for the trip? What is the total time (including the time they were at Mary's house)? What is the average speed for the whole trip? What is the average speed for just the walking portion of the trip?

- Mr. Bryan is taking his dog Tater for a walk. They leave home and walk two blocks down Amherst Street in 3.6 minutes, then stop for 1 minute while Tater watches a neighbor's cat. Next, they walk four blocks on Parkside Avenue in 7.5 minutes, three blocks on Tillinghast Place in 5.1 minutes, stop for 45 seconds so Tater can sniff a tree, and finally walk five blocks back home in 8.5 minutes. Assume that a block is 1/10 of a mile. What is the total distance for the trip? What is the total time (including the time when Tater was busy)? What is the average speed for the whole trip? What is the average speed for just the walking part of the trip?

Teacher Notes

Activity 3: Take the Train



Math Concepts

- Internet Data Collection
- List Manipulation
- Direct Variation
- Linear Function

Activity Notes

- If you do not have Internet access, you can still do this activity by obtaining a bus schedule from a local station.
- Be sure to take some time to carefully explain the idea of converting standard time values to decimal form. This concept is confusing for many students.

Sample Data

City	Time (hours)	L1=Times (decimal hours)	L2=Elapsed Time (decimal hours)	L3=Total Distance (miles)
Chicago, IL	8:00	8	0	0
Homewood, IL	8:51	8.85	0.85	25
Kankakee, IL	9:24	9.4	1.40	57
Champaign-Urbana, IL	10:37	10.61667	2.62	129
Mattoon, IL	11:18	11.3	3.30	174
Effingham, IL	11:43	11.71667	3.72	201
Centralia, IL	12:35	12.58333	4.58	254
Carbondale, IL	1:30	13.5	5.50	310
Fulton, KY	3:40	15.66667	7.67	406
Newbern-Dyersburg, TN	4:25	16.41667	8.42	442

Analysis and Questions - Key

1. $k = 54$.
2. k represents the speed of the train in miles per hour.
3. Slowest = Chicago to Homewood, 29.4 mph; fastest = Champagne-Urbana to Matoon, 66.2 mph.
4. $a = 54.860$, $b = -8.998$. The a-value is very close to the value found by guess-and-check. The value of b in the regression equation should equal zero.
5. No, since the train stops for a period of time when it arrives in Memphis. You need to consider layovers, stopping times, and variations in speed if you wish to accurately predict arrival times.