

Name _____

Date _____

EXPLORATIONS

ACTIVITY 3

Solving Systems of Equations

Solutions to real-world problems can't always be represented by a single variable from a single equation. Consider two trains that are traveling in opposite directions, leaving different stations, and passing each other at some point. If the trains are traveling at constant speeds, their motions can be represented by a pair of linear equations, each expressing *distance* as a function of *time*. The common solution to the equations is the time and distance coordinates where the trains pass.

A set of equations with the same variables is called a *system of equations*. For example, the solution to a system of two equations, with two variables, is the ordered pair that satisfies both equations. If all the equations in the system are linear, then it is called a *linear system*.

This activity explores three different methods for solving linear systems on the Voyage™ 200 PLT.

Instructions

Part A—Solving Systems with the Elimination-by-Addition Method

1. A tennis coach buys lunch for the team at a fast food restaurant. He orders 8 hamburgers and 5 french fries at a cost of \$24. Because some of the tennis players are still hungry, the coach buys 6 more hamburgers and 2 more french fries for \$16.60.

The equations for the purchases can be expressed mathematically as a pair of linear equations with x representing the price of a hamburger and y representing the price of an order of french fries.

Write the equation for the second purchase of food.

$$\text{1st purchase} = 8x + 5y = 24$$

$$\text{2nd purchase} =$$

Solving Systems of Equations *(Continued)*

1. (continued)

What is the cost of each food item? This question can be answered by finding the ordered pair (x, y) that makes both equations true. One method, called elimination by addition, involves manipulating the equations algebraically so that one variable is eliminated when the equations are added together.

2. Because the equations are already written in standard form, $Ax + By = C$, they are ready to be stored in the Voyage™ 200 PLT.

Press $\overline{\text{ON}}$ to turn on the Voyage 200 PLT. To reset to the default settings, press $\overline{\text{2nd}} \overline{\text{MEM}} \overline{\text{F1}}$, select 1:RAM, select 2:Default, and then press $\overline{\text{ENTER}} \overline{\text{ENTER}}$.

Press $\overline{\blacktriangledown} \overline{\text{[CALC HOME]}}$ to access the Home screen. Press $\overline{\text{F6}}$, select 1:Clear a–z, and press $\overline{\text{ENTER}}$ to clear all values stored to one-character variables.

To enter the first equation in the entry line, press $\mathbf{8} \overline{\text{X}} \overline{\text{X}} \mathbf{+} \mathbf{5} \overline{\text{X}} \overline{\text{Y}} \overline{\text{=}} \mathbf{24}$. Then press $\overline{\text{STO}} \overline{\text{EQ1}} \overline{\text{ENTER}}$ to store the equation. Repeat these steps to store the second equation to eq2.

3. To eliminate a variable, the coefficients of one variable must be opposites, or additive inverses.

To eliminate x , multiply the first equation by 3 using the Voyage 200 PLT as follows:

Press $\overline{\text{F2}}$ and select 3:expand(.

Press $\mathbf{3} \overline{\text{X}} \overline{\text{EQ1}} \overline{\text{D}} \overline{\text{STO}} \overline{\text{EQ1}} \overline{\text{ENTER}}$ to store the equivalent equation in expanded form back to equation 1. Record this equation below.

eq1 =

4. Determine the constant multiplier that will yield an opposite coefficient for x in equation 2.

multiplier =

Repeat the procedure described above to store the modified version of equation 2; record the equation below.

eq2 =

5. Add the equations from step 3 and step 4 by entering $\overline{\text{EQ1}} \overline{+} \overline{\text{EQ2}} \overline{\text{ENTER}}$ at the entry line. The resulting answer is an equation in terms of y .

equation =

Solving Systems of Equations (Continued)

6. To solve for y , press $\boxed{F2}$ and select 1:solve(. Press \odot to highlight the equation in step 4. Press \boxed{ENTER} to paste this equation in the entry line, and then press $\boxed{\downarrow}$ \boxed{Y} $\boxed{\downarrow}$ \boxed{ENTER} .

Write the solution for y in the space provided.

Recall that the value of y corresponds to the price of an order of french fries.

$y =$ _____

Store this value of y in the Voyage™ 200 PLT by entering it on a clear entry line and then pressing $\boxed{STO\rightarrow}$ \boxed{Y} \boxed{ENTER} .

7. The corresponding x -solution to the system is determined by substituting the value of y found in step 5 into eq1 or eq2.

To solve equation 1 for x in terms of the y -value you stored, press $\boxed{F2}$ and select 1:solve(. Enter **EQ1** $\boxed{\downarrow}$ \boxed{X} $\boxed{\downarrow}$ \boxed{ENTER} .

Record the solution for x below. This value represents the cost of a hamburger.

$x =$ _____

Part B—Solving Systems with the Substitution Method

In *Part A*, you solved a system of two equations with two unknowns using the elimination-by-addition method. You also can determine a solution in the form (x, y, z) to a system of three linear equations with three unknowns. The procedure for solving this system is called the substitution method.

In the substitution method, you solve one equation for a single variable and then replace that variable in the other two equations with the expression for the same variable. The result is a pair of equations with two unknowns. The process of solving and substituting is continued until you have determined the values of all three variables.

Consider the following system of equations:

$$\begin{aligned} -3x + 2y + z &= 27 \\ 5x - y + 9z &= 45 \\ 4x + 3y - 6z &= -62 \end{aligned}$$

Solving Systems of Equations *(Continued)*

1. To start, press $\boxed{\text{F6}}$, select 1:Clear a–z, and press $\boxed{\text{ENTER}}$ to clear all values stored to one-character variables. Enter the first equation in the entry line, and then press $\boxed{\text{STO}} \rightarrow \boxed{\text{EQ1}} \boxed{\text{ENTER}}$ to store the equation.

Repeat these steps to store the second equation to eq2 and the third equation to eq3.

2. Begin by solving the first equation.

Press $\boxed{\text{F2}}$ and select 1:solve(. Press $\boxed{\text{EQ1}} \boxed{\text{Z}} \boxed{\text{ENTER}}$. Write the resulting equation for z in terms of x and y in the space below.

$z =$

3. By substituting the z -expression from step 2 into equations 2 and 3, the system can be reduced to two equations with two unknowns, x and y .

The “with” operator feature on the Voyage™ 200 PLT, $\boxed{[1]}$ (accessed by pressing $\boxed{2\text{nd}} \boxed{\text{K}}$), is displayed as a “bar” on the screen. It substitutes the expression that follows into the expression that precedes the “bar.”

Press $\boxed{\text{EQ2}} \boxed{2\text{nd}} \boxed{[1]}$. Press \odot to highlight the z -equation in the history area, and press $\boxed{\text{ENTER}}$ to paste it into the entry line. Press $\boxed{\text{STO}} \rightarrow \boxed{\text{EQ2}} \boxed{\text{ENTER}}$ to store the modified equation back to eq2. Record the equation below.

$eq2 =$

4. Press $\boxed{\text{EQ3}} \boxed{2\text{nd}} \boxed{[1]}$. Press \odot to highlight the z -equation in the history area, and press $\boxed{\text{ENTER}}$ to paste it into the entry line. Press $\boxed{\text{STO}} \rightarrow \boxed{\text{EQ3}} \boxed{\text{ENTER}}$ to store the modified equation back to eq3. Record the equation below.

$eq3 =$

Solving Systems of Equations (Continued)

5. Solve for x in equation 2 by pressing $\boxed{F2}$ and selecting 1:solve(. Press **EQ2** $\boxed{\square}$, **X** $\boxed{\square}$ $\boxed{\text{ENTER}}$.

To find the y -solution to the system, substitute the x -expression you just found into equation 3 and solve for y as follows:

Press $\boxed{F2}$ and select 1:solve(.

Enter **EQ3** $\boxed{\square}$, **Y** $\boxed{\square}$ $\boxed{2\text{nd}}$ $\boxed{[1]}$.

Use the \leftarrow to highlight the x -equation and press $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$. Record the value of y .

$y =$ _____

Type this value on the entry line, and then press $\boxed{\text{STO}\rightarrow}$ **Y** $\boxed{\text{ENTER}}$ to store it to y .

6. Now you can determine the value of x by solving equation 2.

Press $\boxed{F2}$ and select 1:solve(.

Press **EQ2** $\boxed{\square}$, **X** $\boxed{\square}$ $\boxed{\text{ENTER}}$. Record the value of x below.

$x =$ _____

Type this value on the entry line, and then press $\boxed{\text{STO}\rightarrow}$ **X** $\boxed{\text{ENTER}}$ to store it to x .

7. Finally, press $\boxed{F2}$ and select 1:solve(. Press **EQ1** $\boxed{\square}$, **Z** $\boxed{\square}$ $\boxed{\text{ENTER}}$ to find the value of z . Record this number below.

$z =$ _____

Part C—Solving Systems by the Simultaneous-Equations Method

The Voyage™ 200 PLT has a built-in feature for solving systems of equations using the **simult(** command. This method is especially convenient when more than two equations are involved. For a system of equations written in standard form, $Ax + By = C$, the variable coefficients are entered as a set (matrix) in brackets, followed by the set of result constants (vector) also in brackets. The Voyage 200 PLT returns the solution to the system as a column of numbers (vector).

Solving Systems of Equations (Continued)

For example, in the system shown below, the coefficients and constants are entered into the Voyage™ 200 PLT as shown on the screen image to the right.

$$6x - 2y = 2$$

$$3x + 4y = -19$$

1. You can use the same procedure to solve larger systems. For this exercise, use:

$$10x - 3y + 7z = -55$$

$$-x - 4y + 2z = -29$$

$$6x + 3y - 5z = 55$$

First, clear all the one-character variable values by pressing $\boxed{\text{F6}}$, selecting 1:Clear a-z, and pressing $\boxed{\text{ENTER}}$.

2. To access the **simult**(command, press $\boxed{2\text{nd}}$ $\boxed{\text{MATH}}$ and select 4:Matrix.

Select 5:simult(from the submenu. This places the command in the entry line.

Press $\boxed{2\text{nd}}$ $\boxed{[]}$ and enter 10 $\boxed{,}$ $\boxed{(-)}$ 3 $\boxed{,}$ 7. Then press $\boxed{2\text{nd}}$ $\boxed{[;]}$ and enter the coefficients from the second equation, separating the numbers with commas.

Press $\boxed{2\text{nd}}$ $\boxed{[;]}$ and repeat this procedure for the third equation. Press $\boxed{2\text{nd}}$ $\boxed{[]}$ to close the set.

Next, press $\boxed{,}$ $\boxed{2\text{nd}}$ $\boxed{[]}$ $\boxed{(-)}$ 55 $\boxed{2\text{nd}}$ $\boxed{[;]}$. Enter the constants from the right side of each remaining equation, separating the numbers with semicolons.

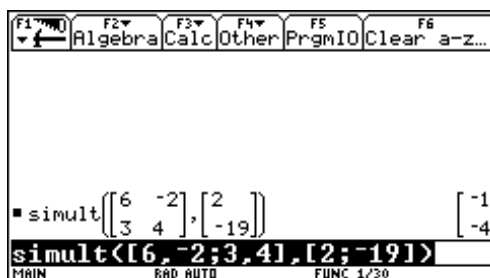
Press $\boxed{2\text{nd}}$ $\boxed{[]}$ to close the set and $\boxed{)}$ to close the command.

Press $\boxed{\text{ENTER}}$ to display the solution to the system. Record the solution to the system below.

$$x = \underline{\hspace{2cm}}$$

$$y = \underline{\hspace{2cm}}$$

$$z = \underline{\hspace{2cm}}$$



Note: In the entry line, columns of numbers are separated by commas and rows are separated by semicolons. The coefficient matrix is also separated from the constant vector set by a comma. The number arrays are displayed in “pretty print” format, with the solution $x = -1$ and $y = -4$ shown on the right side of the history area.

Note: You can also easily solve systems of linear equations with the Simultaneous Equation Solver Application for the Voyage 200 PLT.

Solving Systems of Equations *(Continued)*

Questions

Follow the procedures outlined in this activity to solve the systems of equations below by the indicated methods.

1. You decide to buy two different kinds of candy at the store. In one bag, you put two handfuls of toffees and in another bag you put three handfuls of licorice. The toffees cost \$3.50 per pound and the licorice costs \$1.75 per pound. At the register, the total cost for the candy is \$3.85 for 1.7 pounds.

How many pounds of each kind of candy did you purchase? Solve the system with the elimination-by-addition method. Check your work using the simultaneous equations method.

Let t = number of pounds of toffees

l = number of pounds of licorice

$t =$ _____

$l =$ _____

2. Solve this system with the substitution method.

$$-7x + 5y - 2z = 13$$

$$4x + 2y + 8z = 66$$

$$x - 3y + 6z = 53$$

$x =$ _____

$y =$ _____

$z =$ _____

3. Solve this system with the simultaneous-equations method.

$$2x - y = 12$$

$$-5x + 8y + z = -37$$

$$6x - 3z = 18$$

Notice that the first and last equations are missing a z and y term, respectively. The coefficients of these variables are actually zeros. Be sure to include zeros when entering the coefficients in the Voyage™ 200 PLT.

$x =$ _____

$y =$ _____

$z =$ _____

