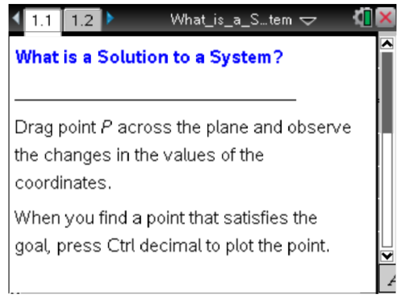
What Is a Solution to a System?



Math Objectives

- Students will understand what it means for an ordered pair to be a solution to a linear equation.
- Students will understand what it means for an ordered pair to be a solution to a system of linear equations.
- Students will look for and express regularity in repeated reasoning (CCSS Mathematical Practice).

Vocabulary

- system of linear equations
- solve

About the Lesson

- This lesson involves solving a system of linear equations.
- As a result, students will:
  - Understand that there are an infinite number of solutions to one equation, but exactly one solution for this particular system of equations.
  - Begin to explore the possibility of having infinitely many or no solutions.

Related Lessons

- After this lesson: Balanced Systems of Equations

TI-Nspire™ Navigator™ System

- Use Quick Polls to check student understanding.
- Use Screen Capture to examine patterns that emerge.
- Use Teacher Edition computer software to review student documents.

TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:
Student Activity
What_is_a_Solution_to_a_System_Student.pdf
What_is_a_Solution_to_a_System_Student.doc

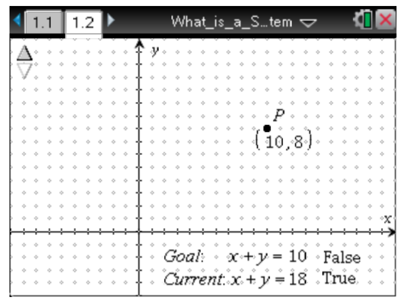
TI-Nspire document
What_is_a_Solution_to_a_System.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

> **Tech Tip:** If students experience difficulty dragging a point, check to make sure that they have moved the cursor (arrow) until it becomes a hand ( ) getting ready to grab the point. Also, be sure that the word *point* appears. Then press ctrl to grab the point and close the hand ( ). When finished moving the point, press esc to release the point.

Move to page 1.2.



1. Move point *P*. Describe how the coordinates relate to the *Current* equation shown in the lower-right corner of the screen.

   <u>Answer:</u> The coordinates of point *P* change. The *Goal* equation stays the same. The *Current* equation displays the result when the current coordinates for point *P* are substituted into the equation.

> **Teacher Tip:** Be sure to emphasize the mathematical relationship between the coordinates of point *P* and the *Current* equation. The value of the *Current* equation is obtained by substituting the coordinates of point *P*. If students do not see this, make sure it is brought out during the discussion.

2. a. In the *Goal* equation, $x + y = 10$, if $x = –3$, what value of y is needed to make the equation true?

   <u>Answer:</u> $y = 13$

   b. Move point *P* so that the first coordinate is –3 and the *Current* equation matches the *Goal* equation. Press ctrl . to mark this point.

   <u>Answer:</u> The point (–3, 13) should be plotted on the graph.

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 1 at the end of this lesson.

3. Move point *P* to a new location where the *Current* equation again matches the *Goal* equation. Press ctrl . to mark this point. Mark at least four more points that make the equations match.

What do you observe about the pattern of the points you have marked?

Answer: The sum of the coordinates is 10. The points form a linear pattern falling from left to right.

Teacher Tip: Students can use different approaches in marking points. Some of them might look for ordered pairs they know satisfy the *Goal* equation; others might notice a pattern using the slope. For example, from one marked point, a slope of –1, moving down one, right one or moving down two, right two will generate additional solutions.

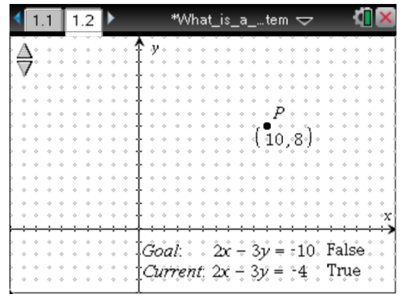
TI-Nspire Navigator Opportunity: *Screen Capture* and *Quick Poll*
See Note 2 at the end of this lesson.

4. A solution to an equation in two variables is an ordered pair (x, y) that makes the statement true. Each point you have marked is one solution to the goal equation $x + y = 10$. How many solutions does this equation have? How do you know?

Answer: This equation has infinitely many solutions. You can choose any value for x and find a corresponding value for y that sums to 10, so there are an infinite number of ordered pairs that will make the equation true.

Teacher Tip: Students can only describe integer solutions as point *P* moves from grid point to grid point due to the design of this file. It is important to make sure students understand that this equation would have non-integer solutions as well. Elicit some solutions that are non-integers. Elicit other solutions that contain negative values.

Click the slider (∆) on page 1.2 to change the problem.



5. Move point *P* to a location where the *Current* equation matches the *Goal* equation. Mark at least two more solutions to the equation.

   a. Describe a pattern you could use to determine two more solutions without randomly moving point *P*.

   <u>Sample Answers:</u> Conjectures that include using the slope, or rate of change, will generate points that satisfy the condition: from one marked point, move right 3 and up 2, or down 2 and left 3. Others can play with numbers until they find additional ordered pairs.

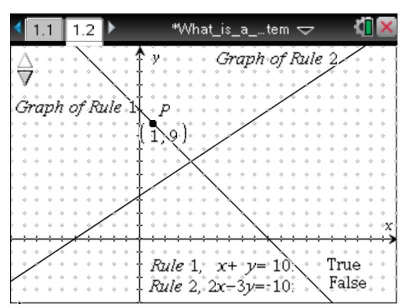
   b. Use your pattern to explain how many solutions you can find for this equation.

   <u>Answer:</u> If students use slope, they can argue that because you can continue moving over 3 and up 2 forever, you can find an infinite number of solutions. They might also argue that you can use any number to replace *x* and then solve for *y* to get an ordered pair (*x*, *y*) that is a solution, so there are infinitely many solutions.

> **Teacher Tip:** Be sure to have students share their strategies. If the notion of rate of change does not emerge, ask them to consider how they could move from point to point on those they have marked. Have them check by using the pattern to go between two points they have marked.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

Click the slider (∆) to change the problem.



6. Move point *P*. Identify a point that satisfies each condition.

   a. *Rule 1* is true and *Rule 2* is false.

   <u>Sample Answers:</u> Any point on the line with *Rule 1*, $x + y = 10$, except the intersection point of the two lines will make rule 1 true and *Rule 2* false. Every point on the line is in the solution set for the equation, so this will make the rule true.

b. Both rules are false.

Answer: Any point that is not on either line will make both rules false because that point will not be in the solution set of either equation.

c. *Rule 1* is false and *Rule 2* is true.

Answer: Any point on the line with rule 2, $2x - 3y = 10$, except the intersection point will make *Rule 1* false and *Rule 2* true. The reasoning is similar to that in part a—a point on the line is in the solution set for the equation of the line and will make the equation true.

d. Both rules are true.

Answer: Only the intersection point will make both rules true because only the intersection point lies on both lines, so it is in the solution set for the equations of both of the lines.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 4 at the end of this lesson.

7. A solution to a system of equations is any ordered pair (*x,* y) that makes both equations true simultaneously.

a. How many solutions are there for the system $\begin{cases} x + y = 10 \\ 2x - 3y = -10 \end{cases}$? Explain your reasoning.

Answer: This system will have only one solution, found at the intersection point of the two lines. These lines can have only one point in common because they have different slopes and after they have crossed once, they cannot cross again.

b. What is the solution to the system?

Answer: The solution is the ordered pair (4, 6).

8. How can you verify your solution in question 7b?

Answer: Substitute x = 4 and y = 6 into both equations.
$x + y = 10 \rightarrow 4 + 6 = 10$
$2x - 3y = -10 \rightarrow 2(4) - 3(6) = 8 - 18 = -10$

9. Candice says that (3, 5) is the only solution to the system $\begin{cases} x + y = 8 \\ x - 2y = -7 \end{cases}$.

Do you agree? Why or why not?

Answer: Agree because 3 + 5 = 8 and 3 – 2(5) = –7, and we know that the system is graphed as two distinct lines that intersect at only one point.

> **Teacher Tip:** Some students might also note that the equations are different so the lines are distinct, and because they have one point in common they are not parallel. In this case, it is important to note that two equations might appear different (for example, $x + y = 8$ and $2x + 2y = 16$) but have the same solution set.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- A solution to an equation in two variables is an ordered pair that makes the statement true.
- A linear equation in two variables has an infinite number of integer and non-integer solutions whose points follow a linear path.
- A solution to a system of equations in two variables is an ordered pair that makes both equations true at the same time.
- A linear system containing two linear equations that intersect in one point has exactly one solution.

Assessment

This activity is suitable for either formal or informal assessment of student understanding and skills.

TI-Nspire Navigator

Note 1

Question 2, *Live Presenter*: Select a student to illustrate his/her answer to Question 2 using *Live Presenter*.

Note 2

Question 3, *Screen Capture* and *Quick Poll*: As students are answering Questions 3 and 4, take screen captures to monitor their progress. Take a quick poll with an oral prompt and use the Open Response feature, "describe a pattern of the points that you plotted." Then look at the responses with the class and discuss what is correct and what is not and why.

Note 3

Question 5, *Quick Poll*: Use the Open Response feature and send this prompt: $2x + y = 7$. Ask students to submit one ordered pair that satisfies this equation.
Option: Require that one of the coordinates must be negative.
Once you have collected the responses, look at the responses and discuss which ones are correct and why.

Note 4

Question 6, *Quick Poll*: Use the Yes No feature and send this prompt:
$x – y = 4$
$2x + y = 8$
(5, 1)
Ask students: Is the ordered pair a solution to this system of equations? Be able to defend your answer.