NUMB3RS Activity: Irregular Polygon Centroids
Episode: "Burn Rate"

Topic: Geometry, Points of Concurrency **Grade Level:** 9 - 10
Objective: Students will be able to find the centroid of irregular polygons.
Time: 20 minutes
Materials: TI-83 Plus/TI-84 Plus graphing calculator

Introduction

In "Burn Rate," when mail bombs kill a series of seemingly unrelated people, Don asks Charlie to help uncover the source and the link that connects the people. Don's team has determined the locations where the letter bombs were mailed, where the envelopes were purchased, a hardware store where some components were bought, etc. Using geo-profiling, Charlie is able to find the likely starting point where the bomber set out to buy his bomb materials.

Geo-profiling is an investigative technique used by law enforcement that uses the locations of connected crimes to determine the most probable area of offender residence. In this activity, students will do their own geo-profiling by finding the centroid of a polygon where the vertices of the polygon will represent the locations that Don's team has discovered. An algebraic method will be determined for finding the centroid of weighted and non-weighted data points.

The extension of this activity details a method for finding the centroid of a non-polygonal region using calculus.

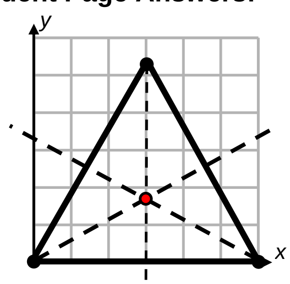
Discuss with Students

Students should be able to construct a median using a compass or paper folding.

To aid in the calculations for finding weighted centroids, download the calculator program CENTROID by going to the Web site **http://education.ti.com/exchange** and searching for "7999." This program will be utilized in Question 6.

Student Page Answers:

1.



2. *(3, 1.7)*

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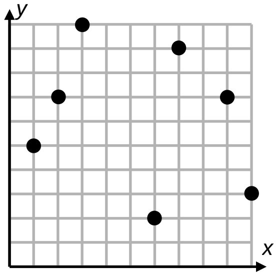
3.



Centroid answers may vary. **4.** *(5.4, 6.1)* **5.** *(4.7, 6.5)* **6.** *(2.9, 5.1)*

Extension Page Answers:

1.

$$m = \int_{-1}^{2} (-x^2 + x + 2)\,dx = 4.5$$

$$M_y = \int_{-1}^{2} (-x^3 + x^2 + 2x)\,dx = 2.25$$

$$M_x = \frac{1}{2}\int_{-1}^{2} (-x^4 + 9x^2 - 4x - 12)\,dx = -10.8$$

2.

$$\bar{x} = \frac{M_y}{m} = \frac{2.25}{4.5} = 0.5$$

$$\bar{y} = \frac{M_x}{m} = \frac{-10.8}{4.5} = -2.4$$

The Centroid is approximately (0.5, –2.4)

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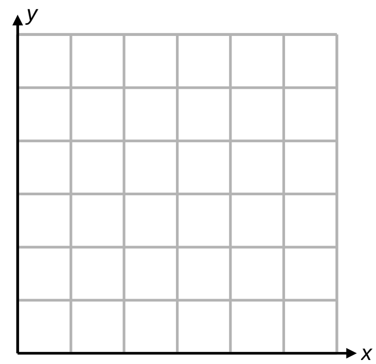
Name: ______________________________ Date: _______________

NUMB3RS Activity: Irregular Polygon Centroids

In "Burn Rate," when mail bombs kill a series of seemingly unrelated people, Don asks Charlie to help uncover the source and the link that connects the people. Don's team has determined the locations where the letter bombs were mailed, where the envelopes were purchased, a hardware store where some components were bought, etc. Using geo-profiling, Charlie is able to find the likely starting point where the bomber set out to buy his bomb materials.

This starting point will be the centroid of the polygon determined by the locations found by Don's team. Remember that a centroid is the balancing point of the polygon. In other words, if the polygon were to be placed on the tip of the pin at the centroid, it would be perfectly balanced.

1. Suppose Don has determined that the locations are (0, 0), (6, 0) and (3, 5.2) when laid out on a map. Plot the points and find the location of the centroid by finding the intersection of the three medians.

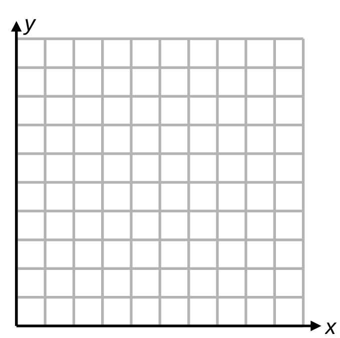


While this works very well, it is dependant upon geometric constructions which can become problematic when dealing with map coordinates given by Don's team. An algebraic method for solving the previously stated problem is to find the centroid using the formula $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

2. Find the centroid using this formula to confirm its accuracy.

This method can not only be used for triangles, but for irregular shaped polygons, which is what the data points supplied by Don's team will result in when graphed.

3. Suppose Don's team has determined the bomber used the following locations (1, 5), (3, 10), (2, 7), (6, 2), (7, 9), (10, 3) and (9, 7). Plot these points on the grid below and estimate the centroid.



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To determine the exact location of the centroid, the original formula will be extended such that:

$\bar{x}$ = average of x-coordinates
$\bar{y}$ = average of y-coordinates

Let x_i be the x-coordinate in the ith data point, y_i be the y-coordinate in the ith data point, and n be the total number of data points. Then the centroid of the data can be summarized by the expressions in the ordered pair $\left(\frac{\sum_{i=1}^{n} x_i}{n}, \frac{\sum_{i=1}^{n} y_i}{n} \right)$.

4. Calculate the centroid using this new formula and see how close your guess was.

In finding the location of the bomber, Charlie decides to weight certain data points more than others because of the importance of them to the bomber. Suppose the data points above were weighted as follows:

(1, 5) –	weight 2	(10, 3) –	weight 1
(3, 10) –	weight 1	(8, 9) –	weight 2
(2, 7) –	weight 3	(9, 7) –	weight 1
(6, 2) –	weight 1		

To compute the weighted centroid, a slight modification to the formula will be needed. Each data point must be multiplied by its weight m before summing, and each sum must be divided by the total of the weights attached to all the data points.

This can be summarized by the expression $\left(\frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}, \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i} \right)$.

5. Calculate the weighted centroid.

6. Using the calculator program supplied by your teacher, compute the weighted centroid for the following data points:

(1, 3) –	weight 3	(4, 8) –	weight 1
(8, 3) –	weight 1	(1, 1.3) –	weight 2
(0, 8) –	weight 2	(0, 6) –	weight 2
(5, 1) –	weight 1	(9, 8) –	weight 1
(3.5, 6) –	weight 1	(9, 11) –	weight 1

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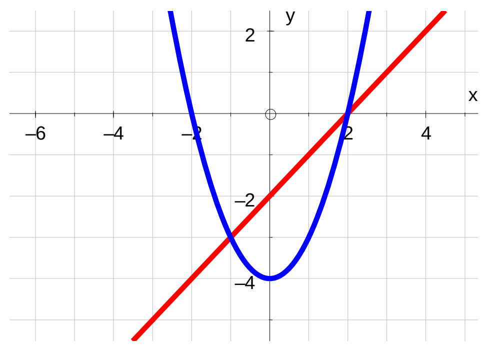
The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extension: Finding the Centroid of a Smooth Curve

The centroid $(\bar{x}, \bar{y})$ for the region bounded by $f(x)$ and $g(x)$ is defined as $\bar{x} = \dfrac{M_y}{m}$ and $\bar{y} = \dfrac{M_x}{m}$, where $M_y = \int_a^b x[f(x) - g(x)]\,dx$, $M_x = \int_a^b \left[\dfrac{f(x)+g(x)}{2}\right][f(x) - g(x)]\,dx$, and $m = \int_a^b [f(x) - g(x)]\,dx$.

To find the centroid of the region bounded by the two equations $f(x) = x - 2$ and $g(x) = x^2 - 4$, we notice from the graph that they intersect at (–1, –3) and (2, 0).



1. Find m, M_y and M_x for $f(x)$ and $g(x)$ given above.

$$m = \int_{-1}^{2} [x - 2 - (x^2 - 4)]\,dx =$$

$$M_y = \int_{-1}^{2} x[x - 2 - (x^2 - 4)]\,dx =$$

$$M_x = \int_{-1}^{2} \left[\frac{x - 2 + x^2 - 4}{2}\right][x - 2 - (x^2 - 4)]\,dx =$$

2. Using the values you found above, what are $\bar{x}$ and $\bar{y}$?

$$\bar{x} = \frac{M_y}{m} =$$

$$\bar{y} = \frac{M_x}{m} =$$

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