

Implicit Differentiation on the TI-89

by Dave Slomer

We do *implicit differentiation* when we are given an *implicit relation* in x and y , such as $x^2 + y^2 = 9$. We usually assume that the independent variable is x and that each other variable (usually only y) is a function of the independent variable. We then apply the Chain Rule to the process of finding the derivative of both sides of the equation with respect to the independent variable, followed by some algebra to solve for dy/dx (refer to figure 1). In “related rates” application problems, we often assume that all variables are functions of *time*, a variable not even necessarily present in the given equation.

If the equation is no more complex than $x^2 + y^2 = 9$, it is not difficult to find $\frac{dy}{dx}$ or $\frac{dy}{dt}$ by hand (once sufficient skill in implicit differentiation has been acquired). But there are those implicit relations for which it would be nice to be able to get a second opinion, if not the answer, from technology [e.g., see the second example in figure 6].

It is possible to do implicit differentiation on the TI-89 using what might be called “The **I**mplicit **D**ifferentiation **F**ormula” (**IDF** for short):

$$\text{If } F(x,y)=0 \text{ defines a differentiable relation of } x \text{ and } y, \text{ then } \frac{dy}{dx} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}.$$

Each “humpbacked d ” (∂) signifies **partial derivative**, an advanced calculus concept. In short, the partial derivative $\frac{\partial F}{\partial x}$ (for example) means to take the derivative of F with respect to x , with all other variables being “held constant”. Meanwhile, $\frac{\partial F}{\partial y}$ is the partial derivative of F with respect to y , found by holding the other variable(s) constant. It’s a long story. But, on the TI-89, there is no difference between the derivative operator, $[d]$ {which is $\boxed{2nd}\boxed{8}$ }, and ∂ . So, to calculate $\partial F/\partial x$ or $\partial F/\partial y$, calculate dF/dx or dF/dy .

Most implicit relations, such as is defined by the equation $x^2 + y^2 = 9$, are not of the required form $F(x,y) = 0$. But it’s always easy to “solve for 0” (that is, to isolate 0—“get everything on one side”). Subtracting 9 from both sides gives $x^2 + y^2 - 9 = 0$, which is in the right form, where $F(x,y) = x^2 + y^2 - 9$. Once you have this, you can find dy/dx either by hand or by ’89.

By hand, the process of finding dy/dx implicitly for $x^2 + y^2 - 9 = 0$ looks like this:

Fig. 1

$x^2 + y^2 - 9 = 0$
$2x + 2y \cdot \frac{dy}{dx} - 0 = 0$
$2y \cdot \frac{dy}{dx} = -2x$
$\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$

On the TI-89, if you first **Define** $f(x,y)=(\dots\text{the given implicit relation, "solved for 0" } \dots)$ and then give the command $-d(f(x,y),x)/d(f(x,y),y)$, the “**IDF**” would shorten the process considerably, as shown in figure 2.

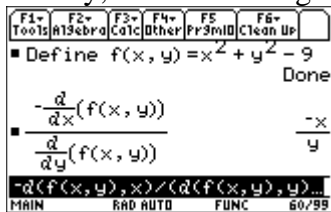


Fig. 2

It's not pleasant or easy to type all that correctly, but improvements are available.

Improvement #1: If you can live with always defining $f(x,y)$ to be the relation for which you want dy/dx , then defining the **IDF** via the command in figure 3 will streamline the process.

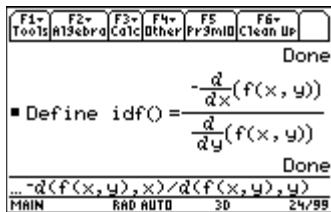


Fig. 3

The command is **Define idf() = -d(f(x,y),x)/d(f(x,y),y)**. Give that command once. To use it, define an implicit function $f(x,y)$ and give the command **idf()**, as shown in figure 4.

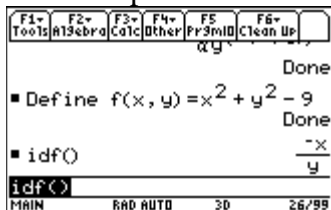


Fig. 4

Then define a new $f(x,y)$ and give the command **idf()** again. Better; can be better still.

Improvement #2: If you can live with the requirement of double quotes around the function $F(x,y)$, giving the command **Define idf(f) = -d(expr(f),x)/d(expr(f),y)** streamlines the process even further, as shown in figure 5.

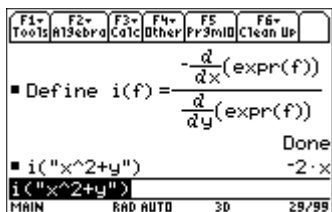


Fig. 5

[Please note that, in order to make the **Define ...** command fit onto the screen, the function was named “i”, not “idf”. Of course, you can name the function anything you want, including i, but two or more letters is recommended.]

Finding dy/dx for $x^2 + y^2 - 9 = 0$ and for another randomly-created horrible mess

[$x^2 + \sin(6y) = \ln(xy)$] would then look like the screen in figure 6, assuming that the name of the function is **idf**.

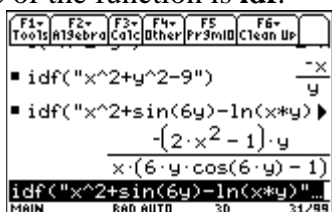


Fig. 6

Note the double quotes around each $F(x,y)$. A little nuisance but the best yet.

Final Improvement: Type in the short program in figure 7. Name it **idf**. To run it, give the home screen command **idf()** and type in the $F(x,y)$ (without double quotes) for which you need dy/dx (see figs. 8 and 9).

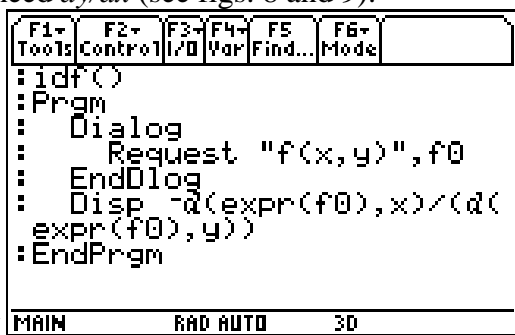


Fig. 7

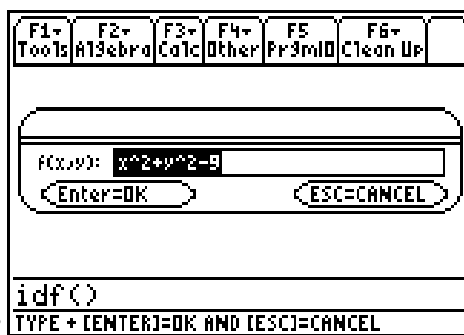


Fig. 8

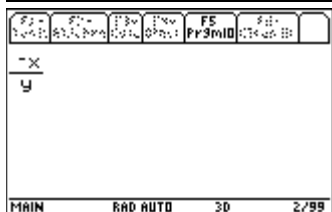
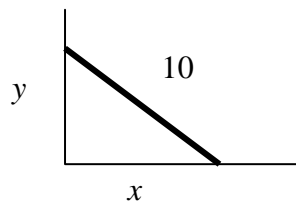


Fig. 9

Now for a problem that would be much harder (if not impossible) to do on the TI-89...

Example: A 10-foot ladder is leaning against a wall. A bug is on the top of the ladder, right up against the wall. If you pull the base of the ladder away from the wall at 1 ft/sec, how fast is the bug descending when the bottom of the ladder is 4 feet from the wall?

The following sketch applies:



We are given that $\frac{dx}{dt} = 1 \text{ ft/sec}$ and need to find $\frac{dy}{dt}$ when $x = 4$. Using the Pythagorean

Theorem and differentiating with respect to t gives:

$$x^2 + y^2 = 10$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$2y \frac{dy}{dt} = -2x \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{-2x \frac{dx}{dt}}{2y} = -\frac{x}{y} \frac{dx}{dt}$$

At the instant at which $x = 4$, $16 + y^2 = 100 \Rightarrow y^2 = 84 \Rightarrow y = \sqrt{84}$, so at the given instant,

$\frac{dy}{dt} = -\frac{4 \cdot 1}{\sqrt{84}}$, which is approximately .44 feet per second downward.

However and whenever you decide to use it, the **IDF** is just a tool. Be sure to compute “reasonable” implicit derivatives by hand, to get a complete understanding of the process. After all, you must do it “by hand” to find derivatives of implicit relations with respect to “third” variables, such as time, as shown in the example.

Exercises: Compute dy/dx for each relation below. Do the first and at least one more by hand. Check it via the **idf** function you defined earlier.

$$x \cdot y = 1$$

$$x^3 - 6x \cdot y + y^3 = 0$$

$$xe^{x \cdot y} = 1$$

$$\sin(x + y) = y^2 + \cos x$$

Further Exercises: Find all points where each implicit function above has a horizontal tangent. (Hint: Set $dy/dx = 0$ and solve for one of the variables. Substitute the result into

the given equation so you can then solve for the remaining variable, possibly exactly, possibly approximately.)

Even Further Exercises: Find software that will draw implicit relations (e.g., Derive™) and see if your answers to the “Further Exercises” look right.

Calculus Generic Scope and Sequence Topics: Derivatives, Applications of Derivatives
NCTM Standards: Number and operations, Algebra, Geometry, Measurement, Problem solving,
Connections, Communication, Representation