EXPLORATIONS

Activity 2

Magic Nines

Concepts/Skills:

Patterns, problem solving

Calculator:

TI-30 Xa SE or TI-34

Objectives:

Students compute multiples of 9, 99, 999, and so forth, search for patterns in the products, and write generalizations of those patterns.

Getting Students Involved

Remind students that mathematics is often thought of as the study of patterns. One of the important things that mathematicians do is to find generalizations of patterns.

Making Mathematical Connections

Start with a discussion of simple computational "tricks" such as "casting out nines."

♦ Do you know any tricks for knowing whether a product involving 9 is not correct?	*The sum of the digits must be divisible by 9. If not, the answer is incorrect.*

You may want to review the [K] key. (This key is available only on the TI-30Xa.)

➠ **Transparency Master K: *Repeat an Operation***

Carrying Out the Investigation

If students are having trouble finding patterns, let them work in pairs or small groups and talk to their partners about the products.

If students are having trouble writing generalizations, let them talk to their partners about the patterns they see in the products. Two generic questions you might use to prompt students' thinking for any of the problems are:

- What is changing in that pattern? What is remaining the same?
- How could you express that pattern in general terms or with a symbolic expression?

Encourage students to think of ways to reduce the number of keystrokes they have to make. For example, in problem 1, store **x 9** in memory on the TI-30Xa by using the [K] key; and in problem 2, compute 79 x 9 =711 and store **x 711** in memory by using the [K] key.

Making Sense of What Happened

Have students share the patterns they found and the generalizations they wrote. Pay special attention to the language that students use in stating generalizations.

Continuing the Investigation

- What patterns do you get when you divide a one-digit number (other than 9) by 9? *The single digit repeats after the decimal point.*
- Why is 9 divided by 9 different? *Because 9/9 = 1.*
- What patterns do you get when you divide a two-digit number by 9? Compare 25 ÷ 9 and 52 ÷ 9. *The digit repeated after the decimal point is the sum of the digits of the original number, provided that sum is a one-digit number. If that sum is a two-digit number, add the sum of the digits again and that number is the number repeated after the decimal point.*
- What patterns do you get when you divide a one-digit number by 99? *The decimal has two digits repeating: 0 and the given number.*
- What patterns do you get when you divide a two-digit number by 99? *The decimal has two digits repeating: the digits in the given number.*
- What patterns do you get when you divide a one-digit number by 999? When you divide a two-digit number? When you divide a three-digit number? *If you think of the given numbers as three-digit numbers (possibly with leading zeros, such as 7=007 or 24=024), the decimal has three digits repeating: the three digits in the given number.*

Solutions

1. **Products** / **Sum of Digits**

Products	Sum of Digits
9 x 1 = 09	0 + 9 = 9
9 x 2 = 18	1 + 8 = 9
9 x 3 = 27	2 + 7 = 9
9 x 4 = 36	3 + 6 = 9
9 x 5 = 45	4 + 5 = 9
9 x 6 = 54	5 + 4 = 9
9 x 7 = 63	6 + 3 = 9
9 x 8 = 72	7 + 2 = 9
9 x 9 = 81	8 + 1 = 9
9 x 10 = 90	9 + 0 = 9
9 x 11 = 99	9 + 9 = 18, but 1 + 8 = 9
9 x 12 = 108	1 + 0 + 8 = 9

2. The sum of the digits (or the sum of the digits of the sum of the digits) is 9.

3. 79 x 1 x 9 = 711

   79 x 2 x 9 = 1422

   79 x 3 x 9 = 2133

   79 x 4 x 9 = 2844

   79 x 5 x 9 = 3555

   79 x 7 x 9 = 4977

   79 x 9 x 9 = 6399

4. The ones and tens digits are each the middle factor in the indicated products; the thousands and hundreds digits form a two-digit number which is 7 times the middle factor in the indicated products.

5. Answers will vary.

6. 79 x 6 x 9 = 4266

   79 x 8 x 9 = 5688

7. The sum of the digits of each product is a multiple of 9, since 9 is a factor.

8. 24 x 99 = 2376

34 x 99 = 3366

74 x 99 = 7326

28 x 99 = 2772

38 x 99 = 3762

78 x 99 = 7722

9. Answers will vary.

10. 53 x 99 = 5247

68 x 99 = 6732

86 x 99 = 8514

11. Each product is 100 times the first factor minus the first factor. If A and B are digits, AB x 99 = AB x (100 - 1) = AB x 100 - AB = AB00 - AB

12. 67 x 99 = 6633

99 x 53 = 5247

94 x 99 = 9306

13. 24 x 999 = 23976

34 x 999 = 33966

54 x 999 = 53946

64 x 999 = 63936

51 x 999 = 50949

75 x 999 = 74925

14. Answers will vary.

15. Each product is 1000 times the first factor minus the first factor.

(Some students may recognize that the hundreds digit is 9 and the other four digits form the same product as when the first factor was multiplied by 99.)

16. 83 x 999 = 82,917

83 x 9999 = 829,917

83 x 99999 = 8,299,917

83 x 999999 = 82,999,917