The Birthday Problem

6698

Introduction

In this activity, students use simulations to answer a popular question involving probabilities.

Grades 6-8

NCTM Data Analysis and Probability Standards

- Understand and apply basic concepts of probability
- Understand and use appropriate terminology to describe complementary and mutually exclusive events

Files/Materials Needed

Birthday1.act, Birthday2.act, BIRTHDAY.73p

PART 1 CHECK STUDENT BIRTHDAYS

1

a. Launch TI-Navigator™ on the computer and start the session.
b. Have each student log into NavNet on their calculator.

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a. Load the **Birthday1.act** activity settings file into Activity Center. This sets up the Activity Center so that students have a list on their calculator.
b. Instruct students to enter and send the month and day of their birthday as a single 3- or 4-digit number. For example, a student whose birthday falls on November 23rd will enter '1123' and a student born on June 5th will enter '605'.
c. Stop the activity and sort the list in Activity Center by clicking on L1 just above the list.

d. Scroll through the list to see if any students share the same birthday. Explain to students that each person's birthday is independent of another person's birthday and therefore the data collected (each student's birthday) in this experiment is considered a collection of independent events.
e. You may also choose to create a histogram directly in Activity Center by clicking the **Frequency Plot** icon. Set the *Begin* value to 100, the *End* value to 1300, and the *Interval* value to 100 to see the distribution of birthdays by month.

PART 2 HOW MANY PEOPLE NEED TO BE IN A ROOM TOGETHER TO HAVE A BETTER THAN 50-50 CHANCE THAT AT LEAST TWO PEOPLE HAVE THE SAME BIRTHDAY?
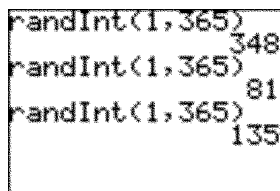
3

a. Ask the question posed above and use **Quick Poll** *Open Response* to collect student results. Most students will assume that a large number of people must be present so that at least two of them share the same birthday.
b. Instruct students to exit NavNet.

The Birthday Problem

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a. To ensure student's have different random number generators, have each student type a different number on the home screen (phone number, student ID, etc), press `STO▸` then `MATH`, arrow over to the **PRB** menu, and select **1: rand** and press `ENTER`.

b. Tell students to enter the random integer command **randInt(1,365)** on the home screen (this command can be found by pressing `MATH` `▸` `▸` **2:randInt**). It will return a random integer from 1 to 365 and therefore can be used to simulate each day of a year. The **randInt** command can be repeated by pressing `ENTER` over and over as shown below.

```
randInt(1,365)
              348
randInt(1,365)
               81
randInt(1,365)
              135
```

c. Students will need to record each number as they see it. Explain to students that they should continue pressing the `ENTER` key until they see any number show up twice. This represents two people with the same birthday.

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a. After students complete this activity, instruct them to log back into NavNet.

b. Load the **Birthday2.act** activity settings file into Activity Center and start the activity.

c. Tell each student to enter and send the number of times they needed to generate a random number before two of the same number occurred.

d. Stop the activity.

e. You may choose to create a frequency plot in Activity Center by clicking the **Frequency Plot** icon.

f. Next, click **Configure** and select the **Existing activity lists** check box. By starting the activity again, each student will receive the entire class data set.

g. Tell students to exit NavNet and press `2nd` `STAT` `▸` `▸` `▸` **1:1-Var Stats** `ENTER` on the home screen to calculate the mean of the data set contained in L1. This result will represent the average number of people that were needed to ensure that at least two people had the same birthday. Use **Screen Capture** to see if each student found this mean correctly. This result should be very close to 23 and very surprising to students!

Getting Started with the TI-Navigator™ System: Middle Grades

EXTENSION

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The concept of complementary events is used to find the theoretical probability of at least two people with the same birthday. Consider that the following two events comprise an entire sample space:

Event 1: "at least two people have the same birthday"
Event 2: "no one has the same birthday"

The probabilities of complementary events have a sum of one. Therefore,

P(at least two people have the same birthday)
= 1 – P(no one has the same birthday)

Depending on the ability level of the students, P(no one has the same birthday) can be calculated by evaluating

P(no one has the same birthday) =

probability that 3 people have different birthdays

$$\frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \dots \cdot \frac{(365 - n + 1)}{365}$$

probability that 4 people have different birthdays

probability that n people have different birthdays

where n represents the number of people.

Using the idea of complements,
P(at least two people have the same birthday)

$$= 1 - \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \dots \cdot \frac{(365 - n + 1)}{365}$$

$$= 1 - \frac{364 \cdot 363 \cdot 362 \cdot \dots \cdot (365 - n + 1)}{365^{n-1}}$$

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a. Have students evaluate this expression on their calculator for the value (rounded to the nearest whole number) found in part II.
b. Use **Quick Poll** to verify that the result is close to 50% (23 people comes out to 50.73%).

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a. If students find the above analysis too difficult, *force send* them the program **BIRTHDAY.73p**. By executing this program, students will be able to input the NO. IN GROUP. The output is the approximate percent chance that at least two people in the group share the same birthday.
b. Students can use the program to plot a graph of *Number in Group* versus *percent chance that at least two people have the same birthday*. This graph provides a nice visual reinforcement to the conclusions found in this activity.