EXPLORATIONS

Activity 9

Power Company

Concepts/Skills:

Powers, roots, problem solving

Calculator:

TI-30Xa SE or TI-34

Objectives:

Students will explore the limits on powers that can be displayed without using scientific notation. Students will look for patterns in the powers; for example, the exponent of the greatest power of 4 that is displayable is approximately half as large as the exponent of the greatest power of 2 that is displayable.

Getting Students Involved

Lead a brief discussion about powers.

- Which is greater, 2^{33} or 33^2? Why? — *2^{33}. There are more factors to multiply.*
- What is another way to write $(2^3)^4$? Explain why. — *2^{12}. When you take a power of a power, you multiply exponents.*
- What power of 4 is equal to the value of 2^6? How do you know? — *$4^3 = 2^6$, since $4^3 = (2^2)^3 = 2^{2x3} = 2^6$.*

Making Mathematical Connections

You may need to review the use of the [y^x] key to compute a power and the [$\sqrt[x]{y}$] key to compute a root.

➠ **Transparency Master H: Powers and Roots**

Carrying Out the Investigation

If students are having difficulty finding the greatest whole number whose square (cube, and so forth) can be displayed without using scientific notation, ask them to compute the square root (cube root, and so forth) of the greatest number displayable on the calculator without scientific notation. That root will not be a whole number, so you may need to help students see that they

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need to experiment with the whole number part of the root and the next greater whole number to find the answers.

- How can you use the square root of the greatest number displayable on the calculator to help you?

  The square of the integer part of that root will be the greatest perfect square displayable on the calculator. (Be careful that rounding doesn't make the root appear to be greater than it actually is.)

- What should you do with the decimal part of the root?

  Ignore it.

- Do you think the number you are trying to find is greater than or less than the square root? Why?

  The number will need to be less than the square root, since any greater number would have a square that is not displayable without using scientific notation.

Making Sense of What Happened

Use the transparency at the end of this activity (page ?) to lead a discussion about how students completed the table. Discuss questions **2**, **3**, and **4** after you have completed the table.

- Were questions **6** and **7** easier after you had answered question **5**? How were your solution strategies similar for these questions? How were your strategies different?

Ask students to explain the changes in the value of the product for the four combinations of base and exponent listed in questions **8**, **9**, **10**, and **11**. Let students use their own words, but then probe their explanations to help them use more precise language. For example, since 9 is 3^2, the greatest exponent for 9 will be about half the greatest exponent for 3.

Continuing the Investigation

Connect the problems here with graphs, perhaps using a graphing calculator or graphing software.

Graph each of the following and describe each graph:

2^x for x = 1, 2, 3, 4, 5, 6, and so forth.

$(0.5)^x$ for x = 1, 2, 3, 4, 5, 6, and so forth.

2^x for x = ⁻1, ⁻2, ⁻3, ⁻4, ⁻5, ⁻6, and so forth.

$(0.5)^x$ for x = ⁻1, ⁻2, ⁻3, ⁻4, ⁻5, ⁻6, and so forth.

Use the fact that $0.5 = 1/2 = 2^{-1}$ to explain the similarities and differences among the graphs.

If you believe your students are ready, look at patterns in which the exponents are negative numbers.

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Solutions

1.

Base	Greatest whole number exponent so that the product is displayed without scientific notation	Product
2	33	8,589,934,592
3	20	3,486,784,401
4	16	4,294,967,296
5	14	6,103,515,625
6	12	2,176,782,336
7	11	1,977,326,743
8	11	8,589,934,592
9	10	3,486,784,401
10	9	1,000,000,000
25	7	6,103,515,625
100	4	100,000,000

2. The exponent for 4 is the greatest whole number less than or equal to one-half of the exponent for 2, because 4 is the square of 2.

3. The exponent for 25 is the greatest whole number less than or equal to one-half of the exponent for 5, because 25 is the square of 5.

4. 3 and 9, 10 and 100.

5. 99,999. To find it, take the integer part of the square root of the greatest number displayable on the calculator (but be careful about rounding).

6. Trial and error will also allow you to find the number.

7. 2,154, with $2154^3 = 9{,}993{,}948{,}264$.

8. Answers will vary. Trial and error will allow you to find the number.

9. 9, with $9^{10} = 3{,}486{,}784{,}401$.

10. Answers will vary. Trial and error will allow you to find the number.

11. The product is always 1.

12. The product decreases; for example, $\frac{1}{4}^{2} = 0.0625$, $\frac{1}{4}^{3} = 0.015625$, $\frac{1}{4}^{4} = 0.00390625$.

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