

## NUMB3RS Activity: Breaking the Code Episode: "Sabotage"

**Topic:** Cryptography

**Grade Level:** 9

**Objective:** In this activity, students will break a code with the aid of linear equations.

**Time:** 25 - 30 minutes

**Materials:** There two versions of this activity; the first version, described on the Teacher pages, uses the TI-Navigator™ system. The second, described on the Student pages does not use the TI-Navigator system.

**For the TI-Navigator activity:** TI-Navigator system (version 3.0), TI-83 Plus/TI-84 Plus graphing calculators, and the following files: *CodedWords.pdf*, *OriginalToCoded.pdf*, *LettersToNumbers.pdf* and *BreakingtheCode.act*. To download these files, go to <http://education.ti.com/exchange> and search for "7792."

**For the activity without TI-Navigator:** TI-83 Plus/TI-84 Plus graphing calculators

### Introduction

In "Sabotage," a series of train wrecks has the FBI puzzled. Until a note is discovered at the scene of the latest train wreck, the FBI has no leads. When they see that the note contains a code written in a series of numbers, they immediately call Charlie. Charlie is able to break the code in the letter, which helps Don and the FBI find the person who caused the train wrecks.

### Notes to the Teacher

This activity is not intended to model the actual process that Charlie goes through in breaking the code in the episode. In the episode, Charlie transforms a sequence of numbers into letters which reveals the message. In this activity, students are presented with a list of words that have been coded into other "words," where one cannot easily discern the coding pattern by just looking at how one word is coded.

This activity leads students to translate the letters in the original word and coded word into sequences of numbers. By looking at a scatter plot, it is anticipated that the students will discern the linear patterns. With the TI-Navigator version of the activity, the whole class quickly sees the results from all of the students. With the other version of the activity, students will need to "plot" 3 or 4 pairs of words before they are able to identify the pattern.

In both versions, students will look for patterns in the plot and attempt to come up with algebraic equations to model the patterns. These equations, once found, can be used to code and decode words.

Note that the code in this activity is an example of a *shift cipher*. The letters A–Z are assigned values 0–25, and then these values are shifted. For each plaintext value  $x$ , the coded value  $y$  is given by  $y \equiv x + k \pmod{26}$ , where  $k$  is a fixed integer.

### **TI-Navigator Activity Instructions**

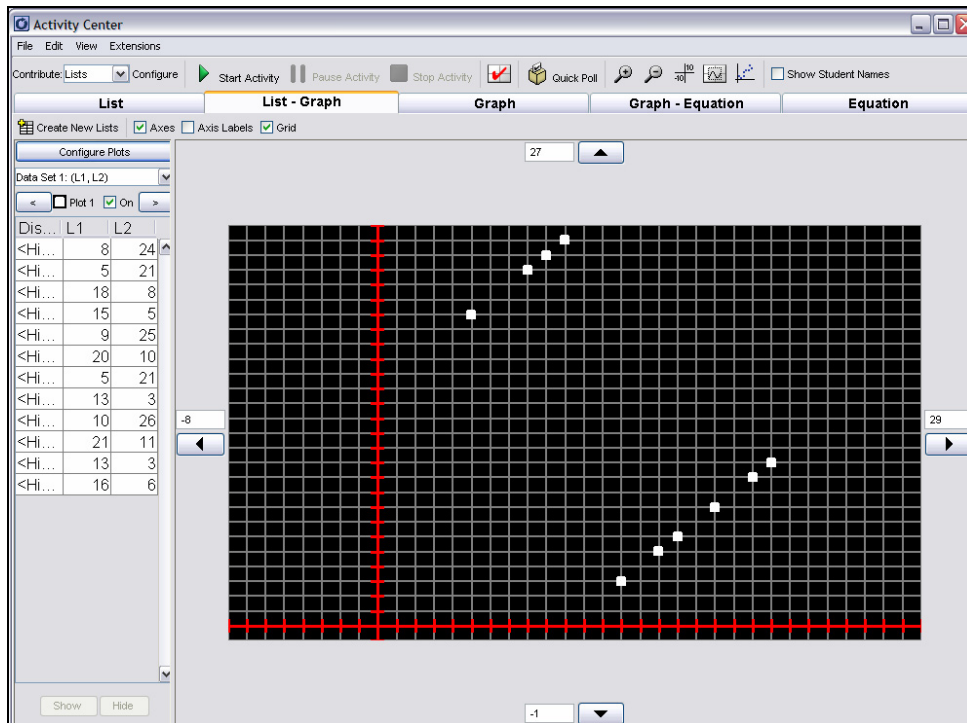
**Description of files:** Each student will need a copy of *Letters2Numbers.pdf* (a list of the alphabet and the number associated with each letter) and *OriginalToCoded.pdf* (a listing of the alphabet with a column for the code for each letter). Each class needs one copy of *CodedWords.pdf* (a list of 26 pairs of original and coded words). The activity settings file *BreakingtheCode.act* configures Activity Center for this activity, and should be downloaded to the teacher's computer.

1.
  - a. Launch TI-Navigator on the computer and press **Begin Class** to start the session.
  - b. Have each student log into NavNet on their calculators and enter the Activity Center.
  
2.
  - a. Give each student one of the pairs of words (original and corresponding coded word) from the *Coded Words* file. For example, one student might get the word HERO and its code, XUHE.
  - b. Have students use the *Letters To Numbers* file to find the numbers that correspond to each of the letters. Have them do this for each of letters in the original and coded words. Thus, if the original word is "HERO" and the coded word is "XUHE" then the numbers for the original word are 8 (for H), 5 (for E), 18 (for R), and 15 (for O) and the numbers for the coded word are 24 (for X), 21 (for U), 8 (for H), and 5 (for E)
  
3.
  - a. Load the *BreakingtheCode.act* activity settings file into Activity Center. The sequence of numbers for the original and coded word will be stored in lists  $L_1$  and  $L_2$ , respectively. This activity produces a scatter plot using the values in list  $L_1$  as the x-values, and the values in list  $L_2$  as the y-values.
  - b. Press **Start Activity**. This will allow students to submit a form. As shown in the screenshot below, the form asks for the Original Input and the Coded Output. For example, the word "HERO" codes to "XUHE," so in Page 1 of the form enter 8 as the original input and 24 for the coded output. Then select **NEXT**. When students complete Page 4 of the form, have them select **SEND**. The scatter plot will then be displayed in Activity Center

PAGE 1 OF 4	
ORIGINAL INPUT:	8
CODED OUTPUT:	24
= 8	
NEXT	

**Note:** If the scatter plot does not appear automatically, click the 'List-Graph' tab and select "Configure Plots." on the top left of the Activity Center's screen. In the menu change X-list to  $L_1$  and Y-list to  $L_2$ . Then click OK.

- c. Press **Stop Activity**. The example below shows sample results from three students. Only ten points are shown because there was some duplication in the letters of the students' words.



4. After all students have sent in their forms, discuss the results of the graph. Ask students to describe the patterns on the graph. Are these patterns linear? Linear functions have the property that the slope between the points is constant. What do students observe about the graph of the points that might help them identify a rule? Note that some students may have made errors in coding their words, so there could be a few points that do not fit the patterns.
5. Ask students to write algebraic expressions for the patterns they see. Ask them how they can use these equations help break the code.
6.
  - a. Have students match each original letter to a coded letter. Have them fill in the table in the *Original To Coded* file to help them organize their data.
  - b. To check for understanding after the students have found the pattern, have them switch their coded word with someone else in the class and ask them to decode the other student's word.
  - c. Have students create a secret message for a classmate and write it in this same code. Have the classmate break the code and check with the author to see if they have decoded it correctly.

**Student Page Answers:**

- 1.** For HERO the numbers are 8, 15, 18, 15 **2.** For XUHE the numbers are 24, 21, 8, 5  
**3.** Answers will vary. **6.** Students could see a number of different patterns in the graph of four points. **7.** Eventually, students should that the points fit two parallel line segments with positive slopes. **8.**  $y = x + 16$  and  $y = x - 10$  are the equations of the lines. **10.** The domain of the first equation is the set of integers from 1 to 10, its range is the set of integers from 17 to 26. The domain of the second equation is the set of integers from 11 to 26, its range is the set of integers from 1 to 16. The domain and range are the original and coded numbers for each letter. The domain determines which equation you would use to code a specific letter.  
**11.**  $x = y - 16, y \in \{17...26\}; x = y + 10, y \in \{1...16\}$  **12.** YOU D IDAG REAT JOB

Name: \_\_\_\_\_ Date: \_\_\_\_\_

## NUMB3RS Activity: Breaking the Code

In "Sabotage," a series of train wrecks has the FBI puzzled. Until a note is discovered at the scene of the latest train wreck, the FBI has no leads. When they see that the note contains a code written in a series of numbers, they immediately call Charlie. Charlie is able to break the code in the letter, which helps Don and the FBI find the person who caused the train wrecks.

The process of breaking a code involves looking for patterns. This method usually involves trial and error; the code breaker tries possible patterns to determine whether they are consistent with the rest of the code. Calculators and computers have made this process much more efficient.

Some codes are based on assigning a number 1–26 to each letter A–Z. A common way of doing this is shown in the chart below. For example, the sequence of numbers for the word BENT is 2, 5, 14, 20.

<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>	<b>F</b>	<b>G</b>	<b>H</b>	<b>I</b>	<b>J</b>	<b>K</b>	<b>L</b>	<b>M</b>
1	2	3	4	5	6	7	8	9	10	11	12	13
<b>N</b>	<b>O</b>	<b>P</b>	<b>Q</b>	<b>R</b>	<b>S</b>	<b>T</b>	<b>U</b>	<b>V</b>	<b>W</b>	<b>X</b>	<b>Y</b>	<b>Z</b>
14	15	16	17	18	19	20	21	22	23	24	25	26

Now, consider a code where the word BENT was coded to RUDJ. This list shows how other words were coded by the same system:

Original Word	Coded Word	Original Numbers	Coded Numbers
HERO	XUHE		
ITEM	YJUC		
JUMP	ZKCF		
KITE	AYJU		
LAMB	BQCR		
MOVE	CELU		

1. Pick one of the original words and convert the letters in the word to a sequence of numbers according to the chart above. Enter your numbers in the third column of the chart.
  
2. Convert the corresponding coded word to a sequence of numbers using the chart and enter those numbers in the fourth column. ( Example: The sequence of numbers for the "word" RUDJ is 18, 21, 4, 10.)
  
3. Do you see any patterns with the code for your word?

4. In your calculator, enter the sequences of numbers for your original word and the coded word in lists  $L_1$  and  $L_2$  respectively.

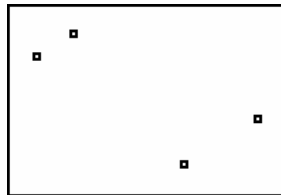
Example:

L1	L2	L3	2
2	18	-----	
5	21		
14	4		
20	10		
-----			
L2(5) =			

5. Construct a scatter plot of the data in lists  $L_1$  and  $L_2$ .

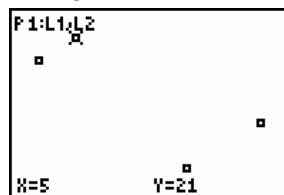
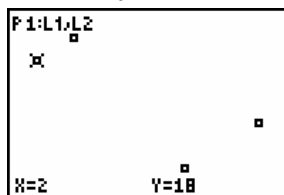
Example:

2nd	Plot2	Plot3
On	Off	
Type:		
Xlist:	L1	
Ylist:	L2	
Mark:		+



6. Do you observe any patterns in your scatter plot? Explain.
7. Choose another one of the original words from the table. Convert it and its coded word to sequences of numbers and add those numbers to the end of the lists in your calculator so that you now have eight entries in both  $L_1$  and  $L_2$ . Re-draw the scatter plot. If necessary, repeat this step with additional words until you can see a pattern in your scatter plot.
8. Now, find two equations for the functions that fit the data in the scatter plot. What can you observe about the points that would indicate the type of function? In your equations, let  $x$  represent the number for the letter in the original word and let  $y$  represent the number for the letter in the coded word. Graph your equations to check that they go through the points.

**Note:** As shown below, you can use the trace function to find the coordinates of the points if you do not want to go back to the list menu.



9. Suppose the word XRAY was coded to NHQO. Convert these words to sequences of numbers and add them to lists  $L_1$  and  $L_2$ . Check to see whether your graphs are consistent with this new information.
  
10. Find the domain and range of each equation you found in question 8. Recall how the variables  $x$  and  $y$  were defined. What do the domain and range of the equations mean?
  
11. The equations found in question 8 can be used to decode words. Solve your equations for  $x$  in terms of  $y$  and identify which values of  $y$  should be used in each.
  
12. Use your results from question 11 to decipher the following code:  
OEKT YTQW HUQJ ZER

*The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.*

## Extensions

### **For the Student**

Repeat the activity using the file *CodedWordsPart2.pdf*. This can be done using either version of the activity, with or without the TI-Navigator™ system.

### **Related Topic**

Another common method for constructing a code is the multiplicative cipher. In this method, each letter is coded by taking the value assigned to a letter and multiplying it by a fixed number. If the result is greater than 26, divide the result by 26 and use the remainder as the coded value.

For example, suppose the fixed number is 7 and you want to code the letter P. Multiply the value for P, 16, by 7:  $16 \times 7 = 112$ . Because  $112 \div 26 = 4 \text{ R}8$ , the coded value is 8, which corresponds to the letter H.

Use this method to code three or four words. Use these words and their codes in the activity and look for patterns in the graph.

### **Additional Resources**

- For an additional activity for the NUMB3RS episode "Sabotage," go to <http://education.ti.com/exchange> and search for "7749."
- For more information about TI-Navigator, go to <http://education.ti.com/navigator>.