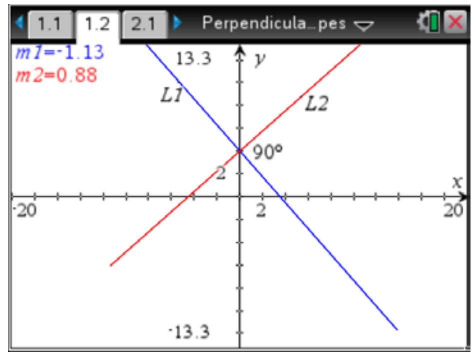
Perpendicular Slopes

PerpendicularSlopes.tns

Name ______________________

Class ______________________

Problem 1 – An Initial Investigation



On page 1.2 of the TI-Nspire document, two lines are displayed: line *L1* with a slope of *m1* and line *L2* with a slope of *m2*.

Notice that the angle formed by the intersection of the lines measures 90°; that is, the two lines are perpendicular.

Grab line *L1* and rotate it. Observe that as the slopes of the lines change, the two lines remain perpendicular. Explore the relationship between the slopes by answering the questions below.

1. Can you rotate *L1* in such a way that *m1* and *m2* are both positive? Both negative?

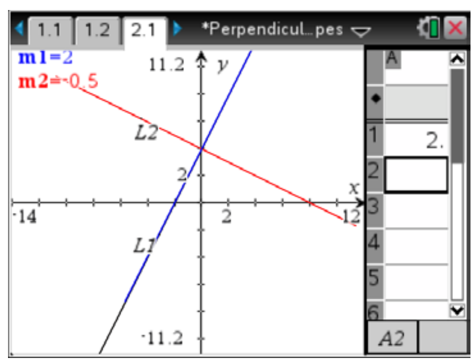
2. Can you rotate *L1* so that *m1* or *m2* equals 0? If so, what is the other slope?

3. Can you rotate *L1* so that *m1* or *m2* equals 1? If so, what is the other slope?

4. Rotate *L1* so that *m1* is a negative number close to zero. What can be said about *m2*?

5. Rotate *L1* so that *m1* is a positive number close to zero. What can be said about *m2*?

Problem 2 – A Closer Examination



Now that you have observed some of the general relationships between the slopes of two perpendicular lines, it is time to make a closer examination.

On page 2.1, you will find a split screen. On the left are perpendicular lines *L1* and *L2* (with slopes *m1* and *m2*, respectively). On the right is a spreadsheet. Change the slope of *L1* by changing the value in cell A1 of the spreadsheet. For each value that you enter, *m1* and its corresponding value of *m2* are recorded in the spreadsheet on page 2.2. Use the graph on page 2.1 to answer question 6–9.

6. Enter **0** into cell A1 to make the slope of *L1* equal to 0. What is the slope of *L2*?

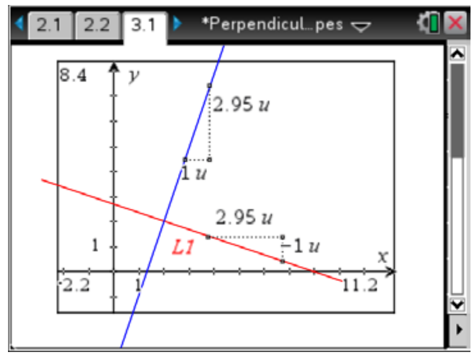
7. What is the slope of *L2* when the slope of *L1* is 1?

8. What is the slope of *L2* when the slope of *L1* is –1?

9. Enter other values for the slope of *L1* and examine the corresponding slope of *L2*. (You can reference a history of your "captured" values on page 2.2.). Conjecture a formula that relates the slope of two perpendicular lines. Enter your formula in the formula cell of Column C (with variables ***slope1*** and ***slope2***) to test your conjecture.

Problem 3 – A Geometric Look

Page 3.1 shows another way to examine the slopes of perpendicular lines, geometrically. Grab line *L1*, rotate it, and compare the rise/run triangles.



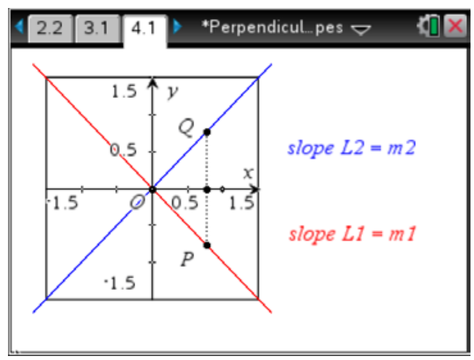
10. What do you notice about the two triangles?

Problem 4 – The Analytic Proof

Now, analytically verify that two lines with slopes *m1* and *m2* are perpendicular if and only if $m1 \cdot m2 = -1$.

(All of the following assumes *m1* ≠ 0. What can be said about the case when *m1* = 0?)

Given two perpendicular lines *L1* and *L2* with slopes *m1* and *m2* respectively, first translate them such that their point of intersection is at the origin. Refer to the static diagram on page 4.1.



11. What are the equations of these translated lines as shown in the diagram?

12. Let *P* be the point of intersection of line *L1* and the vertical line *x* = 1 and let *Q* be the point of intersection of line *L2* and the line *x* = 1. What are the coordinates of points *P* and *Q*?

13. Use the distance formula to compute the lengths of $\overline{OP}$, $\overline{OQ}$, and $\overline{PQ}$. (Your answers should again be in terms of *m1* and *m2*.)

14. Apply the Pythagorean Theorem to triangle *POQ* and simplify. Does this match your conjecture from Problem 2?