Perpendicular and Angle Bisectors

Mathematics Objectives

- Students will investigate and define *perpendicular bisector* and *angle bisector.*
- Students will discover and describe the property that any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment it bisects.
- Students will discover and describe the property that any point on the bisector of an angle is equidistant from the sides of the angle it bisects.

Applications and Skills

Geometry
Measuring angles
Measuring length
Dragging points and rays

Lists and Spreadsheet
Manual data capture
Entering formulas

Materials

- TI-Nspire handheld
- TNS file: lesson03.tns
- *Exploring Perpendicular Bisectors* (pages 58–59; page058.pdf)
- *Exploring Angle Bisectors* (pages 60–61; page060.pdf)
- *Proving Angle Bisectors* (page 62; page062.pdf)

Starting the Lesson

After loading the TNS file (lesson03.tns) on each handheld, begin the exercise by instructing students to do the following:

1. Turn on the TI-Nspire by pressing (on).
2. Press (on) and choose **My Documents**.
3. In the folder *Geometry TCM*, choose *lesson03*.
4. Remind the students how to navigate through the TNS file. To move forward through the pages, press (ctrl) ▶. To move backward through the pages, press (ctrl) ◀. To choose a particular page, press (ctrl) ▲, position the cursor on the desired page, and press (enter). To undo previous steps, press (ctrl) Z or (ctrl) (esc). Show students that any time they are using a menu that they wish to exit, they should press (esc).

Perpendicular and Angle Bisectors *(cont.)*

Note: *Page numbers refer to the TI-Nspire file lesson03.*

Explaining the Concept

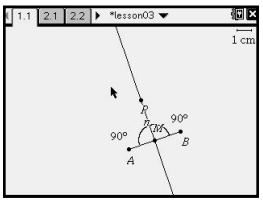
Problem 1—Exploring Perpendicular Bisectors

Step 1 Explain to students that this problem will lead them to investigate the definitions and properties of perpendicular bisectors of segments. Then distribute copies of *Exploring Perpendicular Bisectors* (pages 58–59) to students and have them open to page 1.1.

Step 2 Tell students that the sketch shows $\overline{AB}$ and its perpendicular bisector *n*. Point P is on perpendicular bisector *n*. Point M is the midpoint of $\overline{AB}$.

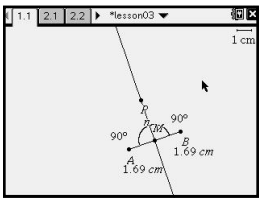
Step 3 Instruct students to measure ∠AMP and ∠BMP. Then, have them complete question 1 on their activity sheets. (See page 199 for a more detailed explanation of measuring angles.)

Step 3



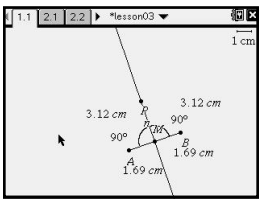
Step 4 Now, have students measure the lengths of $\overline{AM}$ and $\overline{MB}$, and complete question 2 on their activity sheets. (See page 197 for a more detailed explanation of measuring length.)

Step 4



Step 5 Next, students must measure the distance between points A and P, and the distance between points P and B. Tell students that this process is very similar to measuring the distance of a segment. To do this, students must press (menu), select **Measurement**, and then select **Length**. Have students click on the first point and then on the second point. When the "ghost" measurement appears, instruct students to move it to a place that makes sense to them, and press [ ] to release the measurement. Now, have students complete question 3 on their activity sheets.

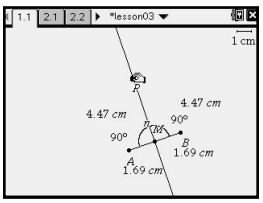
Step 5



Teacher Note: The task does not ask students to make segments for AP and BP, but it is acceptable to do so to measure the length of these segments rather than the distance between the points.

Step 6 Tell students that for the next two questions, they will have to drag point P along perpendicular bisector *n*. (See page 194 for a more detailed explanation of dragging a point.) Remind students to move the point slowly so that they can observe how distances AP and BP change, and have them answer question 4 on their activity sheets.

Step 6



Step 7 Lead a discussion about the meaning of the terms *perpendicular*, *bisect*, and *perpendicular bisector of a segment*. From the discussion, decide on class definitions for these terms, and have students record them on their activity sheets.

Perpendicular and Angle Bisectors *(cont.)*

Note: *Page numbers refer to the TI-Nspire file lesson03.*

Explaining the Concept *(cont.)*

Problem 1—Exploring Perpendicular Bisectors *(cont.)*

Step 8 Write the following statements on the board or overhead. The Perpendicular Bisector Theorem states that any point on the _____ of a segment is _____ from both endpoints of the segment. So, point P is the same distance from A as from B anywhere along perpendicular bisector *n*.

Step 9 Allow students to complete the statements by using their knowledge from this problem. *(perpendicular bisector; equidistant)*

Step 10 Instruct students to complete the rest of the activity sheet in pairs.

Problem 2—Exploring Angle Bisectors

Step 1 Distribute copies of *Exploring Angle Bisectors* (pages 60–61) to the students, and have them press (ctrl) ▶ to tab to page 2.1.

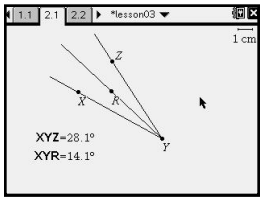
Step 2 As a class, examine the sketch and locate ∠XYZ and $\overrightarrow{YR}$.

Step 3 Explain to the students that ∠XYZ has been partitioned by $\overrightarrow{YR}$. The measures of ∠XYZ and ∠XYR have been given and have been defined as variables.

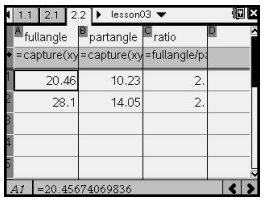
Step 4 Now, have students press (ctrl) ▶ to tab to the spreadsheet on page 2.2. Explain to students that it is set up to capture the measures of ∠XYZ and ∠XYR and to calculate the ratio between them. Discuss each column heading so that students will understand how to interpret the data. Column A will collect measures of ∠XYZ; column B will collect measures of ∠XYR; column C will calculate the ratio $\frac{\angle XYZ}{\angle XYR}$.

Step 5 Have students press (ctrl) ◀ to tab back to page 2.1. Then, instruct students to grab $\overrightarrow{XY}$ (the ray itself, not a point) and drag it to make ∠XYZ larger and then smaller. Tell students that as the ray moves, the spreadsheet will become populated with the simultaneous measures of the two angles. Make sure that students press (esc) to release the ray. Then, have students press (ctrl) ▶ to tab to page 2.2 and examine the information collected.

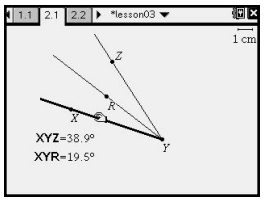
Step 2



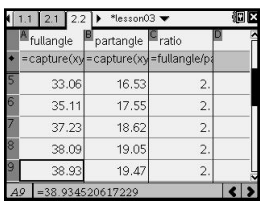
Step 4



Step 5



Step 5



Perpendicular and Angle Bisectors *(cont.)*

Note: *Page numbers refer to the TI-Nspire file lesson03.*

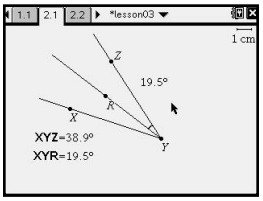
Explaining the Concept *(cont.)*

Problem 2—Exploring Angle Bisectors *(cont.)*

Step 6 Allow students to answer question 2 on their activity sheets.

Step 7 Now, have students press ctrl ◀ to tab back to page 2.1. Instruct students to measure ∠RYZ and then complete questions 3 and 4 on their activity sheets. (See page 199 for more detailed directions on measuring angles.)

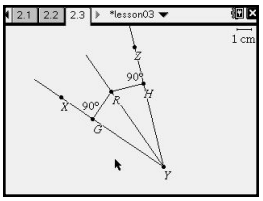
Step 7



Step 8 Now, have students press ctrl ▶ two times to tab to page 2.3. Explain to them that this page shows the same figure that was given in the sketch on page 2.1. In addition, two segments, $\overline{RG}$ and $\overline{RH}$, have been added to show the shortest distance from point R to $\overleftrightarrow{YX}$ and $\overleftrightarrow{YZ}$. The angles formed where the segments intersect the rays are also given.

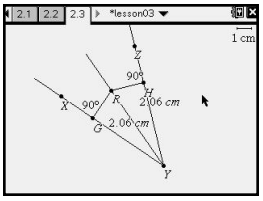
Teacher Note: This sketch shows ∠XYZ bisected by $\overleftrightarrow{YR}$. Behind the scenes, a line has been constructed through point R perpendicular to $\overleftrightarrow{YX}$ to help form $\overline{RG}$. Similarly, $\overline{RH}$ has been formed to be perpendicular to $\overleftrightarrow{YZ}$. These perpendicular distances have been added to show the shortest distance from point R to $\overleftrightarrow{YX}$ and $\overleftrightarrow{YZ}$. The right angles formed where the segments intersect the rays have been measured.

Step 9



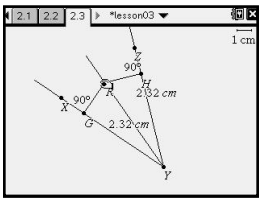
Step 9 Tell students to drag point R and observe what happens to the angle formed by $\overline{RG}$ and $\overleftrightarrow{YX}$. Ask students what they observe about the angle. Then, have them record their observations in question 5 on their activity sheets.

Step 10



Step 10 Next, have students measure the lengths of $\overline{RG}$ and $\overline{RH}$. (See page 198 for more detailed directions on measuring length.) Then, ask them what they observe about the lengths of the two segments. Students should also record their observations in question 6 on their activity sheets.

Step 11



Step 11 Have students slowly drag point R and observe how the lengths of $\overline{RG}$ and $\overline{RH}$ change.

Step 12 Have students work in pairs to complete the rest of the activity sheet. Once students are finished, discuss the answers as a class.

Note: *Page numbers refer to the TI-Nspire file lesson03.*

Applying the Concept

Problem 3—Proving Angle Bisectors

Step 1 Tell students that in order to be a theorem, the Angle Bisector Theorem needs to be proven using definitions, postulates, and theorems that have already been proven.

Step 2 Write the following information on the board or overhead.

- **Given:** any point on the bisector of an angle
- **Show:** the point is equidistant from the sides of the angle
- **Hint:** The proof depends on Side-Angle-Angle (SAA) congruence.

Step 3 Distribute copies of *Proving Angle Bisectors* (page 62) to the students. Instruct them to plan and write the proof of this theorem in any form (paragraph, two-column, or flowchart).

Differentiation

- **Below Grade Level**—Allow students to attempt the proofs with partners or in small groups. Use patty-paper constructions and ruler measurements as concrete models for students.
  Note: *You can find patty paper online or from the butcher at your local grocery store.*
- **Above Grade Level**—Instruct students to complete the proofs in pairs. In groups of four, have students compare their proofs and discuss any differences they find. Allow students to revise their proofs, if necessary. If time allows, invite two or three pairs to share their proofs with the class.

Extending the Concept

- Explore the converse of the Perpendicular Bisector Theorem, and determine if it is true.
- Explore the converse of the Angle Bisector Theorem, and determine if it is true.