

According to the Standards:

**Instructional programs from preK-grade 12 should enable students to:**

- Recognize and use connections among mathematical ideas
- Use the language of mathematics to express mathematical ideas precisely
- Select, apply and translate among mathematical representations to solve problems

**In grades 9-12 students should**

1. Students should develop an increased capacity to link mathematical ideas and a deeper understanding of how more than one approach to the same problem can lead to equivalent results.

**Calculus Scope and Sequence:** Applications of Derivatives

**Keywords:** optimization, maximum, minimum, applications

**Description:** This activity will illustrate the idea of how the derivative is used to find a solution to an projectile motion problem

*A ball is thrown vertically upwards from the ground with an initial velocity of 40 feet per second. Find the instantaneous velocity one second after the ball leaves the ground, and the average velocity over the first 2 seconds and the maximum height the ball reaches.*

- Set up the height function
- Find the slope of the secant line over the first two seconds
- Find the derivative of the height function and evaluate it at 1 second
- Find the critical points on the derivative, check for max, min using the second derivative
- Create and draw the secant line to show the average velocity.

The Derivative is found from the Homescreen in F3-Calc-#1

Syntax:  $d(\text{function}, \text{variable})$

The Solve function is found from the Homescreen in F2-Algebra-#1

Syntax:  $\text{solve}(\text{expression} = \text{expression}, \text{variable})$

The Numerical Derivative (Nder) is found from the Homescreen in F3-Calc-#A

Syntax:  $\text{Nder}(\text{Function}, \text{variable}, \text{value})$

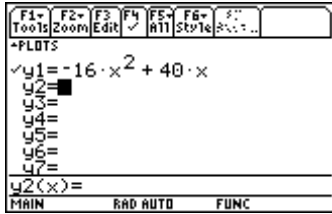
**User Tips:**

1. You can store the function in the Y= screen to make it easier to use
2. You can copy and paste a result from the homescreen by using the Up Arrow to highlight it and then pressing ENTER to paste it into the edit line. (You can also use the copy & paste functions in the F1-Tools menu)

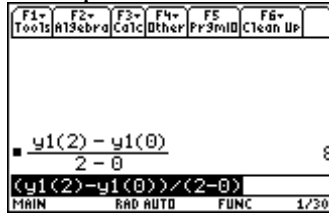
The projectile motion equation (in ft/sec) is:  $h(t) = -16t^2 + \text{initialvelocity}(t) + \text{initialheight}$

So in this problem we'll use:  $y1(x) = -16x^2 + 40x$

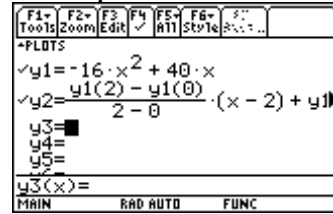
### The function



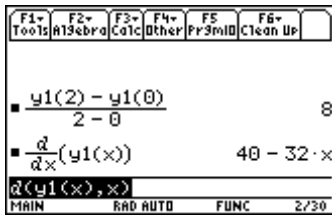
### Slope of Secant



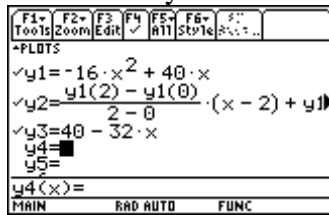
### Equation of Secant



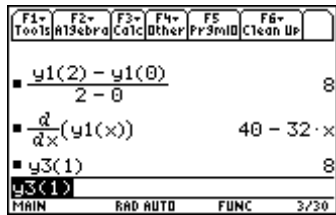
### The derivative



### Stored in y3



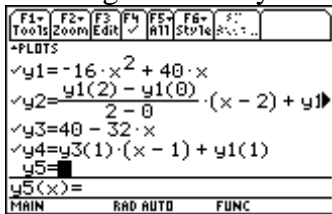
### Evaluated at x = 1



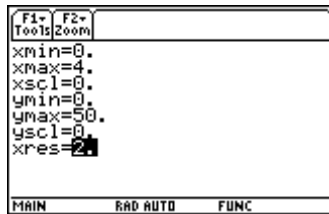
From these results we can see that at the one second mark, the instantaneous velocity was exactly equal to the average velocity over the first two seconds.

We can show that graphically by creating the tangent line at  $x = 1$  and showing it is parallel to the secant line over the first two seconds.

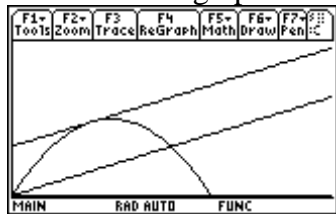
### The tangent line in y4



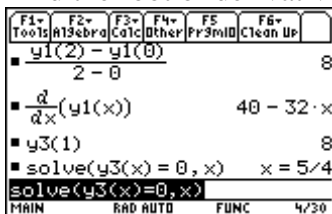
### A reasonable Window



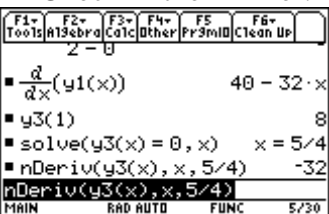
### All three graphs



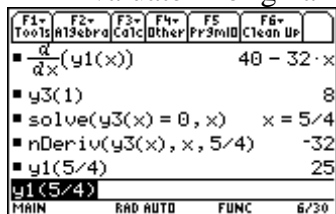
### Find the root of derivative



### Check in the 2<sup>nd</sup> Der.



### Evaluate in original



In Summary:

The average velocity over the first two seconds is 8 ft/sec

The velocity at exactly one second is also 8 ft/sec

The maximum height the ball reaches is 25 ft.