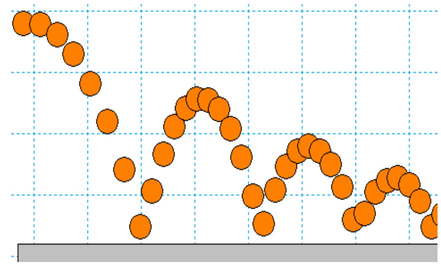
Exploring Geometric Sequences

Student Activity

Name ________________

Class ________________

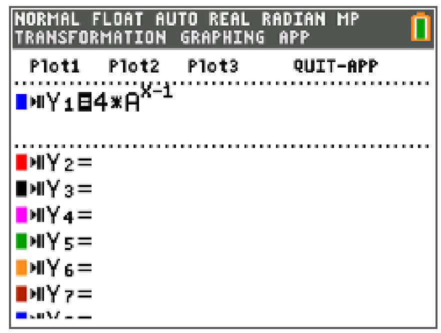
Example of Geometric Sequence



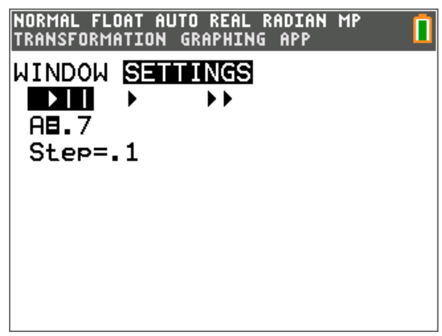
The height that a ball rebounds to after repeated bounces is an example of a geometric sequence. The top of the ball appears to be about 4.0, 2.8, 2.0, and 1.4 units. If the ratios of consecutive terms of a sequence are the same, then it is a geometric sequence. The common ratio *r* for these values is about 0.7.

Problem 1 – Changing the Common Ratio

Explore what happens when the common ratio changes.



Start the **Transfrm App**. Press [y=] and for **Y1**, enter **4∗A^(X–1)**.
Change your settings by pressing [window] and arrow right to go to SETTINGS. Set **A = 0.7** and **Step = 0.1**.
Graph the function by pressing [zoom] and selecting **ZStandard**.
Change the value of the common ratio (**A**).



1. Why do you think the *r*-value is called the common ratio?

2. What did you observe happens when you change the common ratio from positive to negative? Explain why this happens.

3. What would happen if you added all the terms of this sequence? For what common ratio conditions do you think the sum will diverge, (get larger, and not converge to some number)?

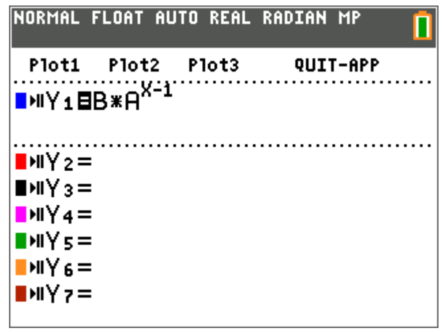
4. When the common ratio is larger than 1, explain what happens to the graph and values of *y*.

5. What *r*-values could model the heights of a ball bounce? Explain.

Problem 2 – Changing the Initial Value and the Common Ratio

Press [y=] and change **Y_1** to **B∗A^(X–1)**.

Change the [window] SETTINGS so that **A=0.7, B=4** and **Step=0.1**.



6. Explain your observations of what happens when **B** changes. What is **B** also known as?

Extension – Deriving and Applying the Partial Sum Formula

The sum of a finite geometric series can be useful for calculating funds in your bank account, the depreciation of a car, or the population growth of a city.

e.g. $S_6 = 4 + 8 + 16 + 32 + 64 + 128$

In this example, the common ratio is 2, the first term is 4, and there are 6 terms.

The general formula:

$$\boldsymbol{S_n = a_1 + a_2 + a_3 + \ldots + a_{n-1} + a_n}$$

Because $a_n = r \cdot a_{n-1}$, substituting gives

$$S_n = a_1 + r \cdot a_1 + r^2 \cdot a_1 + r^3 \cdot a_1 + \ldots + r^{n-2} \cdot a_1 + r^{n-1} \cdot a_1$$
$$r \cdot S_n = r \cdot a_1 + r^2 \cdot a_1 + r^3 \cdot a_1 + \ldots + r^{n-1} \cdot a_1 + r^n \cdot a_1$$

Subtract the previous two lines.

$$S_n - r \cdot S_n = a_1 - r^n \cdot a_1$$
$$S_n(1-r) = a_1(1-r^n)$$

So, $S_n = a_1 \cdot \dfrac{1-r^n}{1-r}$

Use the formula to find the sum of the following finite geometric series.

7. Find S_5 for $a_n = 6\left(\frac{1}{3}\right)^{n-1}$.

8. $\frac{1}{7}+\frac{1}{7^2}+\frac{1}{7^3}+\frac{1}{7^4}+\frac{1}{7^5}+\frac{1}{7^6}=$

9. Find S_{25} for $a_n = 2(1.01)^{n-1}$.

10. $64-32+16-8+4-2+1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}-\frac{1}{32}+\frac{1}{64}-\frac{1}{128}+\frac{1}{256}=$