Parachuting

by Dave Slomer

A parachutist falls from a height of 1000 feet with instantaneous velocity v (measured in ft/sec) defined by $v(t) = 30\dfrac{1-e^{0.6t}}{1+e^{0.6t}}$ ***where t is measured in seconds after she jumps.***

On your TI-89, make **y1=30(1-e^(.6x))/(1+e^(.6x))**, since it only graphs functions of **x**. Keep in mind for the rest of the activity that **y1** <u>is</u> the velocity function defined above.

(a) Graph the velocity function and explain why the shape of the graph is what you'd expect.

(b) The velocity graph has a horizontal asymptote. The asymptote represents *terminal velocity*. Find its equation and discuss its significance for our parachutist.

(c) Is her velocity increasing or decreasing as she falls? Explain. Keep the graph in mind.

(d) *Speed* is usually defined to be the absolute value of velocity. Is her speed increasing or decreasing as she falls? Does the graph bear this out? Explain.

(e) Find a formula for her instantaneous acceleration.

(f) Graph the acceleration function.

(g) How does her acceleration change as she falls? Explain.

(h) Explain why the shape of the acceleration graph and the values you get when you trace it is what you'd expect.

(i) Find her height above ground every 5 seconds until she hits the ground.

(j) To the nearest second, how long does it take for her to hit the ground?

(k) What is her velocity when she hits the ground? (For a successful jump, this should approximately equal the terminal velocity found in part (b).)

(l) For how long does she fall with a velocity that is within 1 ft/sec of the terminal velocity?

(m) Graph the function *h* that gives her height *t* seconds after jumping. Does it look like you know it should from your previous work with this activity?

(n) Find, by graphing or other means, the derivative of the height function, *h*. Where have you seen that graph before in this activity? Does it surprise you? Explain.

Suggestions/hints/solutions/answers/whatever:

(b)   Estimate the equation by tracing. Find its equation algebraically by dividing the numerator and denominator by **e^(.6t)** and finding $\lim_{t\to\infty} y1(t)$, by hand.

[Alternatively, you could factor **e^(.6t)** from numerator and denominator.] Confirm your answer on the home screen by typing **lim(y1(t),t,∞)**. Graph the asymptote. Does it look like it is an asymptote for the curve?
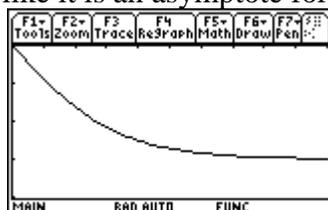

Fig. 1

(e) Instantaneous acceleration is the derivative with respect to time of instantaneous velocity. If you find this by using your TI-89's symbolic power, you will get what may be a strange looking quantity in the answer—**cosh**. See figure 2. To convert it into terms you've seen before, give the command **expand(ans(1))**. The chances are that, if you did the derivative by hand, you did not get either of these results! You can enter your by-hand version after either of the other two versions from the '89 and compare by giving the command **ans(1)=ans(2)**. The '89 will return **true** if (and only if) the last 2 answers match.
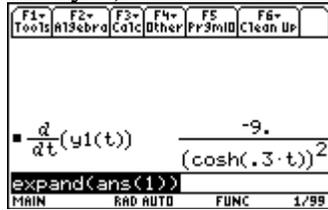

Fig. 2

(i) Fill in the table below with the parachutist's height (h) above ground t seconds after jumping. The integral $\int_0^x v(t)\,dt$ will give the ***total change in distance fallen*** between time 0 and time x. ***(Why?)*** But, if $x = 0$, the integral above gives 0 (Why?) and you want 1000, as the table below shows. So, adjust slightly.

Your '89's $\int$ feature ($\boxed{\text{F3}}$ $\boxed{2}$) gives exact symbolic results when it can, but it can't here. It therefore computes more slowly than the **nInt** (**n**umerically **Int**egrate) ($\boxed{\text{F3}}$ **B**) feature to approximate the distances. So use **nInt** to compute all integrals from here on in this activity.

You could define **y2(x)=1000+nInt(y1(t),t,0,x)**, then and look at your TI-89's **table** starting at 0 (sec) and going up by 5 (sec). [To set up the table, press $\boxed{\text{2nd}}\boxed{\text{F4}}$ and set **tblStart = 0** and **Δtbl = 5**. To see the table, press $\boxed{\text{2nd}}\boxed{\text{TABLE}}$ (above $\boxed{\text{GRAPH}}$).] But this is hard on the batteries (and slow!), since, for each entry, the '89 must start at 0.

So you might find it easier on you and your '89 (and faster!) if, on the home screen, you compute the distance fallen between <u>consecutive</u> table times and add that onto the previous height (previous table entry). For example, to find the height at $t = 10$, find the distance fallen between $t = 5$ and $t = 10$ and add it onto the height at $t = 5$. See figure 3 and the table.
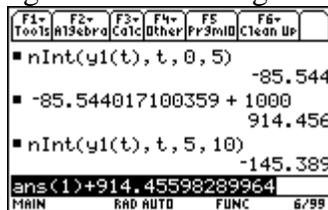

Fig. 3

t	0	5	10	15	20	25	30	35	40	45
h	1000	914	769							

Is it clear that the home screen "trick" in the previous paragraph works simply because of the property: $\int_a^b f(t)dt + \int_b^c f(t)dt = \int_a^c f(t)dt$? (See fig. 4) Specifically, for our parachutist, that property says, in words, "the distance fallen in the first 5 seconds plus the distance fallen in the next 5 seconds equals the distance fallen in the first 10 seconds." In parachuting symbols, $\int_0^5 v(t)dt + \int_5^{10} v(t)dt = \int_0^{10} v(t)dt$.:
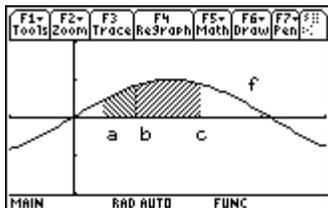

Fig. 4

(j)   When filled in, the table above will show when she "goes through" the ground (height becomes negative). "Zoom" in on the last time period just before she hits ground by setting **tblStart** equal to the last time that you see before she hits and let **Δtbl = 1**.

(m)   You do ***not*** want to use function graph mode to graph *h*! It will <u>really</u> stress your batteries! Instead, go to **parametric** graph mode (press $\boxed{\text{MODE}}$ $\boxed{\text{⊙}}$ $\boxed{2}$). On the $\boxed{\text{Y=}}$ screen, define **xt1 = t, yt1 = y1(t)**. (See figure 5.) Set the $\boxed{\text{WINDOW}}$ so that **t** (the **t**ime variable) starts at 0, goes to the time she hits the ground, and goes up by **5**—this will plot 10 points. (You can make **tstep = 1** and plot 46 points if you want—they're your batteries!)
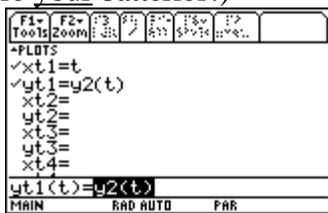

Fig. 5

You might just want to do a **scatterplot** of the points from the table and connect them with a smooth curve. To do so, press $\boxed{\text{APPS}}$ and move the cursor down to **Data/Matrix Editor**. Press $\boxed{\text{ENTER}}$ $\boxed{3}$ $\boxed{\text{⊙}}\boxed{\text{⊙}}$ and enter a name. Once in the editor, cursor up to the **c1** line, press $\boxed{\text{ENTER}}$, type **seq(k,k,0,45,0)** and press $\boxed{\text{ENTER}}$. Then move over to column **c2** and enter the values from the table in part (g).

And then you MIGHT just want to grab a pencil and graph paper and make an accurate sketch!

(n)   If you're going to graph, it would be a good idea to do this in parametric mode, too. It is a REAL battery burner and takes forever! On the other hand, "by other means" includes giving the command shown below in figure 6. You could then graph the result, even if you don't know what **tanh** is (or use **expand(ans(1))** again, as in figure 2).
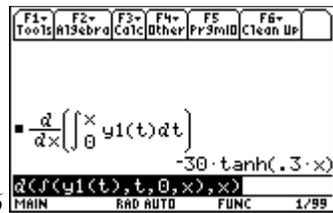
Fig. 6

Calculus Generic Scope and Sequence Topics: Integrals, Applications of Definite Integrals, Definite and Indefinite Integrals, **AREA FUNCTIONS, FUNDAMENTAL THEOREM**

NCTM Standards: Number and operations, Algebra, Geometry, Measurement, Problem solving, Connections, Communication, Representation