NUMB3RS Activity: Shaken, Not Stirred
Episode: "Democracy"

Topic: Application of inductive reasoning in problem solving **Grade Level:** 9 - 12
Objective: To discover a way to solve an interesting problem about handshakes.
Time: 20 - 30 minutes

Introduction

In "Democracy," Charlie tries to convince baseball statistics expert Oswald Kitner (who originally appeared in the *NUMB3RS* episode "Hardball") to join CalSci's academic program by giving him the following problem:

> Suppose there are five couples at a party. People shake hands, but no one shakes hands with the person they came with. At one point, one man asks the nine others how many hands they shook, and gets nine different answers. How many hands did the man himself shake?

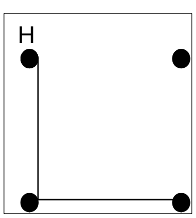
This is not the classic problem of counting the number of possible handshakes (see "Extensions"). This activity introduces the notion of people in couples and gives students the opportunity to "discover" an inductive way to solve the problem Charlie poses. In this activity, students will apply inductive reasoning to solve this problem.

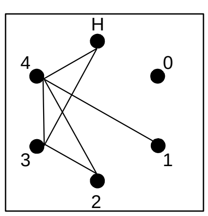
Discuss with Students

This activity is designed to provide a discovery experience for older or more capable students. If possible, provide them with the statement of Charlie's problem the day before doing this activity, so they can think about it. The purpose of the solutions below is to provide the teacher with the means to guide students, or give hints, as they work.

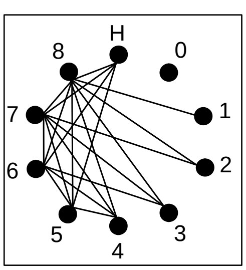
Student Page Answers:

1. 2 **2.** *0, 1, and 2* **3.** *Name the people P0, P1, and P2, by the number of hands they shook. Since P2 shook the maximum number of hands and did not shake hands with P0, these two must be partners. This makes the host and P1 partners. Since the host hasn't shaken hands with P0 and must have already shaken hands with P2, he must have shaken one hand.* **4.** *The maximum number of handshakes is four, so the numbers must be 0, 1 2, 3, and 4.*





5. *(If students have trouble with the graph model, show the graph for the two-couple problem at left after they have solved #3.) The vertices for the three-couple problem are labeled with the number of hands each person shook. Since P0 hasn't shaken any hands, and P4 has shaken the maximum, P0 and P4 must be partners. If P0 and P4 (and their handshakes) are removed from the graph, the 0 and maximum handshake situation now exists for P1 and P3. This means that the host's partner must be P2. The host shook hands with the two people who shook 3 and 4 hands, P3 and P4 respectively.*



6. *By similar reasoning, these 9 people shook from 0 to 8 hands. P0 must be the partner of P8. Removing these two, P1 shook with no one else from the remaining couples, and is the only one that P7 did not shake with, making them partners. Continuing inductively, P2 and P6 are partners, as are P3 and P5. The only person left is P4, who must be the partner of the host. From the graph, the host shook 4 hands, namely P5 through P8.*
7. *For n couples, or 2n people, the host shakes n-1 hands.*

L. Charles (Chuck) Biehl, The Charter School of Wilmington, Wilmington, DE

Name: ________________________________ Date: _______________

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To develop a problem solving strategy, begin with a simpler problem. Suppose there are only two couples, namely two people who each have a "partner."

1. If no one shakes hands with his or her partner, or with himself or herself, what is the maximum number of hands anyone can shake?

2. If one person, call him the "host," asks the other three people how many hands they shook, and gets three different answers, what do these numbers have to be?

3. Use this information to solve the problem for two couples.

4. Extend the problem to three couples. This means there are now five other people besides the host. Recall that no one shakes his or her own hand or his or her partner's hand. Each person reports shaking a different number of hands. The first step in solving the three-couple problem is to determine what the numbers of handshakes must be.

5. Represent each person at the party with a point (called a *vertex*) by labeling the points with the number of hands that person shook (use an *H* for the host). Draw a line segment (called an *edge*) to connect a pair of vertices if those people shook hands. This representation is called a *graph*, although it is different from the graph of an equation or of statistical data. Use the process from solving the two-couple problem, the information gathered from the three-couple problem, and this graph to help solve the three-couple problem. Explain your reasoning, step by step.

6. Use similar reasoning to solve the problem Charlie posed to Oswald Kitner.

7. Generalize the solution for n couples, or $2n$ people.

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The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

Many students are already familiar with the classic counting problem called the "handshake problem," usually stated as, "If there are *n* people in a room and they all shake hands with each other, how many handshakes will there be?" There are several ways to approach this problem.

For the Teacher

The general expression for the number of handshakes in the classic handshake problem is $\frac{n(n-1)}{2}$. This can be determined a number of ways. Time and opportunity permitting, exploring how to produce this answer makes an excellent activity. One approach is to suppose that each of the *n* people takes turns shaking hands with as many people as possible. Person 1 can shake $n - 1$ hands; person 2 can only shake $n - 2$ (he has already shaken person 1's hand), person 3 shakes $n - 3$, etc. The total number of handshakes is $(n - 1) + (n - 2) + \cdots + 1 + 0$. The last person is 0 because everyone else has already shaken his or her hand. For students familiar with arithmetic series, the sum is $\frac{n}{2}(n-1) = \frac{n(n-1)}{2}$.

For less-experienced students, consider that each of the *n* people shakes $n - 1$ hands, for a total of $n(n - 1)$. However, this expression counts every handshake twice, since when someone shakes another person's hand, the other person is shaking back. Therefore, the final answer is $\frac{n(n-1)}{2}$.

A third (more combinatorial) approach is to reword the problem to ask how many pairs (handshakes) there are in a group of *n*. The answer is ${}_nC_2$, or $\frac{n!}{(n-2)!2!} = \frac{n(n-1)}{2}$

For the Student

Here are a few fun problems to think about based on handshakes:

1. If everyone in your math class shook hands with everyone else, how many handshakes would there be? How long would it take?

2. Come up with a reasonable answer to "How long would it take for everyone in the world to shake hands with everyone else?" Explain any assumptions you make.

3. How many different ways can two people shake hands if each person can shake with either hand or even both hands?

4. Answer Question 3 for beings who have 3 hands, 4 hands, or *h* hands.

The act of shaking hands has a long and colorful history. Various forms are associated with different cultures, countries, social groups and secret societies. Brainstorm or research as many different types of handshakes as you can. Make up problems similar to the ones above that use a variety of different kinds and combinations of handshakes.

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