

Step-by-Step Instructions

This investigation guides the students through graphing logistic functions, discovering characteristics, and solving logistic equations in Problem 1. Problems #2-4 invites students to create model logistic functions from data and compare models.

1. Download the logistic functions tns. document and link to student calculators. Note: The teacher version is logistic functions SOL: it includes completed calculations, functions, graphs and tables.
2. Distribute the student worksheet. The nspire document allows students to investigate each logistic function and discover the characteristics of logistic functions.
3. The instructor should make certain students understand how to solve basic equations and solving logarithmic equations by taking the log of each side and solve for x.

Screen 1.2 (page 2 of problem 1) begins with a definition of logistic functions which students can put in their notes if they are using as an introductory investigation activity in Alg.II or as an introductory review in Precalculus.

Page 2

1.1 1.2 1.3 1.4 ▸ RAD AUTO REAL

Logistic functions:

are of the form $y = \frac{c}{1 + ae^{-rx}}$,

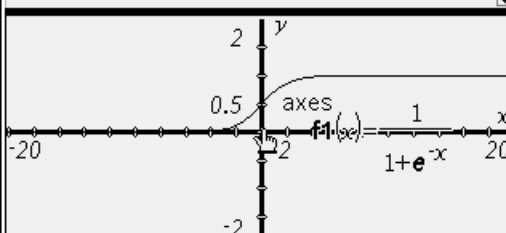
where a, c and r are positive constants.

You will be investigating logistics functions as you graph various functions. You will discover properties of logistic functions.

Page 3

1.1 1.2 1.3 1.4 ▸ RAD AUTO REAL

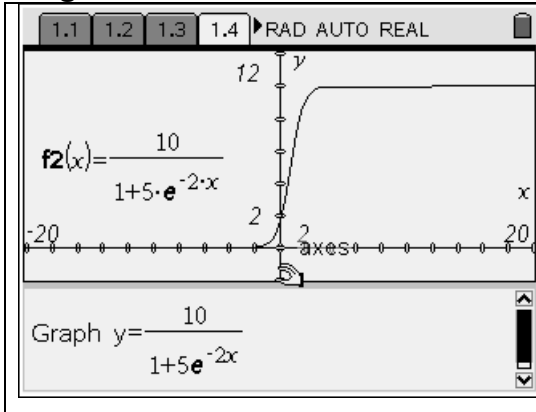
Graph $y = \frac{1}{1 + e^{-x}}$



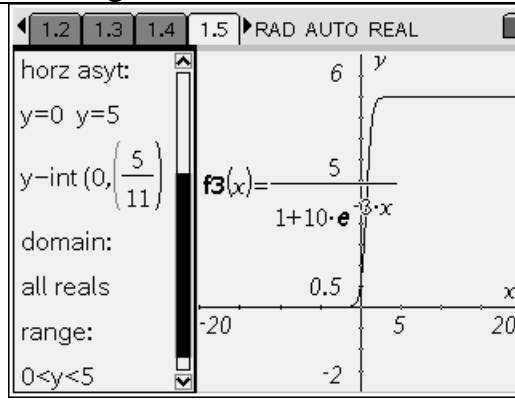
Note the window has been reset, (menu), (4), (1) to reset $-20 < x < 20$ and $-2 < y < 2$. The entry line has been hidden by (ctrl) (G).

On pages 4 and 5 you will reset the window as above with domain the same, but adjust the range according to the function. You will also note the students will be asked to give the horizontal asymptotes, y-intercept, domain and range on page 5. The teacher may have to remind the students to use (ctrl), (tab) to switch from one screen pane to the other and write in the answers in the note page. By this time students should begin to realize how these characteristics are related to the general form of a logistic function.

Page 4

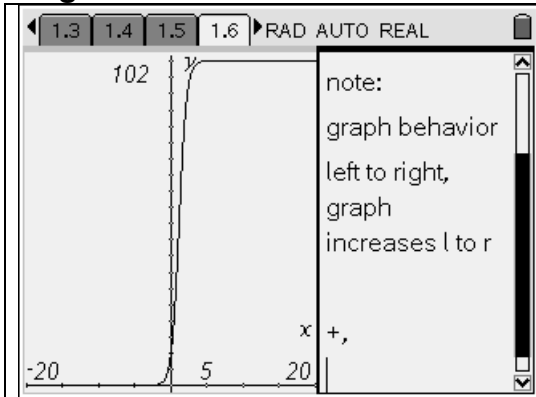


Page 5

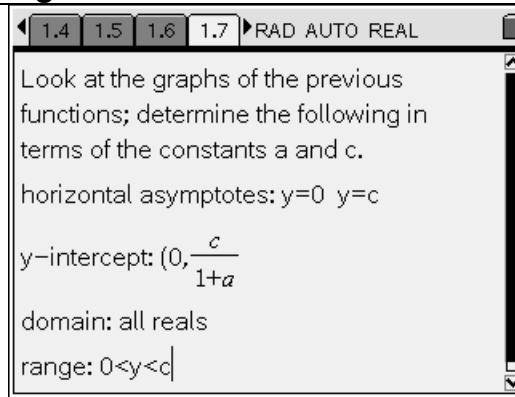


The teacher may need to point out the use of the down arrow to view all of the information in on the notes application on page 5.

Page 6



Page 7



Page 6 requests the students to note the graph behavior and should lead into students realizes the curve increases rapidly and then decreases after a certain point (point of maximum growth) and precursor the concept of an inflection point where we will see a curve change from concave up to concave down. Page 7 is where the student should be able to determine the characteristics in terms of the constants a and c.

Page 8

1.5 1.6 1.7 1.8 RAD AUTO REAL

Create a logistic function that has the following characteristics:
 Horizontal asymptotes $y=0$ and $y=8$.
 y -intercept $(0,2)$
 Point of max growth $(1.098612289,4)$
 Insert a graph and geometry page and graph the created function.

Page 9

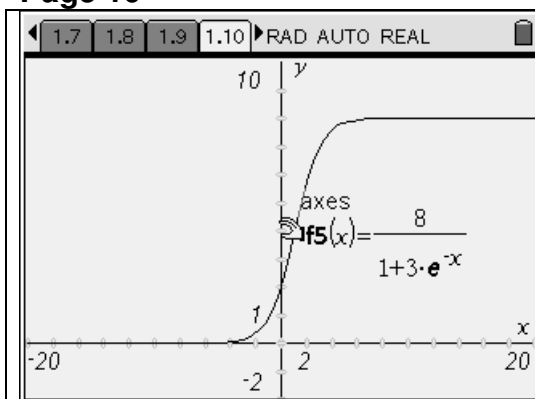
1.6 1.7 1.8 1.9 RAD AUTO REAL

The horizontal asymptote indicates that $c=8$. The y -intercept indicates $a=3$ because $\frac{8}{1+3} = 2$. The point of maximum growth indicates that $r=1$ because $\frac{\ln(3)}{1.098612289} = 1$.

The model would be $y = \frac{8}{1+3e^{-x}}$

Page 8 checks to see that the students really understand the relationship between the characteristics of logistic function and the general form of a logistic function. Page 9 shows the thinking process that students should use to create the model function.

Page 10



Page 11

1.8 1.9 1.10 1.11 RAD AUTO REAL

Question

What is the domain and range of the logistic function graphed on previous page.

Answer

Domain: all reals Range: $0 < y < 8$

Page 10 is the graph page the students should create and page 11 asks questions pertaining to the function they created.

Page 13

1.10 1.11 1.12 1.13 RAD AUTO REAL

Show solution steps on a notes page and a calculator page; then take the natural log of each side to solve the following equation:

$$\frac{50}{1+10e^{-3x}} = 40$$

The next page is partially completed to indicate how to show the steps.

Page 14

1.11 1.12 1.13 1.14 RAD AUTO REAL

$$\frac{50}{1+10e^{-3x}} = 40 \quad | \cdot (1+10e^{-3x}) \text{ each side}$$

result: $50 = 40 + 400e^{-3x}$ $-s(40)$ each side

result: $10 = 400e^{-3x}$ $d(400)$ each side

$$\frac{-1}{3} \cdot \ln\left(\frac{1}{40}\right) = 1.22963$$

1/1

Page 13 instructs the student that they will be asked to solve a logistic equation. The next screen shows the steps so they know what is expected and then allows the student to use a calculator application to calculate the answer in the bottom pane of the screen. The teacher may point out that the student should scroll down in the top note application in order to view all of the steps necessary in solving the equation.

Page 15

1.12 1.13 1.14 1.15 RAD AUTO REAL

Solve $\frac{10}{1+2e^{-4x}} = 9$

Insert a new page. Use a split layout showing step-by-step solution on the top using a notes application and final calculated result on the bottom using a calculator application.

Page 16

1.13 1.14 1.15 1.16 RAD AUTO REAL

$\frac{10}{1+2e^{-4x}} = 9$ $m(1+2e^{-4x})$

result: $10=9+18e^{-4x}$ $s(9)$

result: $1=18e^{-4x}$ $d(18)$

$\frac{1}{4} \cdot \ln\left(\frac{1}{18}\right)$.722593

1/1

Page 15 instructs the student to insert a new notes page and indicate the steps in solving the logistic equation. The expected results are shown on page 16. Note: m means multiply, s means subtract, d means divide. The document proceeds to a new problem, an application problem in which the model function is given. Page 2 of problem 2 will be shown. (Page 1 of problem 2 is a title page).

Page 2 of problem 2

1.15 1.16 2.1 2.2 RAD AUTO REAL

A prepared petri dish has a colony of bacteria growing in it.
The bacteria growth model is:

$A = \frac{49.9}{1+134e^{-1.96t}}$, where t is the elapsed time in days.

On the next page, graph and describe

Page 3

1.16 2.1 2.2 2.3 RAD AUTO REAL

Conclusions:

- h.a.: $y=0$ $y=49.9$
- y-int: $(0, .37)$
- D: all reals
- R: $0 < y < 49.9$
- pt of max growth: $(2.5, 24.95)$

Note that Page 2 gives the student the model of the bacterial growth; remind the student to use the down arrow to view all of the problem. The result of the student instructions is shown on page 3. The student should have adjusted the window appropriately for the model and also answered the question on the note pane of the screen.

Page 4

2.1 2.2 2.3 2.4 RAD AUTO REAL

Question

What does 49.9 probably represent?

Answer ▾

Area of petri dish is probably 49.9 cm^2 , which limits the colony's growth.

Page 5

2.2 2.3 2.4 2.5 RAD AUTO REAL

What would the radius of the petri dish be?

Answer ▾

The radius is about 3.98 cm. The calculation was $\sqrt{\frac{49.9}{\pi}}$

Page 4 and page 5 shows the question and the expected result expected from the student answers.

The document moves to a new problem, problem 3. The first page is a title page so it will not be shown.

Page 2 of Problem 3

2.4 2.5 3.1 3.2 RAD AUTO REAL

The next page provides data, time is in weeks and height is in centimeters. Draw a scatter plot, time versus height, and find the model function. Use discretion as to page layout (single page scatter plot or split page).

Page 3

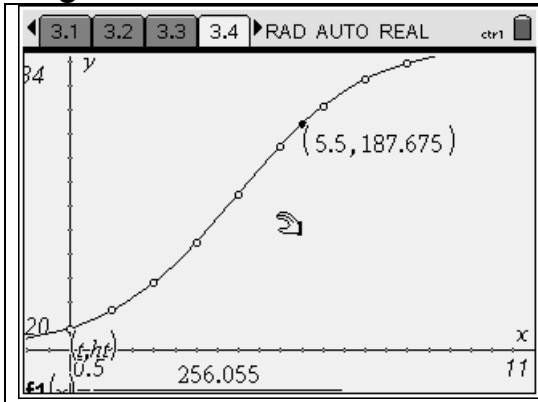
2.5 3.1 3.2 3.3 RAD AUTO REAL

| | A t | B ht | C | D | E | F | G |
|---|-----|------|----------|---------|---|---|---|
| ◆ | | | | =Logi | | | |
| 1 | 0 | 18 | Title... | Log... | | | |
| 2 | 1 | 33 | Reg... | c/(1+.. | | | |
| 3 | 2 | 56 | a | 13.0... | | | |
| 4 | 3 | 90 | b | .650... | | | |
| 5 | 4 | 130 | c | 256.... | | | |

D1 = "Logistic Regression (d=0)"

Students are instructed to insert a page and draw a scatterplot using the data on page 3 and find the model function. Page 3 shows the data and also a logistic regression. The logistic regression was obtained by highlighting the time list and the height list and pressing (menu), (4), (1), (D), and clicking and selecting the appropriate lists, using (tab) to move to the next entry. Note that the first result column must be the first empty list, in this case, c.

Page 4



Page 5

Use the model function and a calculator application to find the y-intercept and the point of maximum growth. y-int:(0,128/7)

| | |
|-----------|---------|
| 256 | 128 |
| 1+13 | 7 |
| $\ln(13)$ | 3.94608 |
| .65 | |
| | 2/2 |

Page 4 shows the result of the scatter plot the students were asked to complete and also the function that was created and stored in f1 as they completed the logistic regression. Note that the teacher needs to have instructed the student regarding menu , $\langle 3 \rangle$, $\langle 4 \rangle$ to create the scatter plot and menu , $\langle 3 \rangle$, $\langle 1 \rangle$ to access f1, enter to graph the regression equation. The calculator application is used in the bottom pane of Page 5 to calculate the y-intercept and the result is written in the note pane of screen page 5. The teacher may need to remind students to use ctrl , tab to move between panes of the layout screen.

Page 6

3.3 3.4 3.5 3.6 RAD AUTO REAL

Question

How tall was the sunflower in $5\frac{1}{2}$ weeks?

Answer \Downarrow

The height would be 187.68 cm in 5.5 weeks.

Page 6 shows the question and answer that the student should give as a response. At this time we move on to the last problem in the document, problem 4, which allows the student to compare different models to a set of data and determine which is the best model.

Page 2 of problem 1

3.5 3.6 3.7 4.1 RAD AUTO REAL

The population, pop, (in millions) of the US from 1800 to 1870 with time representing the number of years since 1800.

| | A time | B pop | C |
|---|--------|-------|---|
| 1 | 0 | 5.3 | |
| 2 | 10 | 7.2 | |
| 3 | 20 | 9.6 | |
| 4 | 30 | 12.9 | |
| 5 | 40 | 17. | |

B pop

Page 3

3.6 3.7 4.1 4.2 RAD AUTO REAL

Insert a new page, copy and paste the lists.

Use statistics regressions to find an exponential growth model and calculate a logistic growth model for the population data.

Insert a graph and geometry page and graph both models.

Use the feature

The data is given on page 2 and page 3 instructs the student to perform an exponential regression and calculate a logistic model.

Note: It is requested that the student calculate the logistic model because logistic regression results were not consistent with the results of the TI-84. The same model is desirable for both models if they are being used in the classroom at the same time.

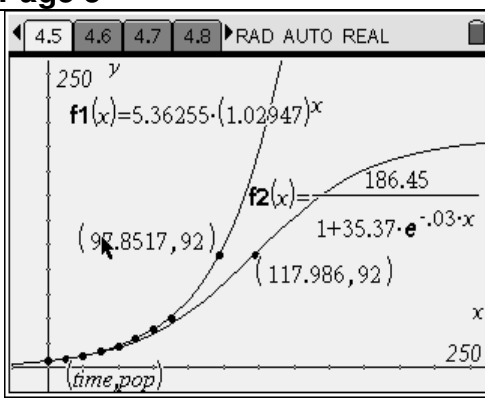
Page 4

4.1 4.2 4.3 4.4 RAD AUTO REAL

| | A time | B pop | C | D | E | F |
|---|--------|-------|----------------|---------|---|---|
| 4 | 30 | 12.9 | b | 1.02... | | |
| 5 | 40 | 17. | r ² | .999... | | |
| 6 | 50 | 23.2 | r | .999... | | |
| 7 | 60 | 31.4 | Res... | {.06... | | |
| 8 | 70 | 39.8 | Res... | {.01... | | |

A8 70

Page 5



The data list and the regression are shown in the page 4 screen. The teacher may need to review how to find the exponential regression with some students. The graph is shown on page 5 for the question in the screen from page 6. This is an opportunity for the teacher to place a point on each graph, menu , $\langle 6 \rangle$, $\langle 2 \rangle$, enter . Place the pointer on one of the y-coordinates, enter , enter , use clear to remove digits, then retype $\langle 9 \rangle$, $\langle 2 \rangle$, enter to determine the corresponding x-coordinate. Repeat with the y-coordinate of the point on the other graph. Students should recall that x-coordinate 0 means the year 1800 when answering the question asked on page 6. This feature is unique to the nspire family.

Page 6

Question

Use both models to find when the population was about 92 million.

Answer

exponential prediction: 1896
logistic prediction: 1918

Page 7

Question

The population was about 92 million in 1910, which model was closer?

Answer

The logistic model was closer.

The expected answer to the question on page 6 is shown and also the expected answer to the question on page 7 is shown.

Page 8

Question

The predicted population is 297.7 million in 2010, which model gives a closer value?

Answer

The logistic gives a value of 175.1 million, which is closer than the

Page 9

| | |
|-----------------------|---------|
| $f1(210)$ | 2390.88 |
| $f2(210)$ | 175.079 |
| $\ln(35.37)$ | 118.862 |
| .03 | |
| $f1(118.86213344058)$ | 169.37 |
| $f2(118.86213344058)$ | 93.225 |
| | 5/5 |

Calculations for answering some of the questions are shown on page 9 and page 10 discusses the point of maximum growth.

Page 10

Discuss the point of maximum growth in the population problem.

The point of maximum population growth is (118.86, 143.23) using algebraic relationship $\left(\frac{\ln(a)}{r}, \frac{c}{2}\right)$. Using $f2(118.86)$ yields a y coordinate 93.23. There is a discrepancy with the results because the graph is "best fit" of the data represented.