

What's Normal, Anyway?

ID: 8556

Time required
60 minutes

Activity Overview

In this activity, students explore the normal distribution and several of its most interesting properties. First, they use a histogram of data from a binomial experiment to examine the general shape of a normal curve. Then, they use a dynamic illustration to make observations, using sliders to change the mean and standard deviation. Lastly, they examine the empirical rule for normal distributions, describing the percent of data values falling within different standard deviations from the mean.

Topic: The Normal Distribution

- *Graph the probability density function of the normal distribution for a given mean and standard deviation.*
- *Explain how the mean of a random variable is changed under a linear transformation.*
- *Explain how the standard deviation of a random variable is changed under a linear transformation.*

Teacher Preparation and Notes

This activity is designed for use in a Precalculus or Statistics classroom. It is best used to ease into the introduction of the normal distribution, after discussion of probability, independent events, and binomial experiments and distributions.

- *Prior to beginning the activity, students should know how to interpret histograms and be familiar with the concepts of mean as a measure of center and standard deviation as a measure of spread.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- **To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "8556" in the keyword search box.**

Associated Materials

- *WhatsNormal_Studentdoc*
- *WhatsNormal.tns*
- *WhatsNormal_Soln.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Exploring the Normal Curve Family (TI-Nspire technology) — 13273*
- *Plotting a Normal Curve (TI-84 Plus family) — 8573*
- *Looking Normal (TI-Nspire technology) — 16924*

Problem 1 – A binomial experiment

In Problem 1, students are asked to consider an experiment in which there are 100 trials of rolling a die 300 times. A “success” in the experiment is rolling a 3.

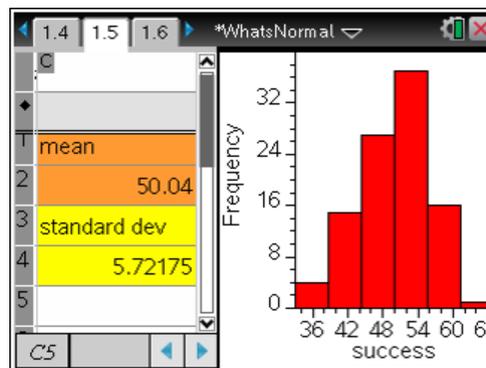
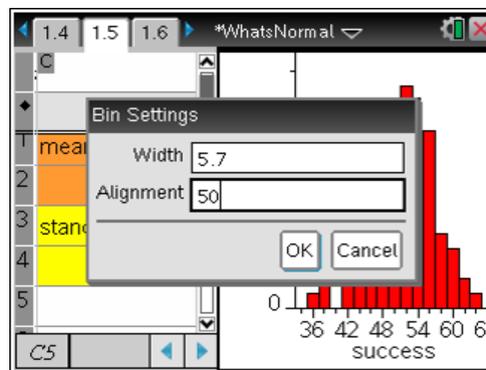
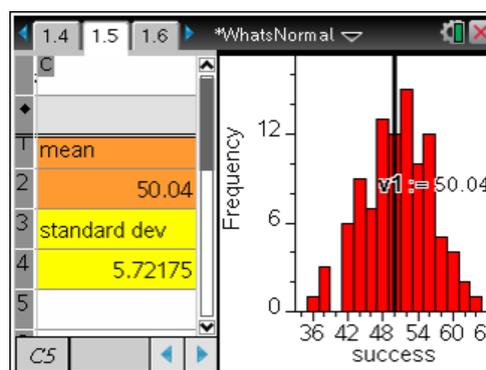
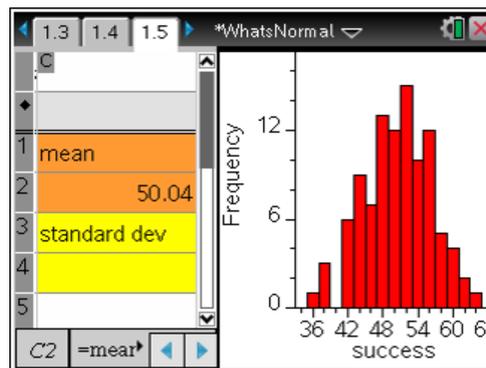
On page 1.5, students are provided with a set of data obtained through a simulation (rather than having students perform the simulation themselves, in the interest of time). They will create a histogram of this data on the right side of page 1.5, and use cell C2 to calculate the mean via the command **mean(success)** and then use **Plot Value** to display the value on the histogram.

Students are prompted to make the observation that the line representing the mean (for this data set, 50) appears to be a line of symmetry for the histogram.

Next, they use the command **stdevpop(success)** to calculate the standard deviation of the data set (5.7) in cell C4.

The next step in this pre-investigation of the normal curve is for students to reposition and resize the intervals on the histogram. This is achieved by selecting Plot Properties > Histogram Properties > Bin Settings and setting the width equal to the value of the standard deviation and setting the alignment to the value of the mean. Or students can grab the left side of one of the middle bars and align it with the mean, 50. The other side of the bar is then dragged to align with the value **mean + stdev**, or 55.7. The width of the bars are now equal to 5.7, or one standard deviation of the data set. Examining the counts in each of these intervals provides a foundation for introducing the empirical rule in Problem 3.

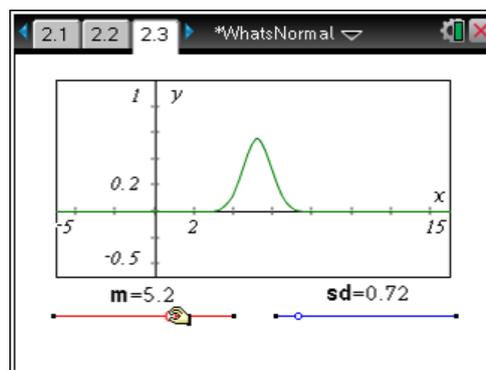
Note: Students will need to adjust the vertical scale by grabbing and dragging the scale values.



TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 1 at the end of this lesson.

Problem 2 – Properties of the normal curve

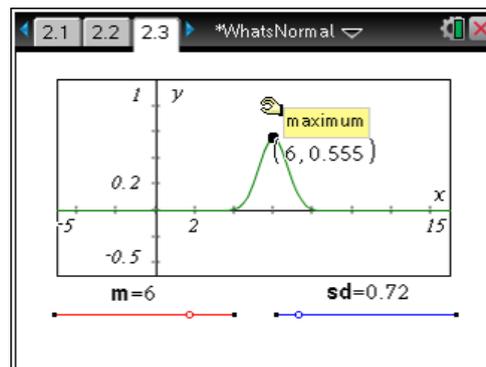
Presented with the general equation for a normal curve, students find that it has two parameters: the mean and the standard deviation. Page 2.3 provides a model with which students can adjust the value of the mean and standard deviation using sliders. It should be clear from the model that the value of the mean affects the horizontal position of the curve, while the value of the standard deviation affects the vertical stretch.



TI-Nspire Navigator Opportunity: Quick Poll, Live Presenter

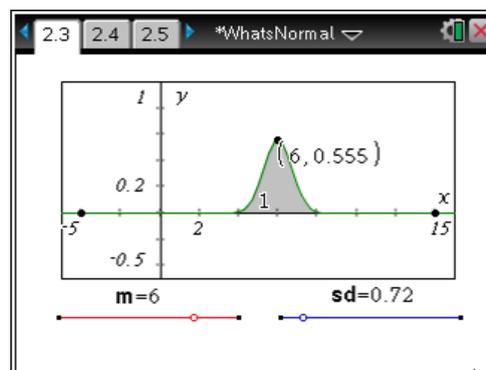
See Note 2 at the end of this lesson.

Next, students explore two properties involving the mean. Using the **Point On** tool (from the Points & Lines menu), students place a point on the curve, and drag it to the function’s maximum point. Comparing the value of the x-coordinate of the maximum with the value of **m** on the slider, students should conclude that the maximum value of a normal curve occurs at the mean.



Additionally, a **Perpendicular** line (from the Construction menu) to the x-axis through the function’s maximum point reveals that the line $x = m$ is a line of symmetry for the curve.

The directions on page 2.6 call for using the **Integral** tool to measure the area between the curve and the x-axis, in an effort to have students observe that, regardless of the values of **m** or **sd**, the area is always equal to 1. While students may not be familiar with the term “integral,” the concept of “area under a curve” is easily understandable.



Be sure, here, that students understand that the curve does not “end” at the ends of the axis where the integral measurements were taken, but rather that the x-axis is an asymptote. The integral from $-\infty$ to $+\infty$ is equal to 1; the value of the integral here is **approximately** equal to 1. The area under the curve *outside* of the viewable window is almost entirely negligible.

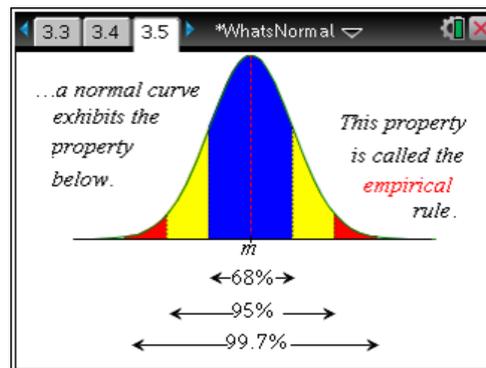
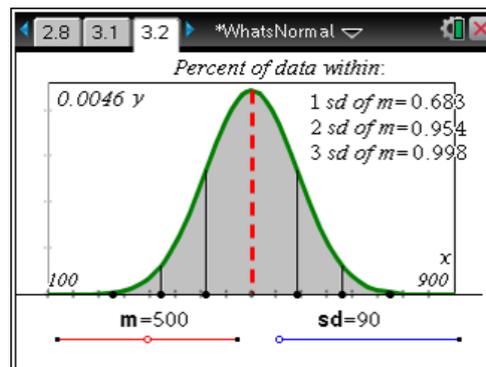
Problem 3 – The empirical rule

In setting up for a discussion of the empirical rule, students once again use the **Integral** tool, this time to measure the regions under the curve that are within one, two, and three standard deviations of the mean.

Though the measurements are given in decimals, they can easily be interpreted as percents. Students should find the areas under the curve to be as shown in the screenshot to the right. With the measurements displayed, they should alter the value of **sd** using the slider. As **sd** changes, the widths of the cross-sections change accordingly, but the **area** of the cross-sections remain the same! This very interesting property is known as the empirical rule.

The empirical rule is depicted in a static diagram on page 3.5. On the page that follows, students are shown how these three percentages can be used to find the percentages of different combinations of the cross-sections.

Lastly, students bring it all together, as they are again directed to the questions on page 1.8. The power of the empirical rule should become obvious: the percentages may also be viewed as the *probability* that a randomly chosen trial is within a certain distance from the mean—a measure of how close it is to “norm”!



TI-Nspire Navigator Opportunities

Note 1

Problem 1, Live Presenter

It may be helpful to students if *Live Presenter* is used to help demonstrate how to manipulate the width and location of the histogram intervals.

Note 2

Problem 2, Quick Poll, Live Presenter

After the students have explored the effects of dragging both sliders, send a *Quick Poll* asking what effect each slider has on the shape and position of the curve. Use *Live Presenter* to clear up any misunderstandings the quick poll shows.