NUMB3RS Activity: Guarding the Goods Part I
Episode: "Obsession"

Topic: The Art Gallery Problem **Grade Level**: 6 - 10
Objective: Explore geometry through The Art Gallery Problem
Time: 10 - 15 minutes

Introduction

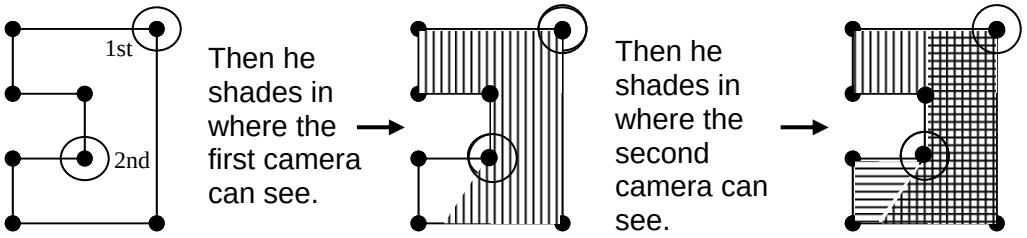
"The Art Gallery Problem" was first posed in 1973 by mathematician Victor Klee. Though simple in concept, the general solution to the problem is found in computational geometry. The problem as originally posed is this: What is the minimum number of guards that would be needed at the corners of an art gallery to guarantee that every point of the gallery is always in sight of at least one guard. On an elementary level, simple Art Gallery problems can be solved by experimentation. Making minor changes to the shapes of the art galleries or the capabilities of the guards can transform elementary problems into much more complex problems.

Discuss with Students

In **"Obsession**,**"** Agent Don Eppes is investigating a murder in a home that had security cameras installed in it. His brother Charlie helps him figure out where the cameras could see and, more importantly, where they couldn't. He does this by using a model of the home and then applying his knowledge of "The Art Gallery Problem". Depending on the shape of the home, a certain number of cameras are needed to guard the interior.

To solve the problem, rules need to be established. In "The Art Gallery Problem," cameras (or guards) can see in all directions at once, but they can't see through walls. Also, they must be placed in the corners (vertices) of the gallery so they don't block anyone's view of the art.

***NUMB3RS* Example** Charlie is trying to verify that the security cameras could indeed observe an entire floor of a home. To test this, he makes a diagram of the floor, marks the corners, and then circles where the cameras are:



Because the entire shape ends up shaded, Charlie knows that the security cameras can see the entire floor. He also knows that parts of the floor are seen by both cameras.

Student Page Answers: **1**. *All convex galleries can be guarded by one guard placed at any vertex.* ***2****. The segment drawn must have both endpoints lying in the interior of the polygon, and part of the segment must be in the exterior.* **3**. *This can be guarded by 2 guards in many ways. Students will have various correct answers. For example, one at A and one at C.* **4**. *3* **5**. *3 [There are many possible locations for the guards in both* ***4*** *and* ***5****.]*

Bro. Patrick Carney, DePaul Catholic High School, Wayne, NJ

Name: _____ Date: _____

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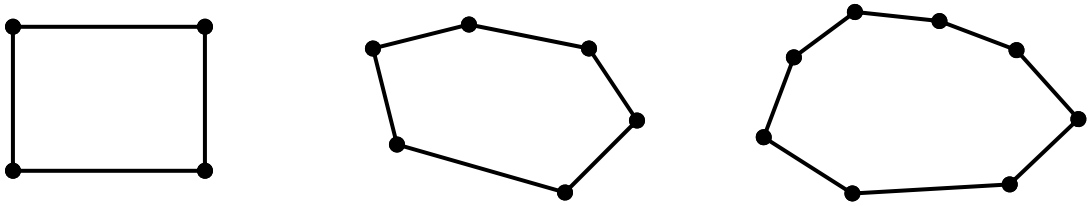
Security at an art gallery demands that guards be able to see the entire exhibit area and they are placed in the corners so they don’t block the view of the patrons. Galleries prefer to hire as few guards as possible in order to save money. Consider the galleries below. How many guards would you need to cover the floor? Circle the corner(s) where you would place them.



These floor plans are all convex. If a shape is *convex* it means that a line segment connecting any two points in the interior will not go outside the shape.

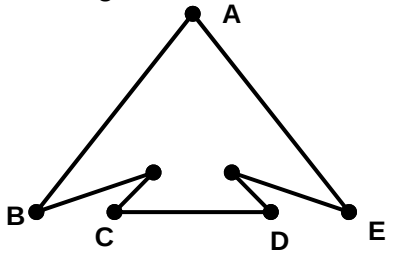
1. What conclusion can you draw about guarding an art gallery that is convex?

_____

2. Real art galleries want to have a lot of wall space so that they can display more art. One way to do this is to use non-convex polygons. In *non-convex* polygons, it is possible to connect two points that are both inside the shape, with a line segment that goes outside the shape. Draw a segment in the polygon shown in #3 that shows it is non-convex.

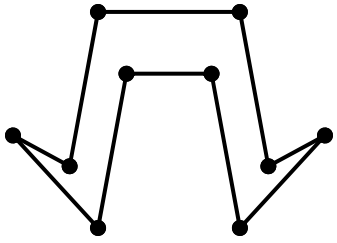
Consider the art gallery below. It could be protected with guards at **B**, **C**, and **E**.

3. Can you find a way to protect it with only two guards? _____
_____
_____



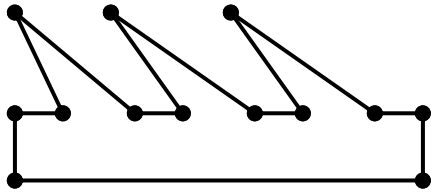
Find the minimum number of guards for each of the following shapes. Circle the vertex where each guard will stand.

4.



Number of guards: _____

5.



Number of guards: _____

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The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

There are many ways to make "The Art Gallery Problem" more complex. Some of these are explored in a separate ***NUMB3RS*** activity. Students can change the parameters of the problem to see how this will affect the possible solutions.

For the Student

- Consider the situation that guards may be placed along the sides and not only at vertices. How will this affect your solutions?
- What if the guards could walk along a wall from one corner to the next and then back again? Would this affect your solutions?
- Suppose guards may be placed in the interior of the gallery instead of only at the corners or along the walls. How will this affect your solutions?
- Draw a gallery space with pillars in the interior of the space. See how this will affect your solution to the problem.

Additional Resources

http://www-cgrl.cs.mcgill.ca/~godfried/teaching/cg-projects/97/Thierry/thierry507webprj/507applet/triangulize *(this is one web address on two lines)*
A program that allows users to draw their own shape and see where guards should be placed to minimize the number of guards used.

http://www.cut-the-knot.org/Curriculum/Combinatorics/Chvatal.shtml
This is a treatment of "The Art Gallery Problem" in language for advanced students. It includes an applet for users to explore the application of diagonals in this problem.

Activity: Keeping an Eye on the School

For the Student

Work in teams. Obtain a blueprint of your school (or make one through observation) and determine the best place for security cameras to be positioned so that the entire school is viewable. Since each classroom would likely need its own camera, limit your investigation to the hallways and public areas. Explore your school to make sure there are no obstructions to the line of sight of the camera. See if you can use fewer cameras than your classmates.

If your school already has cameras installed, determine if they can see the entire school. If your school is interested in installing security cameras, or if your school already has cameras, present your findings to the Principal or school board.

Related Topics

"The Art Gallery Problem" has applications outside of art galleries and museums. Sporting events are arranged such that officials and television cameras have access to all of the action from many angles. Lighting fixtures are set up such that an entire area is illuminated. What other situations could benefit from a geometric approach similar to the one used in this activity?

Bro. Patrick Carney, DePaul Catholic High School, Wayne, NJ