Introduction of the Fundamental Theorem

ID: 9778

Time required
45 minutes

Activity Overview

This activity builds student comprehension of functions defined by a definite integral, where the independent variable is an upper limit of integration. Students are led to the brink of a discovery of a discovery of the Fundamental Theorem of Calculus, that $\frac{d}{dx}\int_0^x f(t)dt = f(x)$.

Topic: Fundamental Theorem of Calculus

- *Graph a function and use Measurement > Integral to estimate the area under the curve in a given interval.*
- *Use Integral (in the Calculus menu) to obtain the exact value of a definite integral.*

Teacher Preparation and Notes

- *This investigation should follow coverage of the definition of a definite integral, and the relationship between the integral of a function and the area of a region bounded by the graph of a function and the x-axis.*
- *Before doing this activity, students should understand that if a < b and f(x) > 0, then:*
  - $\int_a^b f(x) > 0$
  - $\int_a^b -f(x) < 0$
  - $\int_b^a f(x) < 0$
  - $\int_b^a -f(x) > 0$
- *Before starting this activity, students should go to the HOME screen and select* ***F6:Clean Up > 2:NewProb****, then press* ENTER*. This will clear any stored variables.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "9778" in the keyword search box.***

Associated Materials

- *FundamentalTheorem_Student.doc*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Sum Rectangles (TI-89 Titanium) — 12099*
- *Exploring the Fundamental Theorem of Calculus (TI-Nspire CAS technology) — 9205*

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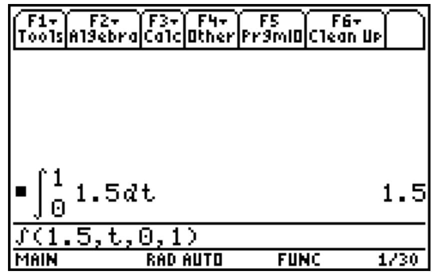
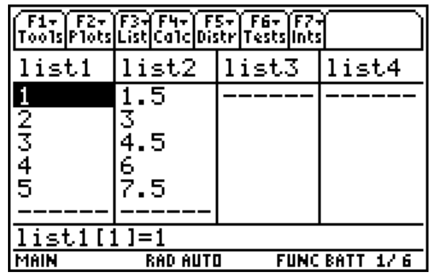
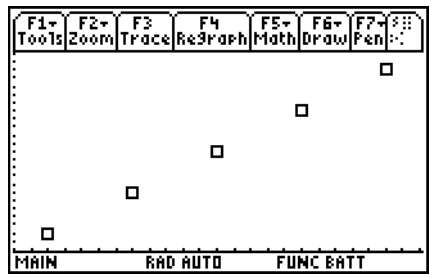
Problem 1 – Constant Integrand

Students explore the function $\int_0^x 1.5dt$. They should notice that there is a constant rate of change in the graph of $f(x) = \int_0^x 1.5dt$. This rate of change is 1.5.







1. The table looks like the one below.

x	$\int_0^x 1.5dt$
1	1.5
2	3
3	4.5
4	6
5	7.5

2. $\int_0^0 1.5dt = 0$; There is zero area under the graph of $y = 1.5$ from $x = 0$ to $x = 0$.
3. 1.5 units
4. The graph will be a line through the origin with slope 1.5.

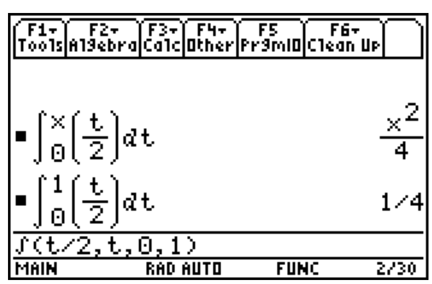
Students will enter their data into the lists and then view the graph of $\left(x, \int_0^x 0.5dt\right)$.

5. A line; yes (student answers may vary)
6. The same as before, except the slope would be 0.5 instead of 1.5.

Problem 2 – Non-Constant Integrand

Students investigate the behavior of $f(x) = \int_0^x \frac{t}{2}dt$. Students should note that this function changes at a non-constant rate and are asked to explain why this is so (from a geometric point of view).

x	$\int_0^x \frac{t}{2}dt$
1	0.25
2	1
3	2.25
4	4
5	6.25



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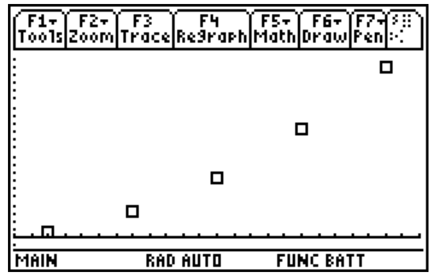
8. $\int_0^0 \frac{t}{2}dt = 0.$ The height and the length of the triangle are 0 so the area is 0.

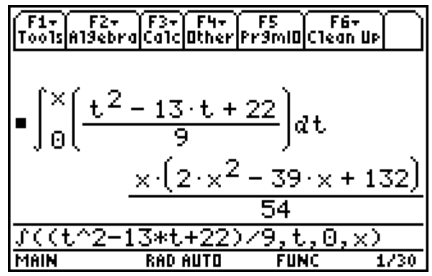
9. The area changes by a different amount each time because both the height and width are increasing.

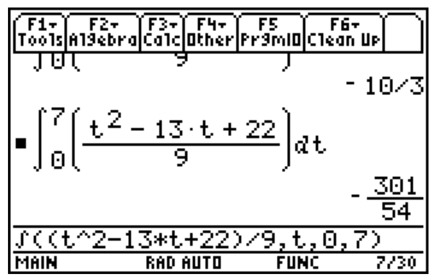
10. The graph is not linear. It is a parabola as seen by the formula in the above screen shot.



Problem 3 – An Integrand That Changes Sign

x	$\int_0^x \frac{t^2 - 13t + 22}{9}dt$
1	95/54 = 1.76
2	62/27 = 2.29
3	11/6 = 1.83
4	16/27 = 0.59
5	–65/54 = –1.20
6	–10/3 = –3.33
7	–301/54 = –5.57
8	–208/27 = –7.70
9	–19/2 = –9.50
10	–290/27 = –10.74
11	–605/54 = –11.20
12	–32/3 = –10.67
13	–481/54 = –8.91
14	–154/27 = –5.70





The fractions are approximated to the nearest hundredth.

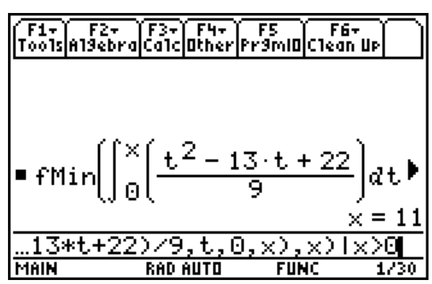
11. After $x = 2$, the integral value begins to decrease.

12. The values for x in which the integral decreases are $2 < x < 11$; the function is negative.

13. The values for which the integral is increasing are $x < 2$, $x > 11$; the function is positive.

14. The table seems to indicate $x = 11$. To find out for sure, use **fMin** on the integral. You have to restrict the domain to $x > 0$, to get the answer.

15. Yes, we have seen a similar situation. The minimum occurs on a function where the function stops decreasing and starts increasing.



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