Special Cases of the Product Rule

Teachers Teaching with Technology™ Professional Development from Texas Instruments

Student Activity

7 8 9 10 11 **12**

TI-Nspire™ | Activity | **Teacher** | 3 hours

Introduction

If $f(x)$ and $g(x)$ are both differentiable functions, then their product $f(x)g(x)$ is also differentiable, and using the product rule then:

$$\left(f(x)g(x)\right)' = \frac{d}{dx}\left(f(x)g(x)\right) = f'(x)g(x) + f(x)g'(x) = g(x)\frac{d}{dx}\left(f(x)\right) + f(x)\frac{d}{dx}\left(g(x)\right)$$

In this activity we will meet examples of functions that also satisfy $\left(f(x)g(x)\right)' = f'(x)g'(x)$ and examine and explore the patterns in the resulting differentials.

PART 1

Question: 1.

a) Consider the functions $f(x) = -(x+1)$ and $g(x) = \frac{1}{x}$, show that $\left(f(x)g(x)\right)' = f'(x)g'(x)$

b) Change $f(x)$ slightly so that $f(x) = x+1$ and check if it still satisfies: $\left(f(x)g(x)\right)' = f'(x)g'(x)$

Question: 2.

a) Consider the functions $f(x) = (x+2)^2$ and $g(x) = \frac{1}{x^2}$, show that $\left(f(x)g(x)\right)' = f'(x)g'(x)$

b) Change $f(x)$ slightly so that $f(x) = x+2$ and check if it still satisfies: $\left(f(x)g(x)\right)' = f'(x)g'(x)$

Question: 3.

a) Consider the functions $f(x) = -(x+3)^3$ and $g(x) = \frac{1}{x^3}$, show that $\left(f(x)g(x)\right)' = f'(x)g'(x)$

b) Change $f(x)$ slightly such that $f(x) = (x+a)^3$, determine the value for a such that it satisfies the condition:

$$\left(f(x)g(x)\right)' = f'(x)g'(x)$$

Question: 4.

Consider the functions $f(x) = -(1)^n(x+n)^n$ and $g(x) = \frac{1}{x^n}$, use CAS to verify $\left(f(x)g(x)\right)' = f'(x)g'(x)$

For the cases when $n = 4$ and $n = 5$.

Question: 5.

Consider the functions $f(x) = (x+10)^{10}$ and $g(x) = \frac{1}{x^{10}}$, use CAS to verify $\left(f(x)g(x)\right)' = f'(x)g'(x)$

Can you predict the general result? Hint use a slider for *n*.

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PART 2

Question: 6.

Consider the functions $f(x)=\frac{1}{1-x}$ where $x\in R\setminus\{1\}$ and $g(x)=x$, show that $\left(f(x)g(x)\right)'=f'(x)g'(x)$

Question: 7.

Consider $f(x)=\frac{1}{(2-x)^2}$ where $x\in R\setminus\{2\}$ and $g(x)=x^2$, show that $\left(f(x)g(x)\right)'=f'(x)g'(x)$

Question: 8.

Consider $f(x)=\frac{1}{(3-x)^3}$ where $x\in R\setminus\{3\}$ and $g(x)=x^3$, show that $\left(f(x)g(x)\right)'=f'(x)g'(x)$

Question: 9.

Consider $f(x)=\frac{1}{(b-x)^m}$ where $x\in R\setminus\{b\}$ and $g(x)=x^n$, given that $\left(f(x)g(x)\right)'=f'(x)g'(x)$

Express both *b* and *m* in terms of *n*.

Hence write generalised sets of functions $f(x)$ and $g(x)$ which satisfy $\left(f(x)g(x)\right)'=f'(x)g'(x)$,

If $f(x)=\frac{1}{(10-x)^{10}}$ where $x\in R\setminus\{10\}$ and $g(x)=x^{10}$ can you predict $\left(f(x)g(x)\right)'=f'(x)g'(x)$.

Using your conjecture is it true for non-integer values of *n*? Prove your conjecture in general.

PART 3

Question: 10.

Consider the two non-constants functions $f(x)$ and $g(x)$, where $g(x)\neq g'(x)$ if

$\left(f(x)g(x)\right)'=f'(x)g'(x)$ then show that $\frac{f'(x)}{f(x)}=\frac{g'(x)}{g'(x)-g(x)}$

Question: 11.

Consider the case when $g(x)=e^{kx}$, $k\in R\setminus\{1\}$, solve the differential equation in Question 10 and hence find a function $f(x)$ which satisfies $\left(f(x)g(x)\right)'=f'(x)g'(x)$.

Question: 12.

Can you find other sets of functions for example non-polynomial functions $f(x)$ and $g(x)$, for example trigonometric exponential or logarithmic functions $f(x)$ and $g(x)$ which satisfy $\left(f(x)g(x)\right)'=f'(x)g'(x)$

Generalize your results.

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