# **Special Cases of the Product Rule**



# **Student Activity**

7 8 9 10 11 12









### Introduction

If f(x) and g(x) are both differentiable functions, then their product f(x)g(x) is also differentiable, and using the product rule then:

$$\left(f(x)g(x)\right)' = \frac{d}{dx}\left(f(x)g(x)\right) = f'(x)g(x) + f(x)g'(x) = g(x)\frac{d}{dx}\left(f(x)\right) + f(x)\frac{d}{dx}\left(g(x)\right)$$

In this activity we will meet examples of functions that also satisfy (f(x)g(x))' = f'(x)g'(x) and examine and explore the patterns in the resulting differentials.

#### PART 1

#### Question: 1.

- a) Consider the functions f(x) = -(x+1) and  $g(x) = \frac{1}{x}$ , show that (f(x)g(x))' = f'(x)g'(x)
- b) Change f(x) slightly so that f(x) = x + 1 and check if it still satisfies: (f(x)g(x))' = f'(x)g'(x)

## Question: 2.

- a) Consider the functions  $f(x) = (x+2)^2$  and  $g(x) = \frac{1}{x^2}$ , show that (f(x)g(x))' = f'(x)g'(x)
- b) Change f(x) slightly so that f(x) = x + 2 and check if it still satisfies: (f(x)g(x))' = f'(x)g'(x)

## Question: 3.

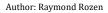
- a) Consider the functions  $f(x) = -(x+3)^3$  and  $g(x) = \frac{1}{x^3}$ , show that (f(x)g(x))' = f'(x)g'(x)
- b) Change f(x) slightly such that  $f(x) = (x+a)^3$ , determine the value for a such that it satisfies the condition: (f(x)g(x))' = f'(x)g'(x)

## Question: 4.

Consider the functions  $f(x) = -(1)^n (x+n)^n$  and  $g(x) = \frac{1}{x^n}$ , use CAS to verify (f(x)g(x))' = f'(x)g'(x)For the cases when n = 4 and n = 5.

#### Question: 5.

Consider the functions  $f(x) = (x+10)^{10}$  and  $g(x) = \frac{1}{x^{10}}$ , use CAS to verify (f(x)g(x))' = f'(x)g'(x)Can you predict the general result? Hint use a slider for n.





## PART 2

## Question: 6.

Consider the functions  $f(x) = \frac{1}{1-x}$  where  $x \in R \setminus \{1\}$  and g(x) = x, show that (f(x)g(x))' = f'(x)g'(x)

#### Question: 7.

Consider 
$$f(x) = \frac{1}{(2-x)^2}$$
 where  $x \in R \setminus \{2\}$  and  $g(x) = x^2$ , show that  $(f(x)g(x))' = f'(x)g'(x)$ 

## Question: 8.

Consider 
$$f(x) = \frac{1}{(3-x)^3}$$
 where  $x \in R \setminus \{3\}$  and  $g(x) = x^3$ , show that  $(f(x)g(x))' = f'(x)g'(x)$ 

#### Question: 9.

Consider 
$$f(x) = \frac{1}{(b-x)^m}$$
 where  $x \in R \setminus \{b\}$  and  $g(x) = x^n$ , given that  $(f(x)g(x))' = f'(x)g'(x)$ 

Express both b and m in terms of n.

Hence write generalised sets of functions f(x) and g(x) which satisfy (f(x)g(x))' = f'(x)g'(x),

If 
$$f(x) = \frac{1}{(10-x)^{10}}$$
 where  $x \in R \setminus \{10\}$  and  $g(x) = x^{10}$  can you predict  $(f(x)g(x))' = f'(x)g'(x)$ .

Using your conjecture is it true for non-integer values of *n*? Prove your conjecture in general.

#### PART 3

#### Question: 10.

Consider the two non-constants functions f(x) and g(x), where  $g(x) \neq g'(x)$  if

$$(f(x)g(x))' = f'(x)g'(x)$$
 then show that  $\frac{f'(x)}{f(x)} = \frac{g'(x)}{g'(x) - g(x)}$ 

#### Question: 11.

Consider the case when  $g(x) = e^{kx}$ ,  $k \in R \setminus \{1\}$ , solve the differential equation in Question 10 and hence find a function f(x) which satisfies (f(x)g(x))' = f'(x)g'(x).

## Question: 12.

Can you find other sets of functions for example non-polynomial functions f(x) and g(x), for example trigonometric exponential or logarithmic functions f(x) and g(x) which satisfy (f(x)g(x))' = f'(x)g'(x) Generalize your results.

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