Charged Up

ID: 12369

Time Required
20 minutes

Activity Overview

*In this activity, students will be asked some exam-like multiple-choice questions to introduce separable differential equations. Students will use the **deSolve** command to find general and particular solutions to differential equations. They will also dynamically explore the family of particular solution to a differential equation.*

Topic: Differential Equations

- *Differential equations solved algebraically. Solutions shown graphically.*
- *Use CAS to find solutions to DEs.*

Teacher Preparation and Notes

- *The syntax for **deSolve** is **deSolve(y'=f(x,y) and y(s)=t,x,y)**, where the initial condition y(s) = t is optional, x is the independent and y is the dependent variable. This activity will help students see the application of DEs, the process of solving separable DEs, and how to use TI-Nspire CAS to check or find the solution to DEs.*
- *Students will write their responses directly into the TI-Nspire handheld or on the accompanying handout. On self-check questions, students can then press (menu) and select **Check Answer** (or (ctrl) + ▲). If desired, by using the TI-Nspire Teacher Edition software, teachers can change the self-check questions to exam mode so students cannot check their answer. On any question click the Teacher Tool Palette and select Question Properties. Change the Document Type from Self-Check to Exam.*
- *Students should know how to navigate between pages with (ctrl) + ► or ◄, and navigate between applications on a split screen with (ctrl) + (tab).*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "12369" in the quick search box.***

Associated Materials

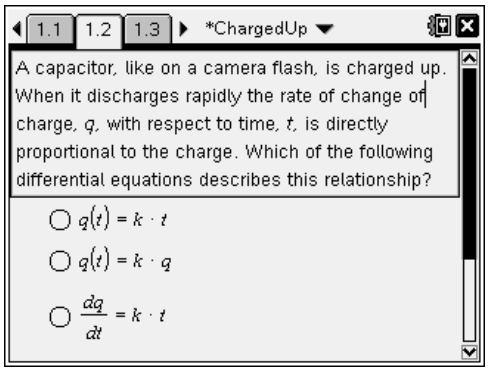
- *ChargedUp_Student.doc*
- *ChargedUp.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Differential Equations (TI-Nspire CAS technology) — 8998*
- *Exponential Growth & Decay (TI-Nspire CAS technology) — 9992*
- *Logistic Growth, Differential Equations, Slope Fields (TI-89 Titanium) — 5514*

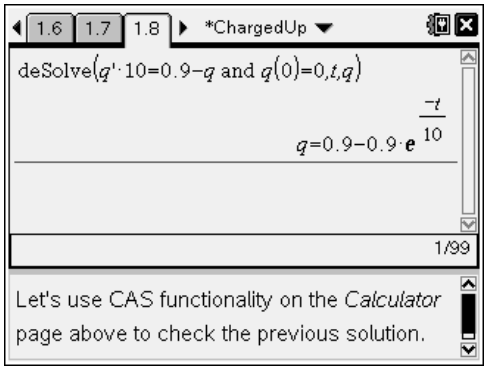
Part 1 – Separable Differential Equation Introduced



On page 1.2 and 1.3, students are asked exam like questions about differential equations. These foundational (practically review questions) can serve as an introduction to separable differential equations. Teachers can use the question on page 1.3 to formatively assess how well students are making the connection to their previous knowledge. If students appear to be struggling, teachers can use the opportunity to state the following steps: STEP 1 separate variable, STEP 2 integrate both sides, STEP 3 apply initial conditions to solved for the constant of integration and find a particular solution, STEP 4 answer the question.

DEs are applicable for all sorts of physical phenomena, including radioactive decay, economics, biology, chemistry, population growth and electric circuits with capacitors.

The steps for solving a separable DE are outlined and reinforced. Some students may need help with the algebra involved in the steps.



Then, **deSolve** is used to check the solution. The syntax of **deSolve** is explained so that students can use this as a tool to explore several other DEs.

Student Solutions

1. $\frac{dq}{dt} = kq$

2. $\int dy = \int \sin(x)\cos^2(x)\,dx \Rightarrow y = -\frac{1}{3}\cos^3(x) + C$

and since $\cos\left(\frac{\pi}{2}\right) = 0$, $y(0) = -\frac{1}{3}$.

3. $\frac{dq}{0.9-q} = \frac{1}{10}dt \Rightarrow -\ln|0.9-q| = \frac{1}{10}t + C \Rightarrow \ln|0.9-q| = -\frac{1}{10}t + C$

4. $|0.9-q| = e^{-\frac{1}{10}t+C} = e^{-\frac{1}{10}t}e^{C}$. Let $C_1 = \pm e^{C}$, $0.9 - q = C_1 e^{-\frac{1}{10}t}$ and with $q = 0$ when $t = 0$, $C_1 = 0.9$

$q(t) = 0.9\left(1 - e^{-\frac{1}{10}t}\right)$

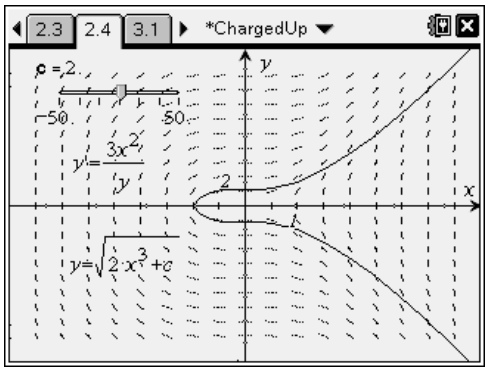
5. They are equivalent. In the solution using **deSolve**, the 0.9 was distributed

6. $y = \boldsymbol{c}\cdot x$, where the bold $\boldsymbol{c}$ is an arbitrary constant
$c = 1$ if $y(1) = 1$, so the particular solution is $y = x$.

Part 2 – Homework/Extension – Practice with deSolve and Exploring DEs

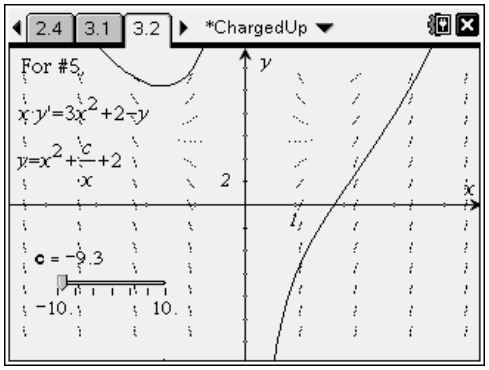
Students first use **deSolve** to find the general solution to four DEs. The fourth situation is then graphically explored.

On the worksheet students will write the solution to each DE. You may wish for students to algebraically show their work and use **deSolve** check their answers.



A non-separable DE is solved with deSolve and this one is also graphically explored. The slope field is shown on the screen. It can be removed using the **Hide/Show** tool and clicking on the slope field, or deleting the first scatter plot.

Finally, **deSolve** is used to find the particular solution for three DEs with initial conditions. Students are asked to solve for *y*. With CAS, this can be done with **solve**(). Again, students may show their work and use **deSolve** to check their answers.



Another extension/exploration activity would be to have students come up with their own DE and find the general solution. Have them discuss solutions that surprised them.

Student Solutions

1. $y = Ce^{kx}$
2. $y^2 = x^2 + C$
3. $y^3 = 3x^2 + C \rightarrow y = (3x^2 + C)^{1/3}$
4. $y^2 = 2x^3 + C$
5. It looks like a parabola because $y = x^2 + \frac{c}{x} + 2$ is $y = x^2 + 2$ when $c = 0$.
6. $1 - \frac{1}{y} = \frac{x^2}{2} \Rightarrow y = \frac{-2}{x^2 - 2}$
7. $\tan^{-1}(y) + \frac{\pi}{4} = x \rightarrow y = \tan\left(x - \frac{\pi}{4}\right)$
8. $y = e^{7x}$