

Expanding Binomials

ID: 11139

Time required
45 minutes

Activity Overview

In this activity, students will explore the link between Pascal's Triangle and the expansion of binomials in the development of the Binomial Theorem.

Topic: Sequences & Series

- Combinations
- Pascal's Triangle
- Binomial Theorem

Teacher Preparation and Notes

- Load the *ExpandingBinomomials.tns* file onto student handhelds.
- Remind students to use **ctrl** + **▶** to move to the next page and **ctrl** + **tab** to move between applications on the same page.
- To reduce time required for this activity, consider dividing up the tasks among students and sharing results on a board or projection screen.
- Consider the following ideas as extensions to this activity:
 - Consider exploration of Bernoulli probability experiments as a related concept.
 - Explore the additional number patterns studied by Blaise Pascal.
 - Relate the given activity to a binomial with two variables, $(ax + by)^n$
- Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.
- **To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "11139" in the keyword search box.**

Associated Materials

- *ExpandingBinomomials_Student.doc*
- *ExpandingBinomomials.tns*
- *ExpandingBinomomials_Soln.tns*

Suggested Related Activities

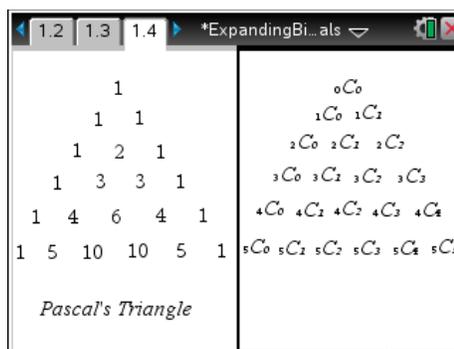
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Exploring the Binomial Theorem (TI-89 Titanium)* — 5376
- *Using the TI-83/84 to Explore the Binomial Theorem (TI-84 Plus family)* — 5768

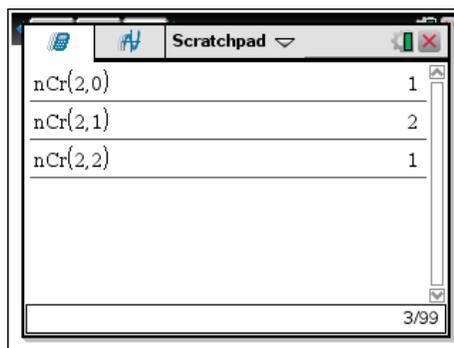
Problem 1 – Exploring $(x + b)^n$

Introduce the activity and discuss the pattern observed in Pascal’s Triangle on page 1.4. Students will then explore how this pattern relates to combinations by completing the right side of the screen.

At this point, a review of the meaning of the combination notation as well as alternative notation may be discussed as needed.

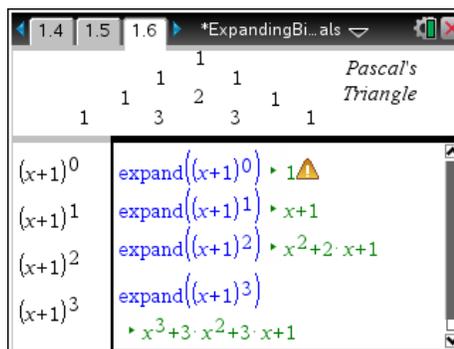


Using the **Scratchpad** () , you can show students that any combination will equal the number in the same position in Pascal’s Triangle. (i.e., ${}_2C_1 = 2$). Encourage students to test a few of these combinations to verify that they indeed represent the pattern in Pascal’s Triangle.



On pages 1.6 and 1.8, students are to expand a very basic binomial to see that the resulting pattern is connected to Pascal’s Triangle. Remind them that the first and last term have a coefficient of 1. Students should record the expansions on their worksheet.

Discuss with students how the exponent of the binomial relates to the row number of Pascal’s triangle.

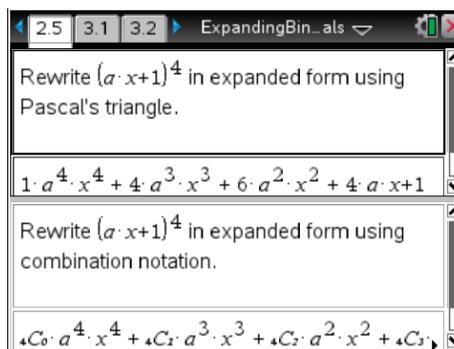


TI-Nspire Navigator Opportunity: Quick Poll
See Note 1 at the end of this lesson.

Problem 2 – Exploring $(ax + 1)^n$

On pages 2.2–2.3, students use the **Expand** command to expand the given binomials. The binomials on page 2.2 have specific integers. On page 2.3, the coefficient of x is replaced with a . They should see that a affects the coefficient whenever there is an x term.

After exploring the patterns in the binomials, students are asked to use what they’ve learned to expand $(ax + 1)^4$ by hand with the numbers from Pascal’s Triangle and by using combination notation for the binomial coefficients.



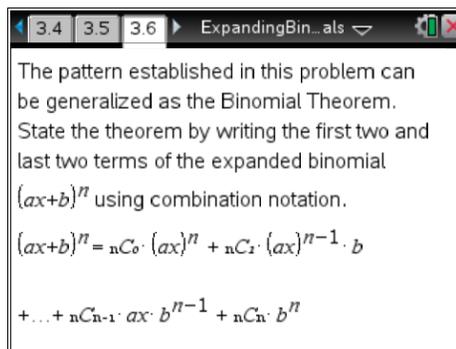
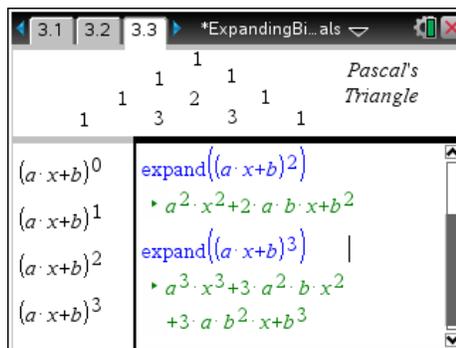
Problem 3 – Exploring $(ax + b)^n$

Once students have explored the effects of a and b separately, they will expand specific and general binomials with a and b together.

Students will rewrite the expanded binomials from page 3.3 using combination notation. They can refer back to page 1.4 if necessary. Encourage students to compare their results with a partner. This will help students identify errors and explore the resulting patterns that develop.

Students are to then write the general formula for an expanded binomial, $(ax + b)^n$, otherwise known as the Binomial Theorem.

The worksheet gives four extra problems that students can complete if there is extra time at the end of class or for homework. They should do it by hand and then check the work with the handheld.



Extra Problems

Students are to use the Binomial Theorem to expand the binomials given on the worksheet.

1. $(6x + 1)^5 = (6x)^5 + 5(6x)^4(1) + 10(6x)^3(1)^2 + 10(6x)^2(1)^3 + 5(6x)(1)^4 + 1^5$
 $= 7,776x^5 + 6,480x^4 + 2,160x^3 + 360x^2 + 30x + 1$
2. $(x + 7)^6 = (x)^6 + 6(x)^5(7)^1 + 15(x)^4(7)^2 + 20(x)^3(7)^3 + 15(x)^2(7)^4 + 6(x)^1(7)^5 + 7^6$
 $= x^6 + 42x^5 + 735x^4 + 6,860x^3 + 36,015x^2 + 100,842x + 117,649$
3. $(3x + 5)^4 = (3x)^4 + 4(3x)^3(5) + 6(3x)^2(5)^2 + 4(3x)(5)^3 + 5^4$
 $= 81x^4 + 540x^3 + 1,350x^2 + 1,500x + 625$
4. $(7x + 4)^8 = (7x)^8 + 8(7x)^7(4)^1 + 28(7x)^6(4)^2 + 56(7x)^5(4)^3 + 70(7x)^4(4)^4 + 56(7x)^3(4)^5 + 28(7x)^2(4)^6 + 8(7x)^1(4)^7 + 4^8$
 $= 5,764,801x^8 + 26,353,376x^7 + 52,706,752x^6 + 60,236,288x^5 + 43,025,920x^4 + 19,668,992x^3 + 5,619,712x^2 + 917,504x + 65,536$

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Note 1

Problems 1–3, Quick Poll

You may choose to use *Quick Poll* to assess student understanding. The worksheet questions can be used as a guide for possible questions to ask.