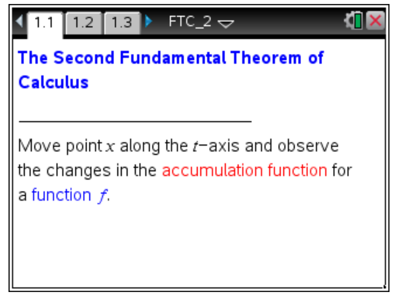
The Second Fundamental Theorem of Calculus
Student Activity

Name ________________

Class ________________

Open the TI-Nspire document *FTC_2.tns*.

The accumulation function, $A(x)$, measures the definite integral of a function f from a fixed point a to a variable point x. In this activity, you will explore the relationship between a function, its accumulation function, and the derivative of the accumulation function. These observations will help you better understand the consequences of the second Fundamental Theorem of Calculus for computing definite integrals.



Move to page 1.2.

1. The graph shown is of the function $y = f(x)$. The **accumulation function** of $f(t)$ from a to x is given by $A(x) = \int_a^x f(t)\,dt$. The accumulation function gives the value of the definite integral of $f(t)$ between a and x. Set $a = -3$ and find the following:

   a. $A(3) = \int_{-3}^{3} f(x)dx =$ __________________

   b. $A(0) = \int_{-3}^{0} f(x)dx =$ __________________

2. Without changing the value of a, how could you use the values of the accumulation function in question 1 to find $\int_0^3 f(t)\,dt$? Explain your thinking.

3. Without changing the value of a, use the accumulation function and your results from question 2 to find the following. For each, be sure to explain your thinking.

   a. $\int_1^4 f(t)dt =$ __________________

   b. $\int_{-2}^{2} f(t)dt =$ __________________

   c. $\int_0^{-1} f(t)dt =$ __________________

Move to page 1.3.

4. The top graph shows the original function, *f*, and the measurement of an accumulation function as the point *x* is dragged along the *t*-axis. The bottom graph shows the accumulation function as a function of *x*. What relationship, if any, do you notice between the original function and the accumulation function? Explain.

Move to page 1.4.

5. The top graph on page 1.4 is the graph of the accumulation function, $y = A(x)$, for the function *f* from the previous pages, and the bottom graph shows the graph of its derivative, $y = A'(x)$.
   a. Choose several values of *x* and find the corresponding values of $A'(x)$. For each of these, how do they compare to the value of $f(x)$ for that *x*? What do you observe? Does this make sense? Explain.

   b. Given your response to a, complete the following:
      $f(x)$ is ________________________________ of $A(x)$.
      $A(x)$ is ________________________________ of $f(x)$.

6. Drag point *a* on the top graph on page 1.4.
   a. What are you changing in the accumulation function when you change *a*? What are you changing in the graph of the accumulation function? Explain.

   b. Using what you know about the accumulation function, why do you think the bottom graph doesn't change when you change the value of *a*? Explain.

7. Suppose you are given that an accumulation function for a continuous function $f(x)$ can be expressed as $A(x) = x^2 + 3$. Explain how you can use this to find $\int_2^4 f(x)\,dx$.

8. Based on your answers to questions 5 and 6, how do you think you would find a formula for an accumulation function of a continuous function without using the integral? Explain.

9. Using your response to question 8, describe how you would find the value of a definite integral for a continuous function *f*.

10. Use your response to question 8 to find $\int_0^3 2x\,dx$. Explain your solution. How can you check your work?