## Math Objectives

- Students will identify the amplitude, period, horizontal and vertical translations in the equation $f(x) = a\sin\big(b(x + c)\big) + d$, and the effect that these parameters have upon the graph of the function.
- Students will utilize their knowledge of the effect of the parameters of a sine function to rewrite its equation as an equation containing a cosine function.
- Students will write equations for sinusoidal functions by examining their parameters and looking at their graphs.
- Students will use appropriate technological tools strategically (CCSS Mathematical Practice).

## Vocabulary

- horizontal translation
- vertical translation
- amplitude
- phase shift
- period
- parameters

## About the Lesson

- This lesson involves manipulating sliders to change the values of parameters in trig functions and determining the effect that each has upon the shape of the graph.
- As a result, students will:
- Examine graphs to write equations for sinusoidal functions.
- Change parameters to rewrite sine as a cosine function.
- Make connections and find parameters based on sample problems using real world sinusoidal data.

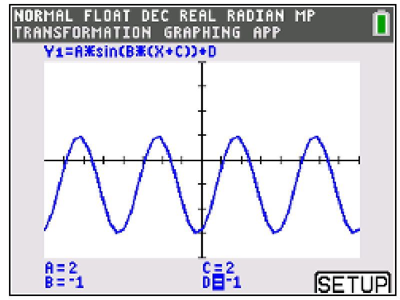
## Teacher Preparation and Notes.

This activity is done with the use of the TI-84 family as an aid to the problems.

## Activity Materials

- Compatible TI Technologies: TI-84 Plus*, TI-84 Plus Silver Edition*, TI-84 Plus C Silver Edition, TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*

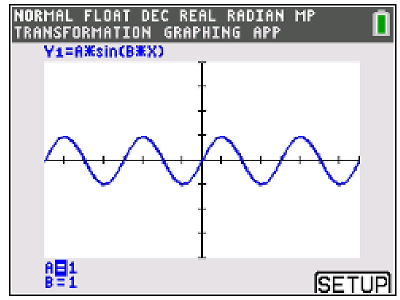



**Tech Tips:**

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at [http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials](http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials)

**Lesson Files:**
*Student Activity*
Basic_Trig_Transformations_84CE_Student.pdf
Basic_Trig_Transformations_84CE_Student.doc

In this activity, you will use the ***Transformation Application*** to change the values of parameters in trigonometric functions and to determine the effect that each change has on the shape of the graph. You will then use this knowledge to write equations for sine and cosine functions.

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Before starting this activity, please make sure that your handheld is in radians, and set you window as follows: Xmin = $-4\pi$, Xmax = $4\pi$, Xscl = $\frac{\pi}{2}$, Ymin = -4, Ymax = 4, Yscl = 1. Once this is set, press **apps**, **:Transfrm**, **any key**. This will set you window and turn on the **Transformations Application**.

1. Press **y =** and type the following function into $Y_1$: $A * \sin(B * x)$. This will allow you to manipulate the parameters $A$ and $B$ on your graph. Press graph. You will see the letters $A$ and $B$ in the bottom left corner. Pressing the up and down arrows toggles you between the two parameters, pressing the left and right arrows allows you to change the value of the parameters. Change the values of *a* and *b* in the function $Y_1 = A * \sin(B * x)$.

   a. Describe how the values of *A* and *B* affect the shape of the graph.

   **Solution:** The sine curve is vertically dilated by a factor of $|A|$. The value of $B$ affects the horizontal dilation of this function and changes the period of the function.

   **Teacher Tip:** If students do not immediately recall how the value of $B$ is related to the period of the function, have them set the value of $b$ to 0.25, 0.5, 1, and 2, and then have them identify the period fo each $(8\pi, 4\pi, 2\pi, \text{and } \pi \text{ respectively})$. After some examination, students should be able to identify the relationship: period = $\left|\frac{1}{B}\right| \cdot 2\pi$.

   b. What happens to the graph if *A* is negative?

   **Solution:** If $A$ is negative, then the curve is reflected over the x-axis.

   c. Complete the following statement:

   **Solution:**

   For *A* ≠ 0 and *B* > 0, the graph of $Y_1 = A * \sin(B * x)$ has an amplitude of ____$|\boldsymbol{A}|$___ and a period

   of ___$\left|\frac{\mathbf{1}}{\boldsymbol{B}}\right| \cdot \boldsymbol{2\pi}$_____.

**Teacher Tip:** Before students change the parameters in question 2, you may want to ask them to predict what will happen by first considering a different function, $y = x^2$. Ask them how to obtain the graph of $y = x^2 + 3$ from the graph of $y = x^2$ (translate the graph up three units).

2. Press **y =** again and change the function in $Y_1$ to $\sin(x) + D$. Go back to the graph and change the value of *d* in the function $Y_1 = \sin(x) + D$.

   a. Describe how the value of *D* affects the shape of the graph.

   **Solution:** $D$ does not affect the shape of the sinusoidal function, but it does affect the function's placement on the coordinate axes in comparison to the parent function $y = \sin x$. There is a vertical translation equal to the parameter $D$; that is, a vertical translation of $D$ units.

   b. Complete the following statement:

   **Solution:**

   The graph of $Y_1 = \sin(x) + D$ has a vertical translation of ___$D$ **units**__.

3. Press **y =** again and change the function in $Y_1$ to $\sin(x + C)$. Go back to the graph and change the value of *C* in the function $Y_1 = \sin(x + C)$.

   **Solution:** Although *C* does not change the shape of the graph, if $C > 0$, there is a horizontal translation of $C$ units to the left. If $C < 0$, there is a horizontal translation of $C$ units to the right. Note: This statement is only true if the coefficient of $x$ is one.

**Teacher Tip:** Students might predict that a change in $C$ will always result in a horizontal shift of $C$ but how that number relates to $C$ will not be immediately clear. This is explored in question #4.

4. Press **y =** one more time and change the function in $Y_1$ to $A * \sin(B * (x + C)) + D$. Go back to the graph and change the values of *A*, *B*, *C*, and *D* in the function $Y_1 = A * \sin(B * (x + C)) + D$.

   a. Which of the four parameters have an impact on the horizontal translation of the graph?

   **Solution:** If the sinusoidal function is in the form above $(B(x + C))$, then only the parameter $C$ affects the horizontal translation, but if the function is in the form $(Bx + C)$, then both parameters $B$ and $C$ affect the horizontal translation.

   b. Complete the following statement:

   **Solution:**

   For *A* ≠ 0 and *B* > 0, the graph of $Y_1 = A * \sin(B * (x + C)) + D$ has a horizontal translation of __$\boldsymbol{-C}$___.

   *Note: But in the form $Y_1 = A * \sin(B * x + C)) + D$, then the function has a horizontal translation of $\boldsymbol{-\frac{C}{B}}$.

**Teacher Tip:** To establish exactly how $B$ and $C$ determine the horizontal translation, students can manipulate the parameters on the graph. Encourage students to conjecture the relationship on their own, but if they need help, have them consider the horizontal translation when $B = 1$ and $C = 2$, when $B = 2$ and $C = 1$, and when $B = 2.5$ and $C = -5$. (It is -2, -0.5, and 2, respectively.) Examining these values, students should conclude that the horizontal translation is $-C$. It is easiest to identify this relationship if parameters $A$ and $D$ are left as initially set ($A = 1$ and $D = 1$). After the relationship is determined, dragging the sliders can verify that neither $a$ nor $d$ affects the horizontal translation.

5. For functions of the form $Y_1 = A * \sin(B * (x + C)) + D$ or $Y_1 = A * \cos(B * (x + C)) + D$, with *A* ≠ 0 and *B* > 0,

   **Solutions:**

   a. the amplitude is ___$\boldsymbol{|A|}$____.

   b. the period is ___$\boldsymbol{\left|\frac{1}{B}\right| \cdot 2\pi}$___.

   c. the horizontal translation is ____$\boldsymbol{-C}$___.

   d. the vertical translation is ____$\boldsymbol{D}$___.

**Teacher Tip:** Although some mathematics textbooks use the term "horizontal translation" synonymously with "phase shift", engineers and physicists make a distinction between the two terms. The phase shift enables us to calculate the fraction of a full period that the curve has been shifted to determine if two waves reinforce or cancel each other. For the sinusoidal function written in the form $f(x) = a\sin\big(b(x + \emptyset)\big) + d$ or $f(x) = a\cos\big(b(x + \emptyset)\big) + d$, $\emptyset$ is the phase shift.

6. Press **y =**, in $Y_3$, type the function $Y_3 = -1.5\sin\left(x + \frac{\pi}{4}\right) + 4$, in $Y_4$ type the function $Y_4 = \cos x$. Knowing that the cosine function is s horizontal translation of a sine function, write an equation for a cosine function that will have the same graph.

   **<u>Possible Solutions:</u>** Two possible equations are $y = -1.5\cos\left(x - \frac{\pi}{4}\right) + 4$ or $y = 1.5\cos\left(x + \frac{3\pi}{4}\right) + 4$.

**Teacher Tip:** Students should observe that the values of |*A*|, *B*, and *D* remain the same for each sine/cosine pair; the only difference occurs in the value of *C*. Because these functions are periodic, there are infinitely many equations that satisfy each condition. Be sure to check students' equations.

7. Repeat the process from question 6 using the equation $Y_3 = 3\sin(2x) - 5$. Write an equation for a cosine function that will have the same graph.

   **<u>Possible Solution:</u>** One possible solution can be written as $y = 3\cos\left(2\left(x - \frac{\pi}{4}\right)\right) - 5$.
   This can also be written as $y = \cos\left(2x - \frac{\pi}{2}\right) - 5$.

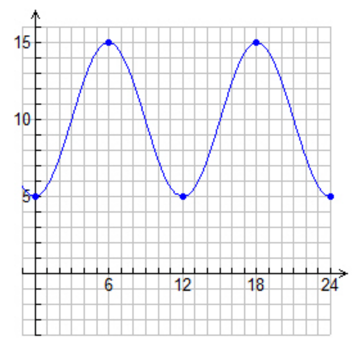
8. a. Write an equation for a sine function with an amplitude of 4, a period of 12, a horizontal translation of 2, and a vertical translation of 3.

**Solution:** One possible solution is $f(x) = 4\sin\left(\frac{\pi}{6}(x-2)\right) + 3$. This solution can also be written as $f(x) = 4\sin\left(\frac{\pi}{6}x - \frac{\pi}{3}\right) + 3$.

b. Write an equation for a cosine function with the same parameters as the sine function in part (a).

**Solution:** One possible solution is $f(x) = 4\cos\left(\frac{\pi}{6}(x-5)\right) + 3$. This solution can also be written as $f(x) = 4\cos\left(\frac{\pi}{6}x - \frac{5\pi}{6}\right) + 3$.

9. a. Write an equation for the sine function whose graph is shown in the figure below.




**Solution:** One possible solution is $f(x) = 5\sin\left(\frac{\pi}{6}(x-3)\right) + 10$. This solution can also be written as $f(x) = 5\sin\left(\frac{\pi}{6}x - \frac{\pi}{2}\right) + 10$.

b. Utilize a cosine function to write an equation for the same graph.

**Solution:** One possible solution is $f(x) = -5\cos\left(\frac{\pi}{6}x\right) + 10$.

**Further Real World Extension**

| **Day ($\theta$)** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** |
|---|---|---|---|---|---|---|---|---|---|
| **Illumination $f(\theta)$** | 0.08 | 0.03 | 0.0 | 0.01 | 0.04 | 0.10 | 0.18 | 0.28 | 0.38 |
| **Day ($\theta$)** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **25** |
| **Illumination $f(\theta)$** | 0.48 | 0.59 | 0.68 | 0.77 | 0.84 | 0.91 | 0.95 | 0.98 | 1.00 |

10. The table above gives the percentages of illumination of the moon on a nightly basis in the month of March 2024. The function $f$ given by $f(\theta) = a\sin\big(b(\theta + c)\big) + d$, where $a, b, c,$ and $d$ are constants, is used to model these data with $\theta$ representing the day of the month (March 1 = 1, March 2 = 2, etc.). $f(\theta)$ represents the percentage of the illumination of the moon on that day, written as a decimal. Assume that the period of $f$ is 29.5 days. Based on the data in the table, find the values for $a, b, c,$ and $d$.

**Solution:** One possible set of solutions are

$a$ = ______**0.5**______ **This is the distance from the midline to the peak or trough of the curve, or the highest value minus the midline: 1.00 – 0.5 = 0.5. You also need to discuss with the students if this value will be positive or negative. This will depend on which direction your horizontal translation (c) is.**

$b$ = __$\frac{2\pi}{29.5} \approx .213$____ **Since the period is assumed to be 29.5 days, use the formula period =** $\left|\frac{1}{b}\right| \cdot 2\pi$**, therefore** $29.5 = \frac{1}{b} \cdot 2\pi$ **or** $b = \frac{2\pi}{29.5}$**.**

$c$ = _______**4**_______ **After sketching the data and drawing the midline of 0.5, to be a sine function, it looks like you can translate the function right roughly 4 days.**

$d$ = ______**0.5**______ **This is the midline, to find it, we average the high and low values** $\frac{1.00+0.0}{2} = 0.5$

**Teacher Tip:** This final problem is a perfect time for you and your students to have fun and make those very important connections to all this activity reviewed. You may split your class into groups where some students use sine and some use cosine. You could also have some use a positive a and some use a negative a. Also, have them play around with different c values. You can also have the students use the regression functions on the handheld to find these values, but after they do it by hand. It can lead to a nice discussion about how the calculator writes the function compared to the students work.

11. Using the information in question 10, describe what each parameter, $a, b, c, and\ d,$ mean in the context of the illumination of the moon.

**Possible Solutions:**

$a =$ **How much the illumination level varies between its max and min values.**

$b =$ **Affects the length of one lunar cycle (period).**

$c =$ **The shift is days to start at a new moon (cos), full moon (cos), or at 50% moon (sin).**

$d =$ **Average illumination level.**

**Wrap Up**

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- The effect that each of the parameters has on the graph of a function.
- How one (or possibly two) of the parameters of a sinusoidal function affect its horizontal translation.
- How to rewrite the equation of a sine function as an equation containing a cosine function.
- How to write equations of sine and cosine functions by examining graphs of sinusoidal functions and information about its parameters.